For these problems, use the simulation “Graphical representation of complex eigenvectors” (Complex Eigenvectors) in the QuVis HTML5 collection.

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down three things about the controls and displayed quantities that you have found out.

2) (a) Does the blue arrow represent the real part and the green arrow the imaginary part of the vector $\vec{n}$? Explain what is represented by the blue and green arrows in the simulation.

(b) Explain how you can graphically see whether or not the vector $\vec{n}$ is an eigenvector of a particular transformation $\hat{O}$.

(c) Write down a general equation that needs to be fulfilled for a vector $\vec{n}$ to be an eigenvector of a particular transformation $\hat{O}$.

3) (a) For transformation $\hat{O}_1$, use the simulation to find the eigenvectors and associated eigenvalues. Using matrix multiplication, show that the eigenvectors satisfy the eigenvalue equation with the eigenvalues shown in the simulation. For help with the exponential form of complex numbers, click on the Display help button on the Transformation matrix panel.

(b) Choose an angle in the simulation so that the vector shown is not an eigenvector of $\hat{O}_1$. Using matrix multiplication, show that this vector does not fulfill the eigenvalue equation.

4) Do the same as in problem 3 for transformation $\hat{O}_2$. For help with the exponential form of complex numbers, click on the Display help button on the Transformation matrix panel.

5) Try to find an explicit representation for the matrix elements of $\hat{O}_4$. Confirm your result by showing that the eigenvectors and associated eigenvalues for $\hat{O}_4$ given by the simulation satisfy the eigenvalue equation.

6) In quantum mechanics, normalization of the quantum state requires that the length of the vector $\vec{n}$ equals one and does not change through the transformation. Are these two conditions fulfilled for the transformations shown in the simulation? Explain how you can see these results graphically in the simulation.