THOUGHT
A JOURNAL OF PHILOSOPHY
THE NORTHERN INSTITUTE OF PHILOSOPHY

Thought ISSN 2161-2234

ORIGINAL ARTICLE

Beyond Atomism

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Contemporary metaphysicians have been drawn to a certain attractive picture of the structure of the world. This picture consists in classical mereology, the priority of parts over wholes, and the well-foundedness of metaphysical priority. In this short note, I show that this combination of theses entails superatomism, which is a significant strengthening of mereological atomism. This commitment has been missed in the literature due to certain sorts of models of mereology being overlooked. But the entailment is an important one: we must either accept superatomism or reject one (or other) of the most widespread theses of contemporary metaphysics.

Keywords ontology; dependence; priority; grounding; mereology; parthood; well-foundedness; atomism; gunk; monism

DOI:10.1002/tht.64

In this note, I present a problem for a certain natural view about the structure of the world. I show that this view is committed to a particular strengthening of *atomism*, which I call *superatomism*. Superatomism has some counterintuitive consequences (e.g., it rules out what Markosian (1998) calls the 'pointy view' of simples). I consider some ways of avoiding this commitment, and draw some additional implications for *priority monism*.

Consider the following picture of the structure of the world, which I will call the default view.

Mereology Necessarily, the parthood relation is governed by the axioms of classical mereology.

Priority Necessarily, the existence of parts is metaphysically prior to the existence of the wholes they compose.

Well-foundedness Necessarily, the metaphysical priority relation is well-founded; there can be no infinite regress of priority.

The mereology thesis is accepted by a great many contemporary metaphysicians—most saliently, David Lewis (1991). Importantly, MEREOLOGY is quite a bit stronger than required for the argument. All that is required is the assumption that the *parthood* relation is a partial order. (For ease of presentation, though, it will be useful to stay within the setting of classical mereology.) The priority thesis also seems to be a predominant view in contemporary metaphysics. There are several kinds of ontological dependence, but

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the notion of *grounding* [introduced by Kit Fine (1995, 1994)] has been gaining traction. I use 'metaphysical priority' here to be fairly neutral with respect to precisely which kind of dependence relation is taken to be in play. The well-foundedness thesis is a standard assumption about priority relations. The thought is that if there were infinite descending priority chains, we would be deferring the 'ground' of things indefinitely. The existence of objects could, in some sense, never get started. Witness Schaffer (2007, p. 184):

> [I]f there actually is an infinite descent, then there are no simple particles that could suffice to explain everything. Indeed, there will be no minima for the minimal nihilist to recognize. Her ontology would drain away down a bottomless pit.

Consider also Lowe (2001, p. 158)

> To that I add here one further assumption concerning 'strong' existential dependency, namely, that there cannot be infinite descending chains of objects standing in relations of strong existential dependency to one another: in short, that all real existence must be 'grounded' or 'well-founded'. Such an 'axiom of foundation' is quite probably beyond conclusive proof and yet I find the vertiginous implications of its denial barely comprehensible.

Although it seems natural to want to hold these theses together, the default view has some problematic consequences. For one thing, the default view rules out the possibility of *atomless gunk*—that is, worlds in which everything has a proper part.[1] It is well-known that there are gunky models of classical mereology.[2] But gunky worlds have infinite descending proper parthood chains. By PRIORITY, we know that the metaphysical priority relation runs in parallel with the proper parthood relation. So, it would follow that there are infinite descending chains of metaphysical priority. But that contradicts WELL-FOUNDEDNESS.

Schaffer (2010, §2.4) recognizes this entailment, and uses it as a reductio of the priority thesis. In rejecting PRIORITY, he suggests that metaphysical priority runs from whole to part rather than the other way around.

However, those sympathetic to the default view will not be persuaded. A reasonable response would be to accept this consequence, and insist on the existence of a fundamental level of mereological atoms—objects with no proper parts.

Atomism Necessarily, the parthood relation is governed by the axioms of classical *atomistic* mereology.

But there is a further entailment of the default view that has typically gone unnoticed. There are models of classical *atomistic* mereology in which the proper parthood relation is not well-founded. That is, there are atomistic models with infinite descending proper parthood chains. Gunky worlds are not the only ones that have this feature.

An example of an atomic model with infinite descending chains is given by considering the set of all subsets of the natural numbers ($\mathcal{P}(\mathbb{N})$), with parthood modelled by the subset relation. The atoms of this mereology are the singletons ($\{n\}$ for each n in $\mathbb{N}$).

Consider an 'object' represented by the set of all m such that $m > n$ for some n. Now, the set of all such objects for every n in $\mathbb{N}$ is an infinite descending chain.[3]

Another such model is given simply by the non-empty regions of three-space ($\mathbb{R}^3$) with parthood modelled by the subset relation. The atoms in this model are singletons (i.e., point-sized regions). But take any cube (e.g., $[0,1]^3$); such a region will have infinitely-many proper subregions (e.g., $\left[0, \frac{1}{n}\right]^3$ for each natural number n). But again, this is an atomistic model.

