For these problems, use the simulation "Phase shifter in a Mach-Zehnder interferometer" (Mach Zehnder Interferometer II) in the QuVis HTML5 collection.

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down three things about the controls and displayed quantities that you have found out.

2) Consider the case where there is no phase shifter present inside the Mach-Zehnder interferometer.

(a) What is the physical meaning of the quantum states $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$? Using the simulation, write down (do not derive) the quantum states of the photon at different points in the interferometer. Explain the physical meaning of these quantum states.

(b) Does the behaviour of the photon as it passes through the interferometer depend on whether or not the filters blocking 1.4 eV light are present? Explain the physical significance of this observation.

(c) Why is the matrix representing the second beamsplitter different from the matrix representing the first beamsplitter?

(d) Using matrix multiplication, show how the quantum states of the photon at different points in the interferometer can be calculated. Verify your results using the simulation.

(e) Explain how the output state in part (d) relates to the detection probabilities shown in the simulation for this situation.

3) Now consider the Mach-Zehnder interferometer with the phase shifter inserted into the lower path.

(a) Describe your observations as you vary the phase shifter. Come up with some general statements relating the phase shift and the detection probabilities that summarize your observations.

(b) Write down (do not derive) the quantum states shown in the simulation of the photon after it has passed the second beamsplitter (but prior to detection) for the particular phase shifts that (i) maximize the detection probability in Detector 1; and (ii) maximize the detection probability in Detector 2. Explain how these states correspond to the detection probabilities shown in the simulation. Hint: $\exp(ix) = \cos(x) + i\sin(x)$.

(c) Consider the quantum state for a photon after passing the second beamsplitter when there is a phase shift in the lower path of $\phi = 3\pi/2$. What are the probabilities for detection in either Detector 1 or Detector 2 for this state? What happens to this quantum state at the moment of detection?

4) Compare the interferometer setup in the simulation (with the phase shifter present) with the two situations (i) and (ii) described below. Explain whether or not the detection probabilities in the simulation would be altered if the changes described in (i) or (ii) were made to the setup.

(i) The phase shifter in the lower beam is placed between the mirror and beamsplitter 2 instead of between beamsplitter 1 and the mirror.

(ii) The phase shifter is placed between the photon source and beamsplitter 1 instead of between beamsplitter 1 and the mirror.

5) Consider the two expressions (a) and (b) below for the state of a photon after having passed through an interferometer. Describe what each matrix in the given expression physically represents, and make a sketch of the interferometer setup that would generate the given output state.

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(b) $\frac{1}{\sqrt{2}}\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{i\chi} \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$