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Morag F. Macpherson, Adam Kleczkowski, John Healey and Nick Hanley

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Morag F. Macpherson[a], Adam Kleczkowski[a], John Healey[b] and Nick Hanley[c]

[a] Computing Science and Mathematics, School of Natural Sciences, Cottrell Building, University of Stirling, Stirling, UK FK9 4LA

[b] School of Environment, Natural Resources and Geography, Thoday Building, Deiniol Road, Bangor, Gwynedd, UK LL57 2UW

[c] School of Geography & Geosciences, Irvine Building, University of St Andrews, North Street, St Andrews, Fife, UK KY16 9AL

Abstract

Effective forest management is crucial to maximising the economic benefits obtained from forests. However, the arrival of novel pests and pathogens may have a negative effect on timber values. We argue that management strategies should be calibrated to consider the effect of disease and in this paper explore the optimal rotation length of a single rotation, even-aged, plantation forest under varying disease conditions. We show that the optimal rotation length, which maximises the net present value of the forest, is reduced when disease decreases the timber value. Moreover, an increase in the rate of disease progression or the effect of disease on timber value has a negative effect on the optimal rotation length. More generally, the effect of forest disease on optimal management depends in a complex way on the interaction of economic, ecological and epidemiological parameters.

1 Introduction

Like many natural resources, forests are experiencing increasing pressure from the emergence of pests and pathogens (Department for Environment, Food and Rural Affairs, 2013b). Changing climate (Netherer and Schopf, 2010; Pautasso et al., 2010; Sturrock, 2012), globalisation of trade and the synonymous increase in the volume and diversity of plant species and products being traded (Department for Environment, Food and Rural Affairs, 2013b) are only a few of the factors leading to an increase in dispersal ranges of pests and pathogens. Recently the UK has seen the arrival of *Phytophthora ramorum* on larch (Brasier and Webber, 2010; Forestry Commission Scotland, 2015); *Dothistroma septosporum*, a needle blight effecting conifers in particular pine (Forestry Commission Scotland, 2013); *Chalara fraxinea* causing dieback on ash (Department for Environment, Food and Rural Affairs, 2013a); and *Thaumetopoea processionea*, a processionary moth causing major defoliator of oak (Netherer and Schopf, 2010; Tomlinson et al., 2015). The arrival of these novel pests and pathogens require management strategies to be reviewed in order to maximise the net benefit obtained from our forests. In this paper we focus on the management strategy of clear felling and address the question of how disease effects the optimal time to harvesting for a plantation (henceforth the 'optimal rotation length'). This is an important question since the arrival of such pests and pathogens can lead to loses in market values through the decrease in timber growth, degradation of timber quality, complex sawmilling requirements and market saturation. By including some of these factors in the modelling framework, we can give insight into how disease alters the optimal rotation length. For simplicity, we focus on the timber values of forestssince the only way to reduce the damage of disease is often to clearfell, the optimal rotation length in the presence of disease risks is an important management strategy to consider.

There are two common approaches used to determine the optimal rotation length. The first is the maximum sustained yield (MSY) which is determined mainly by ecological processes, and will only give the economically optimal rotation under very restrictive economic conditions (Samuelson, 1976). The MSY method defines the optimal rotation length as the age which maximises the timber volume produced per unit of land (Amacher et al., 2009). The second method merges economics and ecology, and was introduced by a German forester, Martin Faustman, in 1849 who derived the optimal rotation length using the principles of discounting (Faustmann, 1849). Faustmann considered a forest as a long-term capital asset and thus the optimal rotation length could be determined by maximising the net present value (NPV) of the land (Amacher et al., 2009).

This area has been extensively studied; a review showed that there has been 313 published books and articles in over sixty journals since Faustmann's revolutionary work (Newman, 2002). Some notable contributions include the addition of the non-market value of forests (Hartman, 1976; Samuelson, 1976); the effect of catastrophic loss, for example from fire (Reed, 1984; Englin et al., 2000) or wind blow (Price, 2011); the effect of including a carbon market (Chladná, 2007; Price and Willis, 2011); the uncertainty and risk of future prices (Alvarez and Koskela, 2006; Loisel, 2011; Sims and Finnoff, 2013); and multiple forests and their interdependent provision of amenity services (Koskela and Ollikainen, 2001). The arrival of disease could be considered a catastrophic event in the case of wide spread epidemics where large areas of forests are felled and market and non-market values (such as ecosystem services) are affected. However, there are many dissimilarities when comparing the effect of disease to the classic catastrophic events such as fire and wind. Some distinctions include the speed of progression (disease can progress at variable time scales, but likely units are years); the symptoms (cryptic infection can result in the disease remaining undetected for long periods of time); the response once detected (there is a large variability in strategies dealing with infected trees); and salvageable timber (infected timber will likely still be marketable, but sometimes for a reduced price). Due to these differences, the lack of previous investigation and the extent of disease presence within the UK, we propose further examination looking specifically at the effect of disease on the optimal rotation length of a forest is required. Moreover, the characteristics, such as rate of spread and impact on the timber value are generally disease specific and thus an in-depth analysis of these attributes and their interaction with the optimal rotation length would be valuable.

To answer the question posed we build on previous optimal rotation framework by merging epidemiology with forest ecology and economics. Mathematical epidemiological models are commonly used to provide an insight into the dynamics of disease spread in addition to evaluating different management strategies Cobb et al. (2012); Boyd et al. (2013). However it has been shown that omitting economic behaviour from animal disease models leads to important failures in our understanding of how to manage disease-prone systems (Fenichel et al., 2010; Horan et al., 2011). This is likely because management strategies are often expensive, and if implemented they can change the course of disease spread, thus creating a dynamic feedback between the economic and epidemiological components. Therefore to examine the effect of disease on the optimal rotation length we extend the classic single-rotation Faustmann model to include the dynamic progression of disease creating an optimal control-type problem.

Without disease, the NPV of a single rotation Faustmann model assumes that there is a one-off cost of establishment and that forest revenue is a product of the stumpage price and the volume of timber of the stand at the end of the rotation (Faustmann, 1849). The optimal rotation length which maximises the NPV is calculated by solving the first order condition. We propose a model which assumes disease will reduce the value of timber from infected trees, therefore the timber revenue and NPV is dependent on the infection state of the forest at the time of harvesting (in addition to the timber growth). In order to solve this new framework we define a system of first order differential equations describing the rate of change of the area of infection over time and proceed as before by finding the first order condition for maximising the NPV. Despite being unable to analytically derive the optimal rotation length, the first order condition and numerical optimisation techniques can provide significant insight in to the systems dynamics and sensitivity to key parameters.

