For these questions, use the simulation “Symmetric perturbation of a one-dimensional infinitely-deep square well” in the QuVis HTML5 collection.

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down three things about the controls and displayed quantities that you have found out.

2) a) Write down an expression for the spatial part of the unperturbed energy eigenfunctions $\psi_n^{(0)}(x)$. Make sketches of the spatial parts of the unperturbed ground state wavefunction $\psi_1^{(0)}(x)$, first excited state wavefunction $\psi_2^{(0)}(x)$, second excited state wavefunction $\psi_3^{(0)}(x)$ and third excited state wavefunction $\psi_4^{(0)}(x)$.

b) Make a sketch of $c_{13}\psi_3^{(0)}(x)$ for $c_{13} = -0.5$. Comparing the graphs of $\psi_3^{(0)}(x)$ and $c_{13}\psi_3^{(0)}(x)$, explain how you can see the magnitude and signs of the mixing coefficients graphically in the simulation.

3) a) Using the graphs shown in the simulation, are the perturbed energy eigenfunctions shifted towards higher or lower potential energy or does this depend on the state?

b) A classical particle of would be moving more slowly in a region of high potential energy (and thus low kinetic energy) and thus be more likely to be found there. Do you see a similar behaviour for the probability density for the perturbed wavefunction?

4) a) Write down explicit expressions for the unperturbed and the perturbed potential energy inside the well shown in the simulation.

b) Calculate the first-order change in the ground state energy in terms of V_0 due to the perturbation (not shown in the simulation).

c) Write down an integral expression for $c_{1n}^{(1)}$. Using symmetry arguments, which coefficients will be zero? Check your result with the simulation.

d) Calculate the first-order mixing coeffient $c_{1n}^{(1)}$ as a function of n.

e) Inserting values for n, determine the first-order mixing coefficients $c_{13}^{(1)}$ and $c_{15}^{(1)}$ in terms of the ratio V_0/E_1 (where $E_1 = \pi^2\hbar^2/2mL^2$) and compare your values with those shown in the simulation.

f) Explain how you can predict the sign of $c_{13}^{(1)}$ without any calculation from the shape of the perturbed ground-state wave function.