For these questions, use the simulation "Density matrices for a two-level spin system" in the QuVis HTML5 collection.
www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/DensityMatrix/DensityMatrix.html

1) Calculate the density matrix ρ and its square ρ^2 explicitly for the following situations shown in the simulation:

a) an equal mixture of particles in eigenstates $|z_+\rangle$ and $|z_-\rangle$

b) particles in the eigenstate $|x_+\rangle$

c) particles in the eigenstate $|x_-\rangle$

d) an equal mixture of particles in eigenstates $|x_+\rangle$ and $|x_-\rangle$.

Which of the states described in a) to d) are pure states, which are mixed states, which are superposition states with respect to the basis $|z_+\rangle$ and $|z_-\rangle$?

Verify your answers by determining the traces $Tr(\rho)$ and $Tr(\rho^2)$.

2) Imagine the Stern-Gerlach apparatus shown in the simulation were oriented at an angle θ to the z-axis, in the xz plane. After passing through the SGA, the beam deflected in the positive θ direction consists of particles in the eigenstate $|\theta_+\rangle = \cos(\theta/2)\,|z_+\rangle + \sin(\theta/2)\,|z_-\rangle$.

a) Interpret this formula for $|\theta_+\rangle$ for the special cases $\theta = 0°$ and $\theta = 90°$ shown in the simulation.

b) Determine the density matrix for the state $|\theta_+\rangle$.