The aim of this paper is thus to examine the relative effect of disease on the optimal rotation length of a single rotation, even-aged, plantation forest. In order to analyse this effect we have excluded many complexities in the system such as thinning, non-market benefits and multiple rotations. Clearly a forest has multiple outputs which may also be effected by disease, for example the reduction in amenity values through the loss of biodiversity, carbon storage and sequestration, and decreased recreation and aesthetic values (Boyd et al., 2013; Forestry Commission Scotland, 2013). Samuelson (1976) discussed the importance of ignoring the non-market values from forestry calculations, and Hartman (1976) showed that in doing so a sub-optimal rotation length would be found. The original Faustmann work considers multiple rotations where trees are perpetually planted and harvested thus synonymously including the benefit of the land ('land rent'). When considering a single rotation, the future value of the land is omitted (we only incorporate the value of the forest) and it has been shown that this leads to a sub-optimal rotation length (Amacher et al., 2009). In this paper we only consider a single rotation since the inclusion of disease in a multiple rotation framework would require a more in-depth detail of the disease system (for example, the level of inoculum that would persist in the environment after harvesting which carries over to the next rotation). However, it is relatively simple to examine the effect of including land rent in our framework, if the land use is changed after the rotation to agriculture, and we can argue that this would be representative of a change in tree species too. Changes in land use could occur for multiple reasons; one such motivation could be that the original tree species is susceptible to the disease in future rotations thus motivating the owner to assess re-planting. We consider this scenario in this paper and show how it would affect the optimal rotation length in the presence of disease.

The structure of this paper is as follows. In section 2 we first deduce the first order condition for a single rotation Faustmann model and then extend the framework to include a general disease system. In section 3 we define a forest growth function and susceptible-infected (SI) disease system which we use to highlight some key results produced by numerical optimisation in section 4. In section 5 we briefly discuss the effect of a control measure which is applied annually.

2 Formulation of the general model

2.1 The model without disease

In this section we review a single rotation Faustmann model for an even-aged forest where the net present value (NPV) includes an establishment cost (planting from bare land) and the benefit from harvesting the timber. We assume that for a forest of area L (in hectares), the establishment costs are linearly dependent on the area $W(L) = cL$ where c is the planting cost per hectare; and the net benefit of harvesting, $M(L, T)$, is a product of the per cubic metre price of timber, p, and the volume of timber produced, $f(T)L$. We extend this model to include an annual payment for land rent *after* harvesting which is linearly dependent on the area of the forest, $A(L) = aL$. Further underlying assumptions include: all costs and prices are constant and known; future interest rates are constant and known; and the growth function of the species is known (Amacher et al., 2009). Thus the NPV is

$$\hat{J}(T) = -W(L) + M(L, T)e^{-rT} + \int_T^\infty A(L)e^{-rt}\, dt. \tag{1}$$

An exponential discount factor, with rate r, is used to discount future revenue (from harvesting and land rent) back to the time of net present value. Parameter definitions and baseline values are given in Table 1. To find the rotation length which maximises the NPV we find the first order condition by differentiating Equation (1) with respect to T, which gives

$$\frac{d\hat{J}(T)}{dT} = \frac{dM}{dT}e^{-rT} - rM(L, T)e^{-rT} - A(L)e^{-rT}. \tag{2}$$

Setting Equation (2) equal to zero and substituting the function for the revenue from harvesting we obtain the first order condition

$$\frac{1}{f(T_{DF})}\frac{df}{dT}\bigg|_{T=T_{DF}} - r = \frac{A(L)}{pf(T_{DF})L}. \tag{3}$$

This implies that the optimal rotation length ($T = T_{DF}$) is given when the marginal gain from the relative timber growth and the opportunity cost of investment (left-hand side) is equal to the future land rent (right-hand side). Clearly Equation (3) shows that the inclusion of future benefits (via land rent) decreases the optimal rotation length which is in line with previous studies (Amacher et al., 2009). Evaluating the second derivative at the optimal rotation length gives

$$\frac{d^2\hat{J}}{dT^2}\bigg|_{T=T_{DF}} = pLe^{-rT_{DF}}\left(\frac{d^2 f}{dT^2}\bigg|_{T=T_{DF}} - r\frac{df}{dT}\bigg|_{T=T_{DF}}\right) < 0 \tag{4}$$

which is negative if the timber growth function is an increasing, concave function and thus T_{DF} maximises the NPV.

We will use Equation (3) as a baseline of which to compare the result from system with disease to.

2.2 General model with disease

We now examine the effect of disease on the optimal rotation length. There are many ways in which disease can decrease the timber value: reduction in growth, for example *Dothistroma septosporum* causes significant defoliation which can greatly reduce growth rate (Forestry Commission Scotland, 2013); reduction in quality, for example *Heterobasidion annosum* decays the wood in the butt end of the log which may reduce the value of the timber (Pratt, 2001; Redfern et al., 2010); or an increase in the susceptibility to secondary infection, for example *Chalara fraxinea* and *Phytophthora ramorum* cause significant damage to the bark and cambium therefore increasing the rate of infection of wood decay fungi (Pautasso et al., 2013; Forestry Commission Scotland, 2015). In the case of an epidemic, large areas of monoculture forest may be felled simultaneously to try to halt disease spread, thus a large influx of material to local sawmills may cause congestion and market saturation (although we do not model this scenario explicitly since that would require a reduced price for all timber independent of its infection status). By suitably scaling the revenue obtained from timber of infected trees, we can represent some of these scenarios in our model. We first introduce the NPV and the general disease system, and finally derive the first order condition which allows us to show the effect of disease on the optimal rotation length.

Equation (1) represents the NPV of a forest of area L which remains in an infection-free state. We build on this model by assuming the revenue obtained from the timber is be dependent on the state of infection at that point in time. Therefore the NPV is

$$\hat{J}(T) = -W(L) + M(\widetilde{L}(T), T)e^{-rT} + \int_T^\infty A(L)e^{-rt}\,dt \tag{5}$$

where $\widetilde{L}(T)$ denotes the effect of the disease on the total area of the forest (explained further below). The establishment cost and land rent remains unchanged, and for the moment we assume that there is no additional cost of disease (for example through control or treatment).

