For these questions, use the simulation "Expansion in eigenstates" in the QuVis HTML5 collection.
www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/ExpansionTheorem/ExpansionTheorem.html

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down three things about the displayed quantities that you have found out.

2) Consider the first input state $\psi(x, t = 0) = u(x) = N \sin^5\left(\frac{\pi x}{L}\right)$, where L is the width of the well.

a) Explain how you can tell that this is a superposition state. Write down an integral expression for the expansion coefficient c_n. Using symmetry arguments, explain which expansion coefficients c_n will be zero.

b) Using a computer algebra system or otherwise, verify that $N = \frac{16}{3\sqrt{7L}}$ and calculate numerical values for the first three non-zero expansion coefficients. Compare your values with those shown in the simulation.

c) Explain qualitatively the signs and magnitudes of the first two non-zero coefficients given the shape of the wave function $u(x)$.

d) Write down an explicit approximate expression for $\psi(x, t)$ given $u(x)$ at time $t = 0$ above. Will the shape of the wave function change with time? Will the values of the coefficients c_n change with time? Explain your answers.

e) If a measurement of energy were made at time $t = 0$ of the particle described by the wave function $u(x)$, with what probability would one find the energy of the second excited state? Would your answer change if the measurement were made at time $t > 0$ instead?