For these problems, use the simulation "Entangled spin ½ particle pairs versus an elementary hidden variable theory".

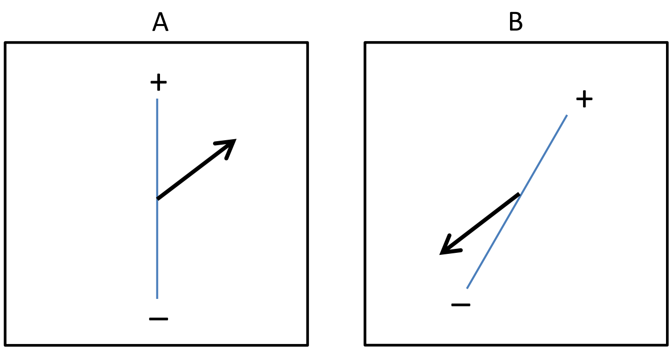
1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down five things about the controls and displayed quantities that you have found out.

2) Hidden variable theories assume quantum mechanics is incomplete, in this case that it does not describe all relevant properties of the spin ½ particles. A **hidden variable** is an additional physical quantity not described by quantum theory and thus unknown to the observer. **Locality** is a common-sense notion that an experiment performed by observer A can have no influence on the outcome of a distant experiment performed by observer B.

(a) What is the hidden variable for the situation shown in the simulation? Why is this a local hidden variable?

(b) Does this hidden variable explanation seem reasonable to you to account for the opposite measurement outcomes when both SGAs are oriented along the same direction? In what way does the quantum theory explanation for these opposite outcomes differ from the hidden variable one? Which of the two explanations (hidden variables and quantum theory) seems more reasonable based on your everyday experience?

(c) The images below show one particular pair of spin vectors for the hidden variable case. What will be the measurement outcomes for observers A and B? Explain your answer. In what directions could the spin vector point for the outcome for A to be +? Shade this region in the image. Do the same for B.



3) Define the probabilities P_{same} and P_{opp} and the correlation coefficient $E(AB)$ shown. See step 3 of the Step-by-step Explanation for an explanation of the correlation coefficient. From the definition, what range of values are possible for the correlation coefficient?

4) Consider the two cases where both SGAs are oriented along the same axis, and where they are opposite to one another (angles for SGA B of 0° and 180°).

(a) Consider the quantum case. What values for the correlation coefficient are predicted by quantum theory for these two angles?

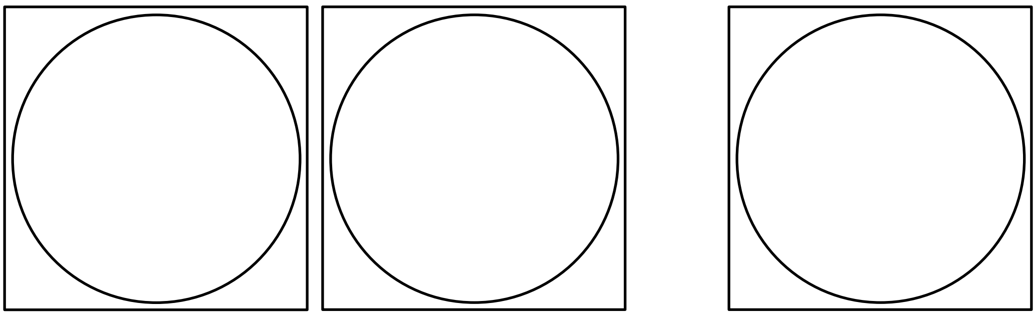
(b) Consider the hidden variable case. Explain physically why the hidden variable theory shown in the simulation gives the same outcomes as the quantum prediction for the two angles for SGA B of 0° and 180°.

For problems 5 and 6, consider the hidden variable case.

5) (a) Step 5 of the Step-by-step Explanation shows the situation for the hidden variable case and the angle of SGA B relative to SGA A of $\theta = 45°$. Sketch the three diagrams shown in this step (circles are already drawn for this below). In your sketches, shade regions of outcome + for the individual outcomes. In the top graphs, add a pair of spin vectors giving the outcome opposite. In the bottom graphs, add a pair of spin vectors giving the outcome same. Shade the regions of outcome same in your paired outcome sketch.