Next we assume a general disease system with N stages of progress is introduced to the forest. We denote the area of the forest in the ith stage by $x_i(T)$ where $1 \leq i \leq N$, and since no partial felling is undertaken to prevent disease spread the area of the forest is unchanged, giving $L = \sum_{i=1}^N x_i(T)$. If the disease had no effect on timber value, the revenue from timber in the ith stage of infection is $pf(T)x_i(T)$. However, we assume that the disease reduces the value of timber (either through reduced quality or growth), so the revenue from timber in each stage is scaled by parameter ρ_i where $0 \leq \rho_i \leq 1$. This means that timber in each stage may be affected differently by disease. We therefore can represent the harvest revenue function for the forest as

$$M(\widetilde{L}(T), T) = pf(T)\left(\sum_{i=1}^{N} \rho_i x_i(T)\right) \tag{6a}$$

$$= pf(T)\widetilde{L}(T) \tag{6b}$$

where the effect of disease on the whole forest at time T is given by

$$\widetilde{L}(T) = \sum_{i=1}^{N} \rho_i x_i(T). \tag{7}$$

Since the disease progresses over time, we specify a system of differential equations (dx_i/dt) which can be solved for $x_i(t)$, and substituted into the harvest revenue function (Equation (6). We are then able to proceed as before and find the optimal rotation length using the first-order condition. We can find a general solution by differentiating Equation (5) which gives

$$\frac{d\hat{J}(T)}{dT} = e^{-rT}\frac{d}{dT}\left(M(\widetilde{L}(T), T)\right) - re^{-rT}M(\widetilde{L}(T), T) - A(L)e^{-rT} \tag{8a}$$

$$= pe^{-rT}\left(\frac{df}{dT}\widetilde{L}(T) + f(T)\frac{d\widetilde{L}(T)}{dT} - rf(T)\widetilde{L}(T) - \frac{A(L)}{p}\right). \tag{8b}$$

Setting Equation (8b) equal to zero and re-arranging we have

$$\frac{1}{f(T_D)}\left.\frac{df(T)}{dT}\right|_{T=T_{DF}} - r = \frac{1}{\widetilde{L}(T_D)}\left(\left|\frac{d\widetilde{L}}{dT}\right|_{T=T_{DF}} + \frac{A(L)}{pf(T_D)}\right). \tag{9}$$

Equation (9) shows that the optimal rotation length ($T = T_D$) is obtained when the relative marginal value of waiting for one more instant of timber growth minus the discount rate (left-hand side) is equal to the relative marginal cost from the disease spreading and the opportunity cost of land rent (right-hand side). We note that in the absence of disease ($\widetilde{L}(T) = L$) Equation (9) reduces to Equation (3), thus showing that the inclusion of disease will reduce the optimal rotation length, since $\widetilde{L}(T)$ is a decreasing function. Additionally, the land rent is positively affected by disease, suggesting that there is an additional incentive to harvest earlier and start accruing rent from the land use change. In summary, Equation 9) highlights the trade-off between harvesting early and preventing disease progression (and the subsequent reduction in forest value), and not achieving further future timber growth.

Establishing whether the optimum rotation length maximises the NPV in Equation (5) is more difficult. Finding the second derivative we obtain

$$\left.\frac{d^2\hat{J}(T)}{dT^2}\right|_{T=T_D} = \left[pe^{-rT}\left(\widetilde{L}(T)\left(\frac{d^2 f}{dT^2} - r\frac{df}{dT}\right) + 2\frac{d\widetilde{L}}{dT}\frac{df}{dT} + f(T)\left(\frac{d^2\widetilde{L}}{dT^2} - r\frac{d\widetilde{L}}{dT}\right)\right)\right]_{T=T_D}. \tag{10}$$

Clearly the overall sign of the first three terms is negative, the sign of the second last term is uncertain and the last term is positive, therefore it is dependent on the relative magnitude of the terms. Once the disease system is specified, we can show that the optimal rotation length at T_D is always a maximum.

3 A Numerical Model

In order to examine the sensitivity of the optimal rotation length to changes in the disease parameters, we specify the timber volume growth, $f(T)$, and the disease system, $\widetilde{L}(T)$, in the following numerical simulation exercise.

3.1 Timber volume function

In our framework the net benefit at the end of the rotation is dependent on the function describing the volume of timber, $f(T)$. In this paper we use yield class 14 of sitka spruce (*Picea sitchensis*) since it is the most dominant conifer species planted in Scotland (Forestry Commission, 2011). Forest Research's timber growth model produces the average timber volume per tree and number of trees (per hectare) over time allowing us to estimate the average volume per hectare. These date points are shown in Figure 1 (a) where the forest timber volume (V_i) is given for each time step (T_i) with (T_1, V_1) being the first data point recorded once $7 - 10$ cm diameter at breast height (DHB) is reached (tree are generally not harvested before this point). These measurements include natural mortality that is expected of an un-thinned stand.

Using the model output we can fit a curve which has the form

$$f(T) = \begin{cases} 0 & \text{if } T < T_1 \\ V_M \left(1 - e^{\bar{b}(T-T_1)}\right) + V_1 & \text{if } T \geq T_1. \end{cases} \tag{11}$$

We have 185 years of output thus so as to capture the shape of the curve over time we fit parameter $\bar{b}$ by setting $f(200) = V_M$. All parameter values are given in Table 1, and Figure 1(a) shows the data points and fitted curve given by Equation (11).

Despite fitting the timber growth function to one species in this paper, we capture the two key characteristics of timber growth curves which are consistent across species: trees are generally only harvested for timber once they have reached $7 - 10$ cm DHB and the timber volume will saturate after a period of time. Therefore we are reasonably certain that the results shown here will be qualitatively similar and representative of other species.

