For these problems, use the simulation "Graphical representation of eigenvectors".

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down three things about the controls and displayed quantities that you have found out.

2) (a) Explain what is represented by the blue arrows shown in the simulation.

(b) Explain how you can graphically see whether or not the vector $\vec{n}$ is an eigenvector of a particular transformation $\hat{O}$.

(c) Write down a general equation that needs to be fulfilled in order for a vector $\vec{n}$ to be an eigenvector of a particular transformation $\hat{O}$.

3) (a) For transformation $\hat{O}_1$, use the simulation to find the eigenvectors and associated eigenvalues.

(b) Show using matrix multiplication that the eigenvectors satisfy the eigenvalue equation with the eigenvalues shown in the simulation.

(c) If $\vec{n}_1$ and $\vec{n}_2$ denote the two eigenvectors of transformation $\hat{O}_1$, what is the direction of $\vec{n}_3 = \vec{n}_1 + \vec{n}_2$? Is the sum of two eigenvectors again an eigenvector? Explain graphically using the simulation. Confirm your result through calculation.

4) For transformation $\hat{O}_3$, solve the characteristic equation $\det(\hat{O}_3 - \lambda I) = 0$ to find the eigenvectors and associated eigenvalues. Here, I is the identity matrix and "det" denotes the determinant. Compare your results with the simulation. (Note that any multiple of an eigenvector is again an eigenvector: choose the eigenvector magnitude to equal one and y to be positive as shown in the simulation.)

5) (a) How does $\hat{O}_4$ transform the x and y components of $\vec{n}$? Using these results, determine the matrix elements for the transformation $\hat{O}_4$.

(b) Confirm your result for $\hat{O}_4$ by solving the characteristic equation to find the eigenvectors and associated eigenvalues, and comparing your results with the simulation. (Again choose the eigenvector magnitude to equal one and y to be positive as shown in the simulation.)

6) In quantum mechanics, normalization of the quantum state requires that the length of the vector remain unchanged through the transformation. Is this fulfilled for the transformations shown in the simulation?