For these problems, use the simulation "Superposition states and mixed states".

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down five things about the controls and displayed quantities that you have found out.

2) Assume all input particles are in the state $|\uparrow>$. Describe what you observe when the SGA is oriented along the z-axis. Describe what you observe when the SGA is oriented along the x-axis. Explain your observations.

3) Assume all input particles are in the state $|\downarrow>$. Describe what you observe when the SGA is oriented along the z-axis. Describe what you observe when the SGA is oriented along the x-axis. Explain your observations.

4) (a) Using the simulation (note the input particles being sent through the SGA), explain what is meant by a random mixture of 50% of particles in the state $|\uparrow>$ and 50% in the state $|\downarrow>$.

(b) Do we encounter similar mixtures of objects in our everyday experience? If so, give an example. If not, explain why not.

(c) For the random mixture from part (a), describe and explain what you observe when the SGA is oriented along the z-axis. Describe and explain what you observe when the SGA is oriented along the x-axis. Include a description of the experimental setup and the measurements you are taking.

5) Now consider the input shown in the simulation where each particle is in a superposition state $\frac{1}{\sqrt{2}}(|\uparrow> + |\downarrow>)$.

(a) Using the simulation, compare this input state to the random mixture in question 4. List similarities and differences in terms of the input and the measurement outcomes.

(b) Explain any differences. Note that $|\uparrow> = \frac{1}{\sqrt{2}}(|+> + |->)$ and $|\downarrow> = \frac{1}{\sqrt{2}}(|+> - |->)$, where $|+>$ and $|->$ are eigenstates of the x-component of spin S_x with eigenvalues $+\hbar/2$ and $-\hbar/2$ respectively.

6) (a) Using the simulation and your results from questions 2 to 5, which of the two inputs labeled "superposition or mixture?" and "superposition or mixture??" is a random mixture of $|\uparrow>$ and $|\downarrow>$? Explain your reasoning.

(b) The mixture consists of a fraction c of particles in state $|\uparrow>$ and a fraction d of particles in state $|\downarrow>$. Find these fractions c and d. Explain your reasoning.

7) (a) Using the simulation and your results from question 5, which of the two inputs labeled "superposition or mixture?" and "superposition or mixture??" consists of particles each in the superposition state $(a|\uparrow> + b|\downarrow>)$? Explain your reasoning. Find the coefficients a and b, assuming they are real and positive.

(b) Show by calculation that the superposition state you have found in part (a) gives the measurement outcome probabilities along z and x shown in the simulation.

8) Do you agree or disagree with or feel neutral about the following statement? Explain your answer, considering the experimental results seen in the simulation.

The quantum state $\frac{1}{\sqrt{2}}(|\uparrow> + |\downarrow>)$ *represents a lack of knowledge about the state of the system. The system actually alternates rapidly in time between being in state* $|\uparrow>$ *and being in state* $|\downarrow>$*. The quantum state tells you that the probability is* $1/2$ *that the system is really in the spin state* $|\uparrow>$ *and* $1/2$ *that it is really in the spin state* $|\downarrow>$*, so that the particle is in each of the two states for one-half of the time.*