3.2 Susceptible-Infected disease system

We now reduce the N state epidemiological equations given earlier to a two state, Susceptible-Infected (SI) system with $x(t)$ representing the area of the susceptible forest and $y(t)$ the area of the infected forest. The total area of forest remains constant over time ($L = x(t) + y(t)$), therefore the SI system can be written as

$$\frac{dx}{dt} = -\beta x(t)\left(y(t) + P\right) \tag{12a}$$

$$\frac{dy}{dt} = \beta x(t)\left(y(t) + P\right) \tag{12b}$$

where disease is introduced to an initially susceptible population ($x(0) = L$) by an external pressure P (for example, by infected spores living in the environment), and disease transmission from infected to susceptible trees is controlled by the transmission coefficient β. Since the area of forest is conserved ($dL/dt = dx/dt + dy/dt = 0$) we can eliminate Equation (12b) by setting $y(t) = L - x(t)$. Thus the system reduces to a one state equation

$$\frac{dx}{dt} = -\beta x(t)\left(L - x(t) + P\right) \tag{13}$$

which can be solved using separation of variables method to give

$$x(t) = \frac{L + P}{\frac{P}{L} e^{(L+P)\beta t} + 1}. \tag{14}$$

In the general framework, $\widetilde{L}(t)$ is used to represent the effect of disease on the whole forest (Equation (7)). For the SI system we have

$$\widetilde{L}(t) = x(t) + \rho(L - x(t)) \tag{15}$$

where ρ scales the revenue from timber that is infected ($0 \leq \rho \leq 1$). Setting $\rho = 1$ means the disease has no effect on the timber revenue from infected trees; conversely $\rho = 0$ means the timber from infected trees is worth nothing since $\widetilde{L}(t) = x(t)$.

The disease dynamics in Equation (14) are governed by the transmission coefficient and an externa diseasel pressure. We select six parameter sets (detailed in Table 2) which aim to capture the characteristics of different infections. It may be possible to estimate transmission coefficients from the data, however interpreting and quantifying an appropriate value of external pressure is more difficult. We therefore introduce another parameter $t_{0.5}$ which is the time taken for half the forest to become infected to describe the external pressure. Using Equation (14) we can find this value by setting $x(t_{0.5}) = 0.5L$ giving

$$t_{0.5} = \frac{\ln(L/P + 2)}{(L + P)\beta}. \tag{16}$$

We can equate $t_{0.5}$ to the disease free rotation length, or proportions of it, to allow for an easy interpretation of the variability in external pressure (when the disease transmission is fixed). Figures 1 (b) and (c) show disease progress curves generated for the parameter sets in Table 2. (Note that we also give $t_{0.5}$ for the first set of parameters when P is constant and β is fixed – this was done so as to find appropriate levels of β.)

4 General results

In this section we use the numerical timber growth and disease system defined in section 3 to give further insight into the general results found in section 2. Many of the results cannot be analytically found when disease is included, however we can highlight key trends and qualitative behaviour demonstrating the relationship between the disease characteristics and the optimal rotation length.

4.1 No disease

First we analyse the system without disease to a provide baseline optimal rotation length which can be used to measure the effect of disease on the system. We show the NPV (given in Equation (1)) against time (or age of the forest) in Figure 2 (a) where it is clear that as the forest ages the NPV initially increases (due to the volume of the forest increasing and thus the net benefit from timber) reaches a maximum and then decreases again (due to the slowed growth rate and discounting). The optimal rotation length is the time (or age of the forest) where the maximum NPV is acheived.

We can analytically find the optimal rotation length by substituting the timber growth function (Equation

(11)) in to the first order condition in Equation (3), obtaining

$$\frac{V_M \bar{b} e^{\bar{b}(T-T_1)}}{V_M(1-e^{\bar{b}(T-T_1)})+V_1} - r = \frac{a}{pf(T)} \tag{17}$$

which holds when $T \geq T_1$. Solving for the optimal rotation length, $T = T_{DF}$, we have

$$T_{DF} = \frac{1}{\bar{b}} \ln\left(\frac{a + rp(V_M + V_1)}{pV_M(r - \bar{b})}\right) + T_1. \tag{18}$$

Using the baseline parameters in Table 1, (which sets the land rent to zero) we find $T_{DF} = 39.3$ years. Figure 2 (a) shows that if this time is missed (for example by the forest being harvested early or late) then the maximum NPV will not be achieved. An alternative method of plotting the optimum rotation length (which we use later) is to plot both sides of the first order condition given in Equation (17) so that the optimal rotation length is at the point of intersection. The left-hand side will tend to $+\infty$ as T approaches T_1 from the right, increasing T results in the left-hand side exponentially decrease and crossing the x-axis before tending to $-r$ (the discount rate). The right-hand side of Equation (17) will be zero if there is no land rent (the optimal rotation length will then be where the left-hand side crosses the x-axis), and strictly positive when $a > 0$, therefore showing that the inclusion of land rent reduces the optimal rotation length. Figure 2 (b) shows this relationship as the land rent is increased and highlights that the trajectory of the optimal rotation length will tend towards the lower harvesting boundary as a increases. This can be explained by the land rent providing an additional incentive to felling earlier and receiving these future payments. This is a similar result when considering multiple rotations.

4.2 Disease

We now find the optimal rotation length which maximises the NPV in Equation (5) when the forest growth function is of the form in Equation (11) and the disease follows a susceptible-infected framework in Equation (12). An analytic solution for the optimal rotation length is intractable, therefore we break the system into two scenarios to carry out sensitivity analysis to the parameters controlling the disease progression (by setting $\rho = 0$) in section 4.2.1, and the relative revenue from timber that is infected (by setting $0 \leq \rho \leq 1$) in section 4.2.2. We set the land rent to zero so as to more clearly determine the relative effect of disease and only consider the parameter space greater than the minimum harvesting boundary ($T \geq T_1$).

