For these problems, use the simulation "Bloch sphere representation of quantum states for a spin 1/2 particle".

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down three things about the controls and displayed quantities that you have found out.

2) Start from the general quantum state $|\psi> = \cos\left(\frac{\theta}{2}\right) \mid \uparrow> + e^{i\phi} \sin\left(\frac{\theta}{2}\right) \mid \downarrow>$. The notations $\mid \uparrow>$ and $\mid \downarrow>$ denote states where the outcome of a measurement of the z-component of spin S_z will give $+\hbar/2$ and $-\hbar/2$ respectively.

(a) Choose the angles θ and ϕ to depict the North Pole and then the South Pole of the Bloch sphere. Determine the quantum states for these two points. Verify your result for the quantum state using the simulation.

(b) Choose θ and ϕ to depict a point on the equator oriented along $+x$. Determine the quantum state for this point. Verify your result for the quantum state using the simulation. Note that $\frac{1}{\sqrt{2}} \approx 0.707$.

(c) Choose θ and ϕ to depict a point on the equator oriented along $+y$. Determine the quantum state for this point. Verify your result for the quantum state using the simulation.

3) (a) Explain how the measurement outcome probabilities shown in the simulation for $S_z = +\frac{\hbar}{2}$ and $S_z = -\frac{\hbar}{2}$ are calculated. Show that the probabilities correctly sum to one.

(b) Choose θ and ϕ to depict a point on the equator oriented along $+y$. Calculate the measurement outcome probabilities for $S_z = +\frac{\hbar}{2}$ and $S_z = -\frac{\hbar}{2}$. Verify your result using the simulation.

4) The quantum states $|+>$ and $|->$ denote states where the outcome of a measurement of the x-component of spin S_x will give $+\hbar/2$ and $-\hbar/2$ respectively. In terms of $\mid \uparrow>$ and $\mid \downarrow>$, these states are $|+> = \frac{1}{\sqrt{2}}(\mid \uparrow> + \mid \downarrow>)$ and $|-> = \frac{1}{\sqrt{2}}(\mid \uparrow> - \mid \downarrow>)$.

(a) Determine θ and ϕ for the states $|+>$ and $|->$.

(b) Where will these states be located on the Bloch sphere?

(c) Verify these positions using the simulation.

5) The quantum states $|n_+>$ and $|n_->$ denote states where the outcome of a measurement of the component of spin along an arbitrary axis $\vec{n}$ in the (θ, ϕ) direction will give $+\hbar/2$ and $-\hbar/2$ respectively. In terms of $|\uparrow>$ and $|\downarrow>$, these states are

$$|n_+> = \cos\left(\frac{\theta}{2}\right) |\uparrow> + e^{i\phi} \sin\left(\frac{\theta}{2}\right) |\downarrow> \text{ and}$$

$$|n_-> = \sin\left(\frac{\theta}{2}\right) |\uparrow> - e^{i\phi} \cos\left(\frac{\theta}{2}\right) |\downarrow>.$$

(a) Assume the vector $\vec{n}$ points in the direction $\theta = 0.2\pi$, $\phi = 0.3\pi$ as shown in step 5 of the simulation. What will be θ and ϕ on the Bloch sphere for the states $|n_+>$ and $|n_->$ for this situation? Verify your positions using the simulation.

(b) Generalize your findings from part (a). If the vector $\vec{n}$ points in the (θ, ϕ) direction, in what directions on the Bloch sphere will the states $|n_+>$ and $|n_->$ be located? Explain your answer.

(c) Show that $|n_-> = \sin\left(\frac{\theta}{2}\right) |\uparrow> - e^{i\phi} \cos\left(\frac{\theta}{2}\right) |\downarrow>$.