If either of these models represent metaphysical possibilities, by PRIORITY there are infinite descending chains of metaphysical priority. But again, that contradicts WELL-FOUNDEDNESS. These models of mereology have been widely overlooked; but it is important to note that the default view cannot accept them. The latter model is particularly significant insofar as one thinks that the mereological structure of the world mirrors the structure of space.[4] For example, this default view would rule out the pointy view of mereological simples, which states that an object is simple iff it occupies a point-sized region of space.[5]

So, we have it that our initially attractive and very natural picture of the world has committed us to *superatomism*, the view atomism plus the further commitment that any subdivision of the world must be atomistic as well.

Superatomism Necessarily, the parthood relation is governed by the axioms of classical *superatomistic* mereology.

Superatomistic mereology has not been previously explored; however, its mathematical models are well-studied.[6] Let B be a Boolean algebra. Then B is *superatomic* iff every subalgebra of B is atomic. Since models of classical mereology are effectively complete Boolean algebras, the models of superatomistic mereology are complete superatomic Boolean algebras.

Clearly, superatomism entails that there are no (and could not be) infinite descending parthood chains. After all, one way of cashing out well-foundedness is as follows: a partial ordering relation R over a domain is *well-founded* iff every non-empty subset of the domain has an R-minimal element. Since metaphysical priority is meant to be a well-founded relation, this displays precisely why we are committed to every substructure of the world-structure being atomistic.

There is another way of avoiding infinite descending parthood chains: accept that the world is (necessarily) finite. If there are only finitely many things, then parthood chains will all be finite. A strict finitist about the structure of the world will also accept superatomism (as every finite Boolean algebra is superatomic). But superatomism is more general; it allows for worlds to be infinite while still ruling out infinite descending chains.

The upshot of the above argument is this: the default view as expressed above has a commitment to superatomism. To those who do not wish to accept superatomism, there are a few options.

The first alternative is to reject WELL-FOUNDEDNESS, and perhaps weaken the thesis from requiring priority to be well-founded, to merely requiring any priority chain to 'bottom out' at some stage (infinite or otherwise).[7] The challenge to defenders of well-foundedness is to decide whether this weakened thesis will do everything one wants it to. (This would be worth deeper exploration in any case since similar issues crop up in other areas of metaphysics. For example, consider the corresponding thesis regarding *causal* chains. Is it plausible that causal chains might extend infinitely far back into the past so long as there is some *first* cause which happened before the infinite causal chain? And what would it mean for that 'first cause' to set the universe in motion if there is no 'first' effect, but only an infinite sequence of effects getting ever and ever closer to the start?)

A second alternative is to reject MEREOLOGY. After all, classical mereology is controversial independently. But one must keep in mind, however, that the argument depends only on the assumption that parthood is a partial order, which holds in nearly all mereologies.[8] Some (e.g., Simons 1987) have even contended that the partial-order axioms for parthood are *analytic*. But this move won't really help solve the problem, since holding onto WELL-FOUNDEDNESS would require giving up PRIORITY as well.[9]

A final alternative would be to reject PRIORITY.[10] One promising proposal along these lines is Schaffer's *priority monism* which reverses the order of mereological dependence.

*Priority** Necessarily, the existence of wholes is metaphysically prior to the existence of the parts they comprise.

But given WELL-FOUNDEDNESS and MEREOLOGY, it will follow that infinite *ascending* proper parthood chains are impossible. So, at the very least, this entails a commitment to the existence of a *universal* object—an object which is not a proper part of anything. Indeed, Schaffer appears to think this is enough to avoid the problem (even though it is clear that Schaffer genuinely intends the stronger well-foundedness thesis, rather than a weaker version which requires merely a fundamental level).[11]

> Grounding is, however, exactly like the classical mereological relation of having as a proper part, which is irreflexive, asymmetric, and transitive, and whose ordering provably is well-founded (in fact it provably has a unique foundation, the whole universe).

> The intuition that being requires a ground [. . .] is the analogue of the set-theoretic axiom of Foundedness [(Schaffer, 2009), p. 376; footnote 35]

But a similar difficulty arises here, analogous to the problem for atomism. There are models of classical mereology that contain both a universal element and infinite ascending proper parthood chains. Consider again the non-empty regions of three-space. Any ascending proper parthood chain from any object (other than the universe) will be infinite. If such models correspond to metaphysical possibilities, Schaffer has an even further commitment to what we might call *noetherianism*, which would require that every substructure of the world have a maximal object.[12]

Priority monists might likewise weaken the well-foundedness thesis and so on, but the retreat to PRIORITY* will not be enough.

