For these questions, use the simulation "Successive measurements in the Bloch sphere representation" in the QuVis HTML5 collection.
www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/Bloch-measurement/Bloch-measurement.html

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down three things about the displayed quantities that you have found out.

2) Consider the initial state prior to measurement $|\uparrow\rangle$ shown in the simulation.

(a) Starting from the general quantum state for a two-level system $|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|\uparrow\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|\downarrow\rangle$, determine the angles θ and ϕ for this initial state in the Bloch sphere representation. Compare your result with the position shown in the simulation.

(b) Now assume a measurement of the z-component of spin S_z is performed. What are the possible outcomes of this measurement? What will be the quantum state in the Bloch sphere representation after the measurement? Does it matter whether an energy measurement is performed instead of a measurement of S_z? Explain why or why not.

(c) Assume that a first measurement of S_z was performed. What will be the result of a second measurement of S_z? Does it matter whether an energy measurement is performed instead of a measurement of S_z? Explain why or why not.

3) Now consider the initial state prior to measurement $|\psi(t)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + e^{i\omega t}|\downarrow\rangle)$ shown in the simulation.

(a) Starting from the general quantum state for a two-level system $|\psi(t)\rangle = \cos\left(\frac{\theta}{2}\right)|\uparrow\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|\downarrow\rangle$, determine the angles θ and ϕ for this initial state in the Bloch sphere representation. Compare your result with the time-dependent position on the Bloch sphere shown in the simulation.

(b) For this initial state, what are the possible outcomes of a measurement of the z-component of spin S_z? Calculate the measurement outcome probabilities. Now assume a measurement of the z-component of spin S_z is performed. What will be the quantum state in the Bloch sphere representation after the measurement? Does it matter whether an energy measurement is performed instead of a measurement of S_z? Explain why or why not.

(c) Assume that a first measurement of S_z was performed. What will be the result of a second measurement of S_z? Does it matter whether an energy measurement is performed instead of a measurement of S_z? Explain why or why not.