Overview
The solid state physics that you have learned so far has relied heavily on the approximation that the electrons do not interact with each other (the 'independent-electron' approximation). This has allowed us to develop a model (the Sommerfeld model) in which the electrons are simply placed into the available single-particle states, with the only constraint that the Pauli exclusion principle must be obeyed. This model explains a good deal of what we see in experiment. However, a back-of-the-envelope calculation shows that, at the electron densities present in typical solids, the energy due to the Coulomb interaction is significantly larger than that due to the individual electrons' kinetic energies. So how does such a non-interacting theory ever work?

This course will answer that question, in the framework of Landau's famous theory of Fermi liquids. We shall learn that the non-interacting objects are not really electrons at all, but quasi-particles made up of a large cloud of electrons and holes. We shall discover how to use the Fermi liquid theory to predict the thermodynamic responses of metals. We shall then investigate what happens when, due to the development of strong correlations, the quasiparticle breaks down. This will take us into some of the most interesting territory of modern condensed matter physics: Mott insulators, spin-density waves, the heavy-fermion materials, and high-temperature superconductivity.

Aims & Objectives
To present an introduction to the physics of electron-electron interactions in solids, Landau's theory of the Fermi liquid, and what happens when the Fermi liquid breaks down. Topics covered will include:

- the susceptibility of the non-interacting Fermi gas to externally applied fields;
- the momentum-space representation of the Coulomb interaction;
- how the Coulomb interaction is screened by the conduction electrons (Thomas-Fermi theory);
- Landau's theory of the Fermi liquid;
- the Fermi surface and how to measure it;
- instabilities of the Fermi liquid: ferromagnetism; spin- and charge-density waves; superconductivity;
- the Mott metal-insulator transition, and the low-temperature behaviour of Mott insulators;
- experimental examples, including the heavy fermion materials and the high-temperature superconductors.

Learning Outcomes
By the end of this module, students will have developed an understanding of the principal types of behaviour exhibited by interacting electrons in solids. This will have been achieved partly through the lectures, and partly through guided self-study via reading assignments. In particular, they will:

- understand the Landau theory of the Fermi liquid, and be able to use the Landau energy functional for practical calculations;
- be able to predict when the Landau Fermi liquid will show a second-order transition into an ordered phase;
- appreciate the Mott metal-insulator argument, and know something of the nature of the Mott transition;
- be able to make practical calculations of the energy spectra of finite-size Hubbard models and of spin waves in the Mott insulating state;
- have developed their ability to distil information from the original research literature.

Synopsis
The many-body problem (1 lecture):
Why the many-body problem is interesting; exponential growth of the Hilbert space; entanglement and correlations.
Models of non-interacting electrons in solids (4 lectures):
The Sommerfeld model, momentum space, and the Fermi surface; the nearly-free-electron model, band gaps, and the Brillouin zone; the tight-binding model, and the Fermi surfaces of simple lattices; the susceptibility

The Coulomb interaction and the Fermi liquid (5 lectures):
Representation of the Coulomb interaction in momentum space; screening of the Coulomb interaction in metals (Thomas-Fermi theory); adiabatic continuity, the Landau quasiparticle, and its decay rate; the Landau energy functional and the Landau parameters; the specific heat capacity of a Fermi liquid.

The Fermi surface and its instabilities (4 lectures):
Measuring the Fermi surface via quantum oscillations; the magnetic susceptibility of the Fermi liquid; the Stoner instability to ferromagnetism; nesting instabilities, spin- and charge-density waves; the Cooper instability and superconductivity.

Mott insulators and their properties (4 lectures):
Mott’s argument for a metal-insulator transition at half filling; the Hubbard model and its phase diagram; the Heisenberg model of magnetism in the Mott-insulating state; spin waves in insulating magnets.

Some experimental examples (3 lectures):
Observing magnetic order and spin waves via neutron scattering; the heavy-fermion materials as extreme Fermi liquids; the high-\(T_c\) superconductors as doped Mott insulators.

Pre-requisites
PH2011, PH2012, MT2001 or (MT2501 and MT2503), (PH3081 or PH3082 or [MT2003 or (MT2506 and MT2507)]), PH3012, PH3061, PH3062, PH4039

Anti-requisites
none

Assessment
Continuous Assessment, written assignments = 50%, Presentation plus Oral Examination = 50%

Additional information on continuous assessment etc
Please note that the definitive comments on continuous assessment will be communicated within the module. This section is intended to give an indication of the likely breakdown and timing of the continuous assessment.

The assessment will consist of:
- three tutorial sheets, each worth 10 per cent of the credit for the module;
- one extended essay, worth 20 per cent of the credit for the module; and
- an oral examination (including a presentation by the student), worth 50 per cent of the module.

Recommended Books
Please view University online record:
http://resourcelists.st-andrews.ac.uk/modules/ph5014.html

General Information
Please also read the general information in the School's honours handbook.