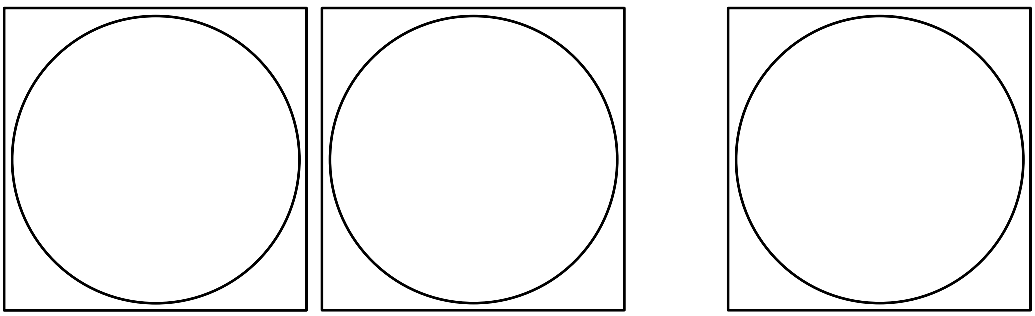
Spin vectors giving outcome opposite

Paired



Spin vectors giving outcome same

Paired



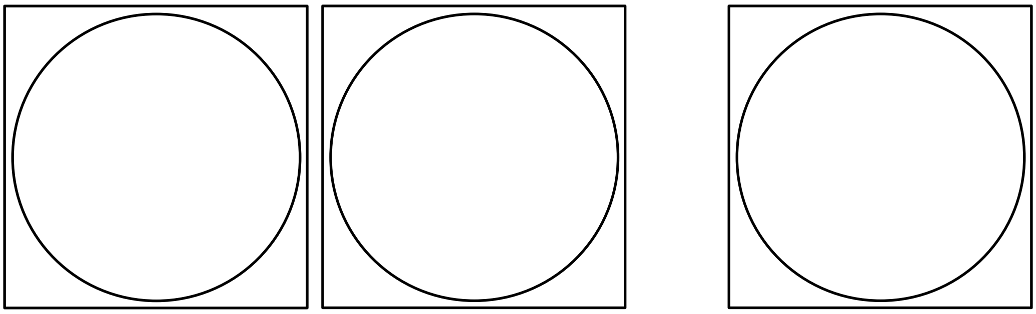
(b) Using the paired diagram, explain how you can from the ratio of areas or the ratio of angles out of 360° obtain the probabilities $P_{same,HV}(45°) = 0.25$ and $P_{opp,HV}(45°) = 0.75$ for this angle $\theta = 45°$. Determine the correlation coefficient $E_{HV}(45°)$ predicted by hidden variable theory. By taking measurements in the hidden variable case, verify that the simulation (within statistical fluctuations) gives the values $P_{same} = 0.25, P_{opp} = 0.75$ and your calculated value for the correlation coefficient for the case $\theta = 45°$.

6. (a) (i) Make diagrams similar to the ones shown in step 5 of the Step-by-step Explanation for angles θ of SGA B relative to SGA A of 22.5°, 67.5° and 90°. Circles are already drawn for you below. In your sketches, shade regions of outcome + for the individual outcomes and regions of outcome same in your paired outcome sketch. For each angle, add a pair of spin vectors giving the outcome same.

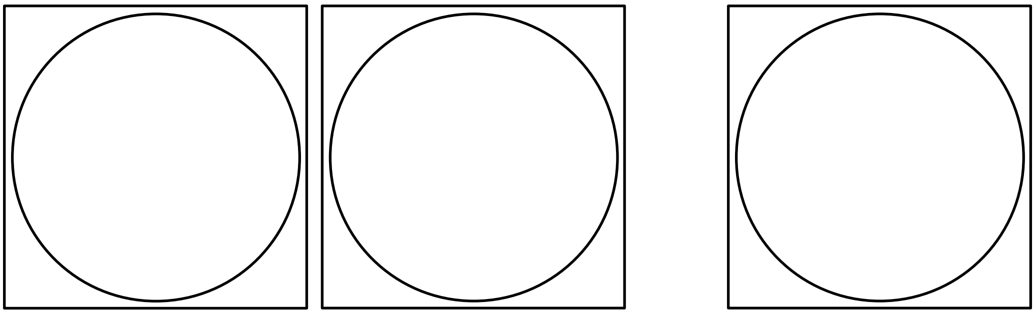
(ii) For each angle θ, determine $P_{same,HV}$ and $P_{opp,HV}$ for the hidden variable case from your diagrams. Also determine the correlation coefficient.

