For these problems, use the simulation "Time development of two-level quantum states in the Bloch sphere representation".

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down five things that you have found out.

2) Go to step 2 of the "Step-by-step Exploration". Explain each of the steps in the derivation of the expression for the time-dependent quantum state $|\psi(t)>$. Describe how you can see this time-dependence in the Bloch sphere representation.

3) Using the simulation, investigate the effect of increasing the magnetic field strength on $|\psi(t)>$ in the Bloch sphere representation. Write down and explain your observations.

4) Investigate the theoretical measurement outcome probabilities for measurements of S_z, the z-component of spin, and S_x, the x-component of spin, for the different input states. Write down your observations, and qualitatively try to explain them (no calculations are needed).

5) Consider the state $|\psi(t)> = \frac{1}{\sqrt{2}}(|\uparrow> + \exp(i\omega t)\,|\downarrow>)$ shown in the simulation.

(a) Start from the general quantum state $|\psi> = \cos\left(\frac{\theta}{2}\right)|\uparrow> + \sin\left(\frac{\theta}{2}\right)\exp(i\phi)\,|\downarrow>$ as given in the simulation. Determine the angles θ and ϕ for this state. Describe the motion of this state in the Bloch sphere representation. Verify your answer using the simulation.

(b) Calculate the theoretical measurement outcome probabilities for a measurement of S_z on this input state. Verify your probability values using the simulation.

(c) The states $|+>$ and $|->$ with $S_x = +\hbar/2$ and $S_x = -\hbar/2$ can be written in terms of $|\uparrow>$ and $|\downarrow>$ as follows:

$$|+> = \frac{1}{\sqrt{2}}(|\uparrow> + |\downarrow>)$$

$$|-> = \frac{1}{\sqrt{2}}(|\uparrow> - |\downarrow>)$$

Calculate the theoretical measurement outcome probabilities as a function of time for a measurement of S_x on the input state $|\psi(t)>$. Verify your result using the simulation, including at which positions the point on the Bloch sphere is located for special values of the probabilities. Show by calculation that the probabilities sum to one for all times.

6) Consider the state $|\psi(t)> = \frac{1}{\sqrt{5}}(2\,|\uparrow> + \exp(i\omega t)\,|\downarrow>)$ shown in the simulation.

(a) Determine the angles θ and ϕ for this state. Describe the motion of this state in the Bloch sphere representation. Verify your answer using the simulation.

(b) Calculate the theoretical measurement outcome probabilities for a measurement of S_z on this input state. Verify your probability values using the simulation.

(c) At what times is the theoretical measurement outcome probability maximal for a measurement of S_x on this input state? Where on the Bloch sphere is the quantum state then located? Calculate this maximal probability, and verify your result using the simulation.