For these questions, use the simulation "Time-development of a free particle Gaussian wave packet" in the QuVis HTML5 collection.
https://www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/gaussian/gaussian.html

For a Gaussian wave packet describing a free particle of mass m, the product of initial position uncertainty $\Delta x(t=0) = \Delta x_0$ and wave number uncertainty is $\Delta x_0 \Delta k = 2$, and the spatial width increases with time as $\Delta x(t) = \Delta x_0 \sqrt{1 + \frac{\hbar^2 (\Delta \mathrm{k})^4 \mathrm{t}^2}{4\mathrm{m}^2}}$. (Note Δx is defined here as the range over which the Gaussian falls off to e^{-1} of its peak value.)

(a) Show that *in the limit of large times*, the position uncertainty of the wave packet can be written as $\Delta x(t) = \Delta x_0 \frac{t}{t_0}$, where t_0 is defined as the spreading time. Determine an explicit expression for the spreading time t_0. Using the simulation, sketch a qualitative graph of $\Delta x(t)$, and using your sketch show the dependence on t for small and for large times. How does the $\Delta x(t)$ curve change if t_0 is larger?

(b) Determine an expression for the dependence of the spreading time t_0 on the initial position uncertainty Δx_0. How does the speed of the spreading depend on the initial position uncertainty, and why is this so? Verify your result qualitatively using the simulation.

(c) The time-dependence of a wave packet describing a free particle can be written as $\psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \phi(p) e^{-ip^2 t/2m\hbar} e^{ipx/\hbar} dp$, where $\phi(p)$ is the momentum amplitude at time t=0. Explain using this expression and an expression for $\psi(x,0)$ why the momentum uncertainty Δp stays constant with time. How is this result shown in the simulation?

(d) How does the spreading time t_0 depend on the momentum uncertainty Δp? How is this result seen qualitatively in the simulation?