(iii) For each angle, verify that the simulation (within statistical fluctuations) gives the values for $P_{same,HV}$, $P_{opp,HV}$ and E_{HV} that you calculated.

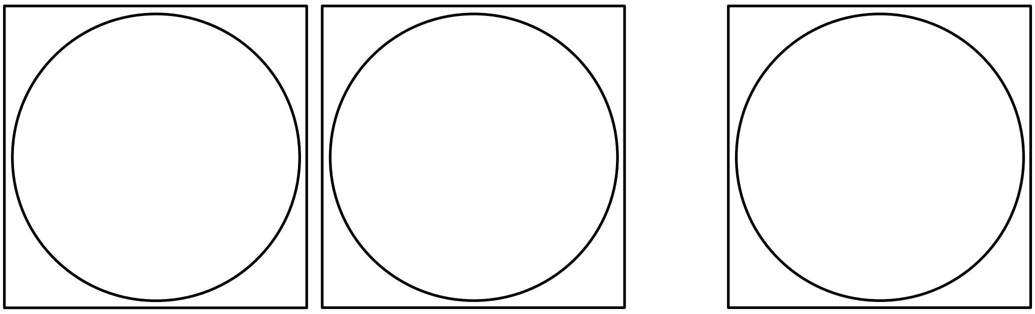
SGA B at $22.5°$



SGA B at $67.5°$

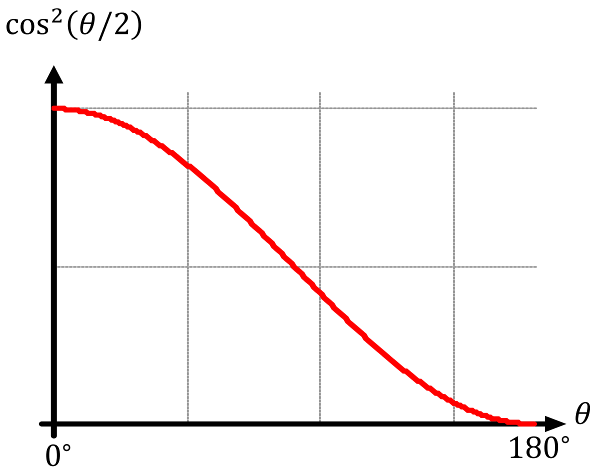


SGA B at $90°$



(b) Generalize your results to make graphs (with θ on the horizontal axis) of $P_{same,HV}(\theta), P_{opp,HV}(\theta)$ and the correlation coefficient $E_{HV}(\theta)$ predicted by hidden variable theory for the range $0 \le \theta \le 180°$ shown in the simulation.

7) (a) The quantum prediction is $P_{opp}(\theta) = cos^2(\theta/2)$, see graph below. Add the quantum predictions for $P_{same}(\theta), P_{opp}(\theta)$ and $E(\theta)$ to your graphs from part 6b). Note that $1 - \cos^2\left(\frac{\theta}{2}\right) = \sin^2\left(\frac{\theta}{2}\right)$ and $1 - 2\cos^2\left(\frac{\theta}{2}\right) = -\cos(\theta)$.



(b) At what angles does the hidden variable prediction maximally disagree with what is actually observed? Explain how you can see this result in the simulation. Interpret this result in terms of your answer to part 2b.

8) Do you agree or disagree with (or feel neutral about) the following statement?

The perfect anticorrelations seen whenever the two SGAs are oriented along the same axis are due to the spins of the particles having pre-determined values at the time of creation. Thus, the measurement is just an increase in our knowledge of the system, not a change in the system itself.

Strongly Disagree   Disagree   Neutral   Agree   Strongly agree

Indicate your confidence in your answer:

Certain   Somewhat certain   Somewhat uncertain   Uncertain

Briefly explain your response: