

For these problems, use the simulation “The expectation value” in the QuVis HTML5 collection.

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down three things about the controls and displayed quantities that you have found out.

2 a) Consider the input state  $\sqrt{\frac{3}{10}}|\uparrow\rangle + \sqrt{\frac{7}{10}}|\downarrow\rangle$  shown in the simulation. Experimentally find the detection probabilities for particles to be detected in the upper path and in the lower path after passing through the Stern-Gerlach apparatus. Explain how these detection probabilities can be theoretically calculated from the input state.

b) Do the same for the input state  $\frac{1}{\sqrt{5}}(-2|\uparrow\rangle + |\downarrow\rangle)$ .

c) Find the absolute value of the coefficients  $|a|$  and  $|b|$  for the third input state. Explain your reasoning. Why can you only find  $|a|$  and  $|b|$  and not  $a$  and  $b$ ?

3) a) Explain the two procedures shown in the Expectation value (I) and Expectation value (II) panels to experimentally determine the expectation value  $\langle \hat{S}_z \rangle$  of the  $z$ -component of spin for a fixed input state. Show that the two procedures must give the identical result and are thus equivalent.

b) Will your value for  $\langle \hat{S}_z \rangle$  obtained experimentally exactly agree with the theoretical value? Explain using the simulation.

4) Consider the input state  $\sqrt{\frac{3}{10}}|\uparrow\rangle + \sqrt{\frac{7}{10}}|\downarrow\rangle$ .

a) What are the possible outcomes for a single measurement of  $S_z$ ?

b) What is the most likely outcome of  $S_z$  for a single measurement? Explain how you can determine this most likely outcome for a single measurement from the theoretical probability  $\text{Prob}_+$ .

c) Explain how you can determine the most likely outcome of  $S_z$  for a single measurement from the theoretical expectation value  $\langle \hat{S}_z \rangle$ .

5) Come up with an input state different to those shown in the simulation for which

a) the theoretical expectation value  $\langle \hat{S}_z \rangle < 0$ .

b) the most likely outcome of  $S_z$  for a single measurement is  $+\hbar/2$ .

c)  $+\hbar/2$  and  $-\hbar/2$  are equally likely outcomes of  $S_z$  for a single measurement.