For these questions, use the simulation "Non-interacting particles in a one-dimensional infinite square well" (Particles In An Infinite Well) in the QuVis HTML5 collection.

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down three things about the controls and displayed quantities that you have found out.

2) Formulate rules about the occupancy of identical spin ½ fermions and identical spin zero bosons across energy levels.

3) a) In units of the single particle ground state energy E_1, what is the ground state system energy E_{System} of a system of $N = 6$ identical spin zero *bosons* in the one-dimensional infinite square well shown in the simulation? What would be the ground state system energy if $N = 10$? Generalize your result to determine a formula for the ground state system energy for N spin zero. Check that your formula works correctly for the cases $N = 3, 4, \dots, 8$ shown in the simulation.

b) In units of the single particle ground state energy E_1, derive formulas for the system energy E_{System} of the first excited state, the second excited state and the third excited state for a system of N identical spin zero bosons in the one-dimensional infinite square well.

4) a) In units of the single particle ground state energy E_1, what is the ground state system energy E_{System} for $N = 6$ identical spin ½ *fermions* (3 spin-up, 3 spin-down) in the one-dimensional infinite square well shown in the simulation? What would be the ground state system energy if $N = 10$?

Generalize your result to determine a formula for the ground state system energy for N identical spin ½ fermions in the one-dimensional infinite square well, assuming that N is an even number. You may use the result

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Check that your formula for E_{System} works correctly for the cases $N = 2, 4, 6$ and 8 (even N) shown in the simulation.

b) By using the approximation $n(n+1)(2n+1) \approx 2n^3$ for $n \gg 1$, determine the ratio of the ground state system energies $\frac{E_{fermion}}{E_{boson}}$ in the limit that the particle number $N \gg 1$. $E_{fermion}$ is the ground state system energy for N identical spin ½ fermions and E_{boson} the ground state system energy for N identical spin zero bosons. Interpret your result.

5) Which of the Challenges did you find most difficult and why? Explain how you solved this challenge. If none of the Challenges were difficult, choose the one you found most interesting and explain how you solved it.

6) How would the simulation need to change if the identical fermions had spin 3/2 instead of spin 1/2?