For these problems, use the simulation "Entanglement: the nature of quantum correlations".

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays.

2) Bring up the correlations panel; more information is available via the "?" button and the "Step-by-step Explanation" tab.

(a) Explain the meaning of the probabilities P_{same} and P_{opp} and the correlation coefficient $E(AB)$ shown.

(b) What does it mean if

$E(AB) = +1$;

$E(AB) = -1$;

$E(AB) = 0$?

3) Orient both SGAs along Z and choose the spin state $|\uparrow_A\rangle|\downarrow_B\rangle$ (the topmost state).

(a) What do Alice and Bob measure individually? What do they find when they compare their results for each pair? Determine the values of P_{same}, P_{opp} and $E(AB)$ for this situation.

Now rotate the SGAs along X.

(b) Using $|\uparrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$ and $|\downarrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$, rewrite the quantum state $|\uparrow_A\rangle|\downarrow_B\rangle$ in the X basis. Compare your result with the simulation. Why is this state called a product state?

(c) Along X, what do Alice and Bob measure individually? What do they find when they compare their results for each pair? Determine the theoretical values of P_{same}, P_{opp} and $E(AB)$ for this situation.

4) Choose the entangled state $\frac{1}{\sqrt{2}}(|\uparrow_A\rangle|\downarrow_B\rangle - |\downarrow_A\rangle|\uparrow_B\rangle)$ (second state, not a product state). Consider both the X and Z orientations of the SGAs.

List two ways in which the results are different to those for the input state $|\uparrow_A\rangle|\downarrow_B\rangle$ (question 3).

5) For this question, use the "Create your own state!" option.

(a) Create a product state using more than one of the terms. Explain why your state is a product state. If a state is a product state along Z, will it also be a product state along X? (you do not need to determine the actual form of your quantum state along X)

(b) Does a product state imply that the spins of the particles have definite values? If so, explain why. If not, create a product state for which this is not the case.

(c) Create an entangled state for which

i) there is perfect correlation along both X and Z

ii) there is no correlation along X and no correlation along Z.

In each case, write down the quantum state. You do not need to determine the form of your quantum state along X.

(d) Is it possible to create a product state for which there is perfect correlation along both X and Z? Explain why or why not.

(e) Entangled states are not product states. Interpret this statement physically.

6) Imagine that instead of the setup shown there are two sources of particle pairs, one emitting particles in the state$|\uparrow_A>|\downarrow_B>$, the other in the state $|\downarrow_A>|\uparrow_B>$. The sources emit particle pairs so that on average, 50% of the particle pairs sent to Alice and Bob are in the state $|\uparrow_A>|\downarrow_B>$, the other 50% are sent in the state $|\downarrow_A>|\uparrow_B>$, but the order of which of these pairs is sent is random. This is called a random mixture of pairs.

Using the setup in the simulation, could Alice and Bob experimentally determine whether they are making measurements on such a random mixture as opposed to making measurements on the entangled state $\frac{1}{\sqrt{2}}(|\uparrow_A>|\downarrow_B>-|\downarrow_A>|\uparrow_B>)$? Explain your reasoning.

7) Show that the entangled state $\frac{1}{\sqrt{2}}(|\uparrow_A>|\downarrow_B>-|\downarrow_A>|\uparrow_B>)$ in the simulation (the second state) cannot be written as a product of two single-particle states. Start by assuming that you could write this entangled state as a product state $\left(a|\uparrow_A>+b|\downarrow_A>\right)(c|\uparrow_B>+d|\downarrow_B>)$ with constants a,b,c,d. Show that this leads to a contradiction in that no values of a,b,c,d can fulfill the resulting equations.