4.2.1 Sensitivity analysis to the disease characteristics

Setting $\rho = 0$ simplifies the model as the net benefit of the timber at the end of the rotation is dependent on the area of healthy forest only, that is $\widetilde{L}(T) = x(t)$ in Equation (15). Substituting this and the timber growth function (Equation (11)) into the first order condition (Equation (9)), we find

$$\frac{1}{f(T)}\frac{df}{dT} - r = \frac{1}{x(T)}\left|\frac{dx}{dT}\right| \tag{19a}$$

$$\frac{-V_M \bar{b} e^{\bar{b}(T-T_1)}}{V_M(1-e^{\bar{b}(T-T_1)}) + V_1} - r = \frac{P\beta(L+P)}{P + Le^{-(L+P)\beta T}}. \tag{19b}$$

That is the NPV is maximised when the marginal benefit of waiting for one more instant of relative timber growth minus the opportunities forgone (left-hand side) is equal to the relative marginal cost of the disease spreading further (right-hand side). Whilst we are unable to solve this analytically we can gain some insight into the dynamics by treating each side of Equation (19b) separately. The left-hand side of Equation (19b) is the same as the disease free case (Equation (17)): it exponentially decreases as the rotation length is increased crossing the x-axis at $T = T_{DF}$ and tending to $-r$ as $T \to \infty$. The right-hand side of Equation (19b) is always positive and saturates to a maximum of $\beta(L+P)$ as $T \to \infty$, therefore there will be one stationary point – and optimal rotation length – of Equation (9) which can be shown to maximise the NPV.

This can be seen in Figure 3 where we plot the left- and right-hand side of Equation (19b) thus the point of intersection gives the optimal rotation length. In the system without disease the right-hand side is zero and the optimal rotation length is given by T_{DF}. Using this as a benchmark, it is clear that as the disease transmission is increased the rotation length shortens ($T_{DF} \to T_{D1} \to T_{D2} \to T_{D3}$ in Figure 3 (a)). Moreover, the left-hand side of Equation (19b) gives the trajectory of the optimal rotation length as the disease transmission, β, is increased. A similar analysis for the external pressure can be from Figure 3 (b): increasing the external pressure decreases the optimal rotation length. Comparing both figures reveals that a slow transmitting disease with a high external pressure behaves similarly to a fast transmitting disease with low external pressure.

The relationship between the optimal rotation length and the disease transmission is highlighted further in Figure 4 (a): as disease transmission (β) increases, the optimal rotation length shortens and tends to the lower harvesting boundary. This highlights the trade-off in waiting for the timber to grow and waiting for the disease to spread: shortening the rotation length allows timber that is not infected to be sold (despite the trees not reaching full growth potential) and some of the costs to be recouped, whereas waiting allows the disease to spread further and subsequently reduces the timber benefit.

As well as reducing the optimal rotation length, the effect of disease on the maximum NPV can be considerable (Figure 4 (b)). When the progression of disease is such that it spreads throughout the forest by the lower boundary for harvesting ($T = T_1$), no benefit can be made from the timber thus the maximum NPV is equal to the establishment costs. Another key point shown in Figure 4 (b) is that the maximum NPV crosses the x-axis, showing a threshold where the maximum NPV is zero. We can analytically find this disease transmission threshold which gives a zero NPV by setting the cost of establishing the forest equal to the revenue from timber at the end of the rotation. Therefore

$$W(L) = pf(T)x(T)e^{-rT} \tag{20}$$

must hold. Applying the substitutions and re-arranging we obtain

$$\beta^{(0)} = \frac{1}{(L+P)T} \ln\left(\frac{L}{P}\left(\frac{p}{c}(L+P)f(T)e^{-rT} - 1\right)\right) \tag{21}$$

where $\beta = \beta^{(0)}$ is the threshold where the maximum NPV is zero. It is common for estimates of the maximum NPV to drive investment decisions and we show here that the characteristics of disease will impact this. Clearly the economic parameters will also drive the investment and from Equation (21) it is clear that a higher the cost of planting, c, or a lower the price of timber, p, returns a smaller threshold, $\beta^{(0)}$. Similar analysis has been carried out for fixed disease transmission and variable external pressure but was omitted here since this shows qualitatively similar results to the analysis for fixed external pressure and variable disease transmission.

To summarise, when the timber that is infected is worth nothing, disease will shorten the optimal rotation length. The magnitude of this reduction is dependent on the time taken for the infection to spread: increasing the rate of disease progression (through disease transmission or external pressure), decreases the optimal rotation length and maximum NPV. Moreover we have established a value of a critical threshold disease transmission which returns a zero maximum NPV (Equation (21)) which in theory could be used to help derive investment decision if parameters could be estimated.

4.2.2 Sensitivity analysis to the value of timber that is infected

In the first scenario we assumed that $\rho = 0$ which means revenue is from uninfected timber only. For some diseases it is likely that timber which is infected will create some revenue, and in this section we aim is to understand how variation in the relative revenue from timber from infected trees ($0 \leq \rho \leq 1$) can affect the rotation length. Using a similar method to before, we substitute the functions describing the infected forest, $\widetilde{L}(T) = x(T)(1-\rho) + \rho L$, and the timber growth (Equation (11)) into the first-order condition (Equation (9)) and find

$$\frac{1}{f(T)}\frac{df}{dT} - r = \frac{1}{\tilde{L}(T)}\left|\frac{d\tilde{L}(T)}{dT}\right| \tag{22a}$$

$$\frac{-V_M\bar{b}e^{\bar{b}(T-T_1)}}{V_M(1-e^{\bar{b}(T-T_1)}) + V_1} - r = \frac{\beta(P/L)(L+P)^2}{(P/L) + e^{-(L+P)\beta T}}\frac{(1-\rho)}{L + P(1+\rho(e^{(L+P)\beta T} - 1))}. \tag{22b}$$

As before, we cannot analytically find the optimal rotation length, however examining the first order condition in Equation (22b) shows that since the right-hand side will remain positive creating one stationary point (and plotting the NPV shows that it is a maximum). We therefore use numerical optimisation techniques to plot the optimal rotation length against the disease transmission coefficient, β, and relative revenue from timber that is infected, ρ, in Figure 5 (a). When $\rho = 1$, the optimal rotation length remains at T_{DF} as the disease transmission is increased since Equation (22b) is equivalent to the system without disease. At the other end of the parameter space when $\rho = 0$ the system reverts to the scenario presented in section 4.2.1 ($\widetilde{L}(T) = x(T)$ in Equation (22b)) where the optimal rotation length decreases to T_1 as the disease transmission increases.

