For these problems, use the simulation "Spin-1 particles in successive Stern-Gerlach experiments".

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Write down five things about the controls and displayed quantities that you have found out.

2) Consider the situation where the two SGAs are both oriented along the z-axis.

(a) Assume the beam deflected upwards by the first SGA is passed on to the second SGA. Using the simulation, what are the probabilities that the <u>second</u> SGA will measure either $S_z = -\hbar$, $S_z = 0\hbar$ or $S_z = +\hbar$?

(b) Calculate the following quantities, and explain how they relate to your answer from part (a):

$$\left|\langle 1_z | 1_z \rangle\right|^2 = \left[\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right]^2 =$$

$$\left|\langle 0_z | 1_z \rangle\right|^2 =$$

$$\left|\langle -1_z | 1_z \rangle\right|^2 =$$

(c) Using the simulation, come up with a general rule about repeated spin measurements that explains what you would expect to see if the beam passed on to the second SGA were instead the undeflected beam or the down-deflected beam.

3) (a) Using the matrix representation of the operator $\hat{S}_x$ given in the simulation, show that the states $|1_x\rangle$, $|0_x\rangle$ and $|-1_x\rangle$ are eigenvectors of the operator $\hat{S}_x$, and determine the corresponding eigenvalues. Explain how these eigenvalues relate to the experimental observations in terms of beam deflections.

(b) What would be the vector representations of the states $|1_x\rangle$, $|0_x\rangle$ and $|-1_x\rangle$ if they were written in the x-basis, instead of the z-basis? What about the $\hat{S}_x$ operator, written in the x-basis? Hint: You should not need to perform any explicit calculations to answer this question.

4) (a) Assume the beam deflected upwards by the first SGA is passed on to the second SGA. Use the simulation to determine the probabilities for the outcomes of spin measurements performed along the x-direction. Describe how you can see these particular results in the simulation, including a description of the experimental setup and the relative orientations of the two SGAs.

(b) Using matrix addition, show that the quantum state $|1_z\rangle$ can be written as a linear combination of the eigenstates of the $\hat{S}_x$ operator written in the z-basis: $|1_x\rangle$, $|0_x\rangle$ and $|-1_x\rangle$

$$|1_z\rangle = \frac{1}{2}\left(|1_x\rangle + \sqrt{2}|0_x\rangle + |-1_x\rangle\right) =$$

(c) Explain how your result from part (b) relates to your answer from part (a).

5) (a) Now assume that the beam that is not deflected by the first SGA is passed on to the second SGA. Use the simulation to find the probability for the second SGA to measure these particles in the state $S_x = 0\hbar$. Verify this result mathematically by explicitly calculating this probability.

(b) Suppose a spin-1 particle is initially in the state $|1_z\rangle$, and when subsequently measured along the x-axis is found to be in the state $|1_x\rangle$. If the spin of that same particle were again measured along the z-axis by a third SGA, what would be the probability that the measurement outcome would again be $S_z = +\hbar$? Justify your answer mathematically by explicitly calculating this probability.