While the default view is a natural and *prima facie* attractive picture of the structure of the world, it is also a stronger collection of theses than one might have thought. When dealing with the infinite it is very easy to overlook possibilities. But these possibilities are crucial to a correct understanding of the structure of the world. If one accepts MEREOLOGY, PRIORITY, and WELL-FOUNDEDNESS, then one is not just committed to atomism but also to a superatomistic picture of the world. Priority monists like Schaffer avoid atomism by replacing PRIORITY with PRIORITY*, but they face a similar unnoticed commitment to *noetherianism*. Whether this provides a good reason to reject the default view is for future work to decide.

Acknowledgments

Thanks to Katherine Hawley, Martin Lipman, and an anonymous referee for very helpful comments on an earlier draft.

Notes

1 See Sider (1993) for more on the debate over gunk.

2 The classic model given by the non empty regular open sets of some topological space is due to Tarski (1956).

3 Thanks to Andrew Bacon.

4 For example, Brzozowski (2008, §1) argues against certain views by using a version of a regress argument which overlooks these non-wellfounded models.

5 At least if one accepts that there are *extended* objects at all, then they would have to be made up of infinitely-many unextended simples. But such a structure would immediately yield infinite descending parthood chains, contradicting the default view. See Markosian (1998) for more discussion of rival views of simples.

6 Superatomic Boolean algebras were first studied by Mostowski and Tarski (1979). For a good introduction see Day (1967).

7 This is the option pursued by Cameron (2008, p. 4), who is (to my knowledge) one of the only people to be explicit about the difference between the well-foundedness of priority vs. the existence of a fundamental level.

> It is false to say that an entity x is ontologically independent iff there is no entity y such that x is directly ontologically dependent on y; this thought depends on the assumption that for any dependent entity there is a finite number of steps taking you from it to its ultimate ontological basis. This should be rejected.

8 For an exception, see Cotnoir and Bacon (2012).

9 Whether non-wellfounded parthood relations and well-founded priority relations play nicely together is an interesting question, one which I hope to explore in future work.

10 A further option would be to reject the modal strength of the default view. It is usually thought that metaphysical principles hold with metaphysical necessity. However, one might reject this and for example think with Cameron (2007) that mereological principles are

metaphysically contingent. Or one might reject that metaphysical priority always tracks the mereological structure of a given world. Or one might think that the well-foundedness of priority is merely a contingent matter. This would open up the possibility of a view which accepts the actual truth of demodalized variants of mereology, priority, and well-foundedness (and hence the actual truth of demodalized superatomism), whilst still accepting the possibility of the various models considered above. While I find this view interesting, I do not have space to consider it further here.

11 See also Schaffer (2010, p. 37).

12 The terminology is again borrowed from algebra where, for example a *noetherian group* is a group that satisfies the ascending chain condition on subgroups.

References

Brzozowski, J. "On Locating Composite Objects," in *Oxford Studies in Metaphysics*, Vol. 4, edited by D. Zimmerman and K. Bennett. Oxford: Oxford University Press, 2008.

Cameron, R. "The Contigency of Composition." *Philosophical Studies* **136** (2007): 99–121.

Cameron, R. "Turtles All the Way Down: Regress, Priority, and Fundamentality." *The Philosophical Quarterly* **58** (2008): 1–14.

Cotnoir, A. J. and A. Bacon. "Non-Wellfounded Mereology." *Review of Symbolic Logic* **5**.2 (2012): 187–204.

Day, G. W. "Superatomic Boolean Algebras." *Pacific Journal of Mathematics* **23** (1967): 479–489.

Fine, K. "Essence and Modality." *Philosophical Perspectives* **8** (1994): 1–16.

Fine, K. "Ontological Dependence." *Proceedings of the Aristotelian Society* **95** (1995): 269–290.

Lewis, D. *Parts of Classes*. Oxford: Basil Blackwell, 1991.

Lowe, E. J. *The Possibility of Metaphysics*. Oxford: Oxford University Press, 2001.

Markosian, N. "Simples." *Australasian Journal of Philosophy* **76** (1998): 213–26.

Mostowski, A. and A. Tarski. "Boolean Rings With an Ordered Basis." *Provability, Computability and Reflection* **2** (1979): 75. (Translated from German 'Boolesche Ringe mit geordneter Basis' *Fundamenta Mathematicae* 32, 69–86 (1939).).

Schaffer, J. "From Nihilism to Monism." *Australasian Journal of Philosophy* **85** (2007): 175–91.

Schaffer, J. "On what grounds what," in *Metametaphysics*, edited by D. M. D. Chalmers and R. Wasserman. Oxford: Oxford University Press, 2009, 347–383.

Schaffer, J. "Monism: The priority of the Whole." *Philosophical Review* **119** (2010): 31–76.

Sider, T. "Van Inwagen and the Possibility of Gunk." *Analysis* **53** (1993): 285–289.

Simons, P. M. *Parts: A Study In Ontology*. Oxford: Clarendon, 1987.

Tarski, A. "Foundations of the Geometry of Solids," in *Logic, Semantics, Metamathematics*, edited by J. Corcoran. Hackett, 1956.