When the relative revenue from timber is not at the extremes ($0 < \rho < 1$) the effect on the optimal rotation length is more intricate. If the disease transmission is fixed, then decreasing ρ (a vertical transect in Figure 5 (a)) will decrease the optimal rotation length. The rate of decrease is largely dependent on the rate of disease progress throughout the forest. For a slow transmitting disease (say $\beta = 0.01$) a large proportion of the forest remains healthy (since the disease progress is slow) and is therefore not subjected to the reduced value in timber. In this case the disease has little effect on the NPV (Figure 5 (b)) and subsequently the optimal rotation length (Figure 5 (a)). As the transmission is increased the forest becomes infected more quickly (for $\beta = 0.1$ roughly 50% of the forest is infected by year 20) therefore the value of timber that is infected has a larger effect on the NPV (Figure 5 (b)) resulting in the optimal rotation length decreasing from $T_D = T_{DF}$ when $\rho = 1$ to $T_D \approx 20$ years when $\rho = 0$. Shortening the optimal rotation length has the benefit of salvaging more healthy timber but has a trade-off in loss of volume. The effect on the NPV can be substantial (Figure 5 (b)).

It is interesting to note that the decrease in optimal rotation length is not linear with the decrease in relative revenue from timber that is infected. This is particularly clear when the disease transmission is increased further: certain values of ρ trigger a sudden switch between the disease free rotation length and much shorter rotation length near T_1. This occurs for fast transmitting diseases because the disease progress is so quick that most of the forest is infected by the minimum harvesting boundary (T_1) thus if the revenue from timber that is infected is sufficient then there is benefit in waiting for the timber to grow (since the effect of disease progress will be relatively small); but if the revenue from timber that is infected is small there is a benefit to harvesting early and salvaging any healthy timber (since timber that is infected is worth very little). For example when $\beta = 0.2$ roughly 95% of the forest is infected by year 20 (Figure 5 (a)), thus there is a benefit on waiting for the timber to grow for most of the ρ parameter space; this allows the disease to progress throughout the rest of the forest (only 5%), but the timber can grow in volume. However when the effect on timber is great (small values of ρ), it is always beneficial to harvest sooner salvaging the heathy timber, since once the timber is infected it is worth very little.

Figure 5 highlights the importance of examining the trade-offs between the ecological, economic and epidemiological drivers; more specifically waiting for the timber to grow and for the disease to spread further. Similar analysis has been carried out for fixed disease transmission and variable external pressure but has been omitted here since they showed qualitatively similar results to the analysis for fixed external pressure and variable disease transmission.

5 Effects of including a "treatment" option

In a standing forest there are limited control methods other than felling which can be applied to prevent the spread of disease or reduce damage caused. One control, which is sometimes available, is the use of chemical or biochemical spray which acts as a protectant and reduces the likelihood of infection occurring. For example a chemical stump treatment is commonly applied to some conifers to protect against Heterobasidion. Another example is the use of copper-based fungicides in some nurseries to protect pines from Dothistroma needle blight. We use these examples as a motivation to explore a hypothetical scenario where a chemical control is available and acts to reduce the disease incidence and/or effect on the timber. The aim of this section is to provide insight into the combined effect of the control and disease on the rotation length; however we do not give in-depth analysis of any particular system.

The purpose of applying the control is either to reduce the effect of disease on the timber or to reduce the spread of disease. First consider that the control reduces the effect of disease on the timber so that when trees become infected they do not experience the same decay in quality or growth as unprotected trees. This means that the control increases the relative revenue of timber from infected trees, which we have already found increases the optimal rotation length and maximum NPV (by exploring the sensitivity to parameter ρ). Second consider that the control reduces the risk of infection; again we know this will increase the optimal rotation length and maximum NPV (by exploring the sensitivity to parameter β and P). However there is now a cost of applying the control which will give rise to a trade-off between the cost of spraying and the benefit (revenue) from the timber which has treated, thus predicting the net affect on the optimal rotation length and maximum NPV of incorporating control and disease is not so straight forward.

To analyse this we extend the NPV given in Equation (5) to consider a forest which is sprayed annually at a cost of $D(L)$ (which is linearly dependent on the area of the forest). Assuming the control occurs throughout the entire rotation, the NPV (with the land rent set to zero) can be written as

$$\hat{J}(T) = -W(L) + M(\widetilde{L}_C(T), T)e^{-rT} - \int_0^T D(L)e^{-rt}\, dt \tag{23}$$

where the effect of disease – and the control – on the whole forest can be written as $\widetilde{L}_C(T)$ (compared to $\widetilde{L}(T)$ in Equation (7) which represents the diseased forest without control). Again we can simplfy the general disease framework by using a SI system in Equations (12) and represent the effect of the control by applying an appropriate scaling to some key parameters dependent on whether the control is reducing the effect of disease on the timber (ρ) or the risk of infection (β and P). Solving the NPV in Equation (23) subject to the disease system we obtain the first order condition

$$\frac{1}{f(T)}\frac{df}{dT} - r = \frac{1}{\widetilde{L}_C(T)}\left(\left|\frac{d\widetilde{L}_C(T)}{dT}\right| + \frac{D(L)}{pf(T)}\right). \tag{24}$$

where the optimal rotation length is a balance of the relative marginal benefit obtained from waiting for one more instant of timber growth minus the discount rate (left-hand side) and the relative marginal cost of the disease spreading further and relative cost of distributing control.

Suppose that the control is completely efficacious (prevents the arrival of disease and/or reduces symptoms so that there is no difference between infected or susceptible timber), then $\widetilde{L}_C(T) = L$ since the disease has no effect on the forest. The cost of applying the control will reduce the optimal rotation length and maximum NPV compared to the disease free system (it will act similar to an increase in discount rate, Equation (24)). However when compared to the system with disease but without control, the optimal rotation length and maximum NPV will be increased if the cost of applying a control is less than the cost of disease spreading further (i.e. if $|d\widetilde{L}(T)/dt| > D(L)/pf(T)L$). This occurs when the benefit of applying the control is greater than the cost of infected trees (i.e. a sufficient number of trees are protected and can be sold at full price).

