

For these problems, use the simulation “Entangled spin $\frac{1}{2}$ particle pairs versus hidden variables”.

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down five things about the controls and displayed quantities that you have found out.

2) Hidden variable theories assume quantum mechanics is incomplete, in this case that it does not describe all relevant properties of the spin $\frac{1}{2}$ particles. A **hidden variable** is an additional physical quantity not described by quantum theory and thus unknown to the observer (and therefore “hidden”). **Locality** is a common-sense notion that the measurement performed by an observer A has no influence on the outcome of a measurement for observer B.

(a) The instruction sets shown in the simulation are an example of local hidden variables. Explain why this is the case.

(b) Does this hidden variable explanation seem reasonable to you to account for the opposite measurement outcomes when both SGAs are oriented along the same direction? In what way does the quantum theory explanation for these opposite outcomes differ from the hidden variable one? Which of the two explanations (hidden variables and quantum theory) seem more reasonable based on your everyday experience?

3) (a) Why could the outcomes shown in the table below not be an instruction set for the experiment shown in the simulation?

SGA A			SGA B		
0°	120°	240°	0°	120°	240°
+	–	–	–	+	–

(b) Explain why there cannot be more than the eight instruction sets shown.

For all of the following problems, choose the “Random orientations” button, so that the orientation of each SGA is chosen at random for each pair of measurements.

4) (a) Define the probabilities P_{same} and P_{opp} shown.

(b) For each of the eight instruction sets separately, calculate the probabilities P_{same} and P_{opp} .

(c) By taking measurements in the hidden variable case, verify your calculated values for P_{same} and P_{opp} from part (b) for each instruction set separately. Explain how you are carrying out this comparison (you may need to choose the button Fixed in the Instruction sets panel).

5) Assume that there are equal numbers of particles with each of the eight instruction sets.

(a) Calculate the probabilities P_{same} and P_{opp} averaged over all instruction sets.

(b) By taking measurements in the hidden variable case, verify your calculated values for P_{same} and P_{opp} from part (a). Explain how you are carrying out this comparison (you may need to choose the button Random in the Instruction sets panel).

(c) Do your probabilities for the hidden variable case from part (a) agree with the quantum prediction?

6) Now assume that there exist different (but unknown) numbers of particles with each of the eight instruction sets. What range of values could P_{same} possibly have? Write this as an inequality $a \leq P_{same} \leq b$ (with numbers a and b) that is valid for any distribution of particles across the eight instruction sets.

What range of values could P_{opp} possibly have? Write this as an inequality $a \leq P_{opp} \leq b$ (with numbers a and b) that is valid for any distribution of particles across the eight instruction sets.

Note that your inequalities are a special case of the more general Bell inequalities, derived by Bell in 1964.

7) From your range of values for P_{same} and P_{opp} from question 6), can there be some distribution across the different instruction sets that is consistent with the quantum prediction for P_{same} and P_{opp} ?

8) Interpret your result to question 7) in terms of local hidden variables and your answer to question 2b).