For these questions, use the simulation "Comparison of the classical and quantum harmonic oscillator" (Quantum Oscillator) in the QuVis HTML5 collection.

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down three things about the controls and displayed quantities that you have found out.

2) a) Using the simulation, how does the fraction of the quantum-mechanical probability density beyond the classical turning points qualitatively change with increasing quantum number (no calculation required)? Is this what you would expect?

b) The simulation shows the classical amplitude A_{CL} for an oscillator with the energy E_n of the nth quantum-mechanical energy eigenstate. Explain how A_{CL} is determined in the lower graph. From the simulation graphs, does A_{CL} increase linearly with n? Derive an expression for A_{CL} as a function of n, $\hbar$, ω and m. Reconcile this expression with the simulation graphs.

c) Defining P_{CL} as the classical probability density, $P_{CL}dx$ is the probability that a measurement of the position of a particle will find it in the region dx. This is equal to the amount of time dt that the classical particle spends in the region dx, divided by the total time needed for one traversal (i.e., half a period).
Using the fact that $dt = \frac{dx}{v(x)}$, show that the classical probability density of the simple harmonic oscillator is $P_{CL}(x) = \frac{1}{\pi\sqrt{A_{CL}^2 - x^2}}$ for $-A_{CL} < x < A_{CL}$, where A_{CL} is the amplitude of the classical oscillator. What is P_{CL} for $x < -A_{CL}$ and $x > A_{CL}$?

d) Interpret the behaviour of P_{CL} for $x \to \pm A_{CL}$. Show explicitly that P_{CL} is normalized.

e) Write down an integral expression for the probability that a measurement of position would find the quantum particle described by the energy eigenfunction $\psi_n(x)$ outside the classically allowed region. You do not need to solve this integral expression.