Now suppose that the control is not fully efficacious: it reduces the spread of infection or increases the revenue from timber that is infected, so that $\widetilde{L}_C(T) > \widetilde{L}(T)$ and $d\widetilde{L}_C(T)/dT < d\widetilde{L}(T)/dT$ but $L > \widetilde{L}_C(T)$. The cost of disease in Equation (24) is now non-zero which means that compared to the system without disease (or to the system with disease and a completely efficacious control which costs the same), the optimal rotation length and maximum NPV will be decreased. When comparing this to the system with disease and without a control the overall effect is dependent on the relative magnitude of the terms: if the net benefit of the control (cost of spraying and the revenue gained) is greater than the cost of disease without control, then the optimal rotation length will be increased compared to the system with disease and without control. Mathematically that is

$$\frac{1}{\widetilde{L}_C(T)}\left(\left|\frac{d\widetilde{L}_C(T)}{dT}\right| + \frac{D(L)}{pf(T)}\right) > \frac{1}{\widetilde{L}(T)}\left|\frac{d\widetilde{L}(T)}{dT}\right|. \tag{25}$$

Alternatively the optimal rotation length will be decreased when the cost of treatment does not outweigh the benefits.

In this section we have briefly highlighted that the inclusion of an annual control which acts to reduce the effect of disease on the timber will reduce the optimal rotation length compared to the disease free system. The effect, when compared to the system with disease but without control, is dependent on the relative cost and benefit of the control.

6 Discussion

Forest management strategies are used to help promote the health and growth of forests which in turn play a vital role in maximising the value of such investments. However tree diseases are threatening the health and profitability of forests worldwide (Department for Environment, Food and Rural Affairs, 2013b). The removal of timber not only impacts the forest owner through revenue loss, but also through the cost of general disease management such as inspections, monitoring and additional licensing. It is therefore of much interest to analyse management strategies of forests threatened by disease.

In this paper we address the management strategy of clear felling, more specifically answering the question: when is the optimal time to harvest a forest when disease arrives during the rotation? This is a key strategy since there is often little that can be done to reduce the effect of disease on a standing forest thus felling is considered a main control. Optimal rotation length analysis has been an important concept in forestry economics since Martin Faustmann's original contribution (Newman, 2002), however at the time of writing, no articles could be found by the authors which address this problem in the context of disease. To answer this question, we construct a model similar to an optimal control problem by combining a function to describe the net present value (NPV) of a single-aged, single rotation plantation forest with a system of differential equations to describe the disease dynamics over time. This framework incorporates the ecological, epidemiological and economic factors which are vital to finding the optimal rotation length. We assume that the net benefit obtained from the timber is dependent on the infection state of the forest at the end of the rotation (when the trees are felled).

We first analysed the NPV for a general disease system and found the optimal rotation length is obtained when the relative marginal benefit of waiting for one more instant of timber growth is equal to the relative marginal cost from the disease spreading further and the cost of opportunities forgone. Thus a key result is that the inclusion of a disease, which reduces the value of timber from infected trees, is likely to reduce the optimal rotation length when compared to the disease free system (section 2.1). We specify a timber growth function and susceptible-infected (SI) disease system which permits sensitivity analysis of the parameters controlling the disease transmission, external pressure of disease and the relatively revenue obtained from timber of infected trees (section 3). We found that when only healthy timber can be sold, an increase in the disease transmission and/or external pressure negatively effects the rotation length: the faster the disease progresses the shorter the optimum rotation length will be (section 4.2.1). This case would apply to diseases where the timber, once infected, cannot be sold due to poor quality, significantly reduced volume, or requires a complex sawmilling not permitting it to be processed.

Disease can, however, have variable effects on the value of timber, and in section 4.2.2 we analysed the sensitivity of the optimal rotation length to the relative revenue obtained from timber that is infected. We found an interesting trade-off between the speed of disease progression and the relative revenue obtained from timber once infected (ρ). For slow transmitting diseases the reducing the revenue from timber that is infected reduces the optimal rotation length since there is a benefit to harvesting more healthy timber. However, the magnitude of change is small, since only a proportion of the forest is infected. Increasing the disease transmission means that a higher proportion of the forest will be infected, thus the effect of the reduced value holds greater weight. One may think that this means the overall effect would be to reduce the optimal rotation length dramatically to salvage as much healthy timber as possible, however we found that in some circumstances waiting may be beneficial: if the disease spreads such that most of the forest is infected by the lower harvesting boundary then there is a benefit to waiting for the timber to grow more (since most of the forest will be subjected to the reduced revenue even if harvested as soon as possible). However when the effect on the timber is very large, it is always better to reduce the optimal rotation length to salvage healthy timber.

This result has interesting implications for large scale epidemics where a significant amount of timber of a single species is being felled within a short time period in a bid to control the outbreak. Whilst this may have the benefit of preventing disease to spread further, it can have a negative effect on the timber market price since there is an large increase in supply. Whilst we do not explicitly model this scenario, our results suggest that when considering a single forest there is a benefit to waiting to harvest at the disease free optimal rotation length (for fast transmitting diseases) and this may perhaps buy time for the timber market to recover.

In section 5 we extend the model to include a control which is applied annually and reduces the disease incidence and/or the effect of disease on the timber. We found that the cost of treatment will reduce the optimal rotation length (and maximum NPV) compared to a disease free case, however a trade-off between the cost (application) and benefit (revenue saved from having less infected timber) of disease prevention arises. The optimal rotation length will be increased compared to the system with disease (and without control) if the net benefit of the control is greater than the cost of disease without control.

In this paper we make some assumptions to help simplify the problem, some of these include: fixed price of timber, the exclusion of multiple rotations, and timber is the only benefit produced by forests. In reality the price per volume of timber is dependent on the volume: smaller volumes are not worth as much as larger (due to the restrictive utility of smaller timber). In this paper we assume a fixed price; however the inclusion of variable price can be achieved using the Timber Price Indices data produced by the Forestry Commission: we have excluded this from the paper for simplicity. Additionally it would be interesting to examine the effect of a declining price for infected timber; this would incorporate the effect of a disease which reduces the value of timber over time (through decreased growth rate or quality). Earlier we discussed that the omission of multiple rotations from the model has been shown to give errors in calculating the optimal rotation length since the value of the land has been ignored (Amacher et al., 2009). Whilst we omit multiple rotation analysis in the traditional sense (calculating the NPV over infinite forest rotations), we show that a land rent can be included in our model by means of an annual payment after the rotation – this can represent changing the land use or changing the tree species planted which can sometimes be necessary after an epidemic. In section 2 we show that the land rent payment negatively effects the optimal rotation length (Equation (9)), thus agreeing with previous analysis that exclusion of land rent reduces the economic pressure to harvest sooner. (Showing the effect of disease on multiple rotations is beyond the scope of this paper since the dynamics of the disease would have to be considered carefully.)

Lastly, we have considered a forest where the owner is driven purely by timber values. Clearly, forests produce a range of non-market benefits such recreation, carbon sequestration and ecosystem services, and omitting such benefits can lead to an incorrect deduction of the optimal rotation length (Hartman, 1976; Samuelson, 1976). Since this is an important area we extend the framework presented here to analyse the optimal rotation age of a single-aged, forest in the presence of disease when non-timber benefits are considered in Macpherson et al. (015b).

This paper presents a theoretical model with the aim of understanding how disease can influence the optimal rotation length when an individual forest owner is maximising their investment. These results are important not only because of the continual arrival of novel pest and pathogens and the importance of plantation forestry to investors, but they highlight the question of geographical scale. A single forest managed optimally under these constraints shortens the rotation length which will have additional positive effects since a potential source of infection will be removed earlier. In turn this may impact on surrounding forests. However for some diseases (very fast transmitting) the optimal solution is to not change the time of harvesting which may promote disease spread to neighbouring forests. An interesting extension to this framework would be to consider this problem in multiple connected forests where disease can spread through a landscape. This would allow different strategies to be analysed in response to a disease outbreak at a landscape scale.

In summary, our model has shown that the inclusion of a disease, which reduces the value of timber that is infected, can reduce the optimal rotation length of a production forest. This result has important implications for the production forestry sector since loses are predicted if disease is not considered and the forest is harvested at the disease-free optimal rotation length.

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Parameter	Definition	Baseline value
L	Area of forest	$L = 1$ ha
c	Forest establishment cost	$c =$£1920 ha^{-1} **
p	Price of timber	$p =$£17.90 m^{-3} ***
r	Discount rate	$r = 0.03$
a	Annual payment after rotation	£0 ha^{-1}
$f(T)$	Timber volume growth (m^3 ha^{-1})	Equation (11)
(T_i, V_i)	Time, T_i (years), and volume, V_i, (m^3 ha^{-1}) from Forest Growth model *	$(T_i, V_i) = (15, 43)$
$\bar{b}$	Fitted parameter in timber growth function $f(T)$	$\bar{b} = -0.01933$ *
$\widetilde{L}(T)$	Impact of disease on the whole forest at time T	Equation (7)
β	Disease transmission coefficient	Table 2
P	External pressure from disease	Table 2
$t_{0.5}$	Time taken for the susceptible area to half	Table 2
ρ	Relative revenue of timber that is infected	$0 \leq \rho \leq 1$

Table 1: The parameter definitions and baseline values used in this paper. Parameters marked * denote values taken from Forest Growth model used by Forestry Commission for yield class 14 of Sitka spruce without thinning and 2m initial spacing. The cost of planting (marked **) is taken to be the same as the planting grant for Sitka spruce (`https://www.ruralpayments.org/publicsite/futures/topics/all-schemes/forestry-grant-scheme/woodland-creation/`). The price of timber (marked ***) is the average standing price (per cubic metre overbark) taken from Coniferous Standing Sales Price Index on 30th September 2014 for Great Britain (`http://www.forestry.gov.uk/forestry/INFD-7M2DJR`).

Disease dynamics (transmission – external pressure)	β	P	$t_{0.5}$
Fast – high	0.1	0.16*	$t_{0.5} = T_{DF}/2$
Medium – high	0.044	0.16	$t_{0.5} = T_{DF}$
Slow – high	0.022	0.16	$t_{0.5} = 2T_{DF}$
Fast – high	0.1*	0.16	$t_{0.5} = T_{DF}/2$
Fast – moderate	0.1	0.019	$t_{0.5} = T_{DF}$
Fast – low	0.1	0.0003	$t_{0.5} = 2T_{DF}$

Table 2: A list of the disease parameter sets with the disease coefficient β (labelled 'fast, medium and slow'), and external pressure P (labelled 'high, moderate and low'). $t_{0.5}$ is the time taken (in years) for the infection to spread to half of the forest as described in Equation (16). Disease progress curves for each parameter set are shown in Figure 1 (b) and (c). T_{DF} is the optimal rotation length in absence of disease and subsidies. * denotes the baseline value for the external pressure and disease transmission.

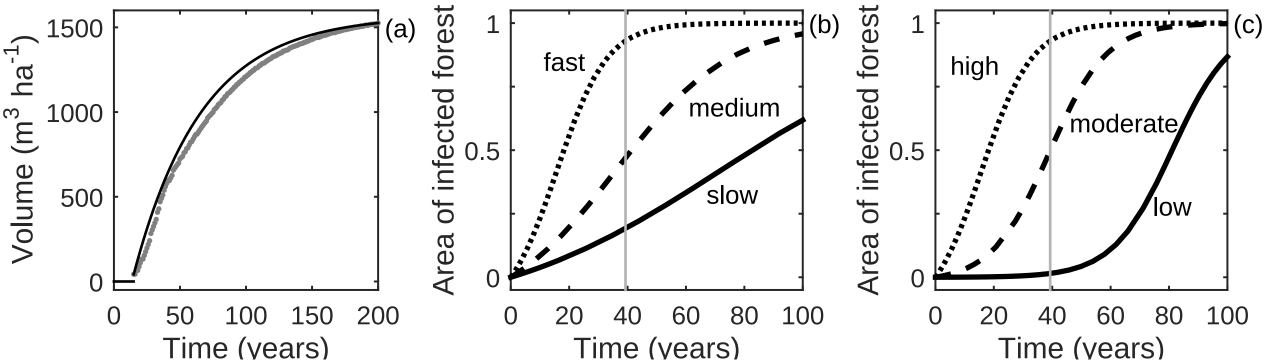

Figure 1: The (a) forest growth function and (b)-(c) disease progress curves. In (a) the data points (grey dots) from the Forest Growth model are given for unthinned, yield class 14 of Sitka spruce (*Picea sitchensis*). The fitted curve (black) is produced using Equation (11) and the parameters are in Table 1. The area of infected forest $(L - x(t))$ against time is plotted for (b) fixed external pressure and three values of disease transmission and (c) fixed disease transmission and three values of external pressure (the parameter sets are in Table 2). The optimal rotation length of the disease free system, T_{DF}, is shown as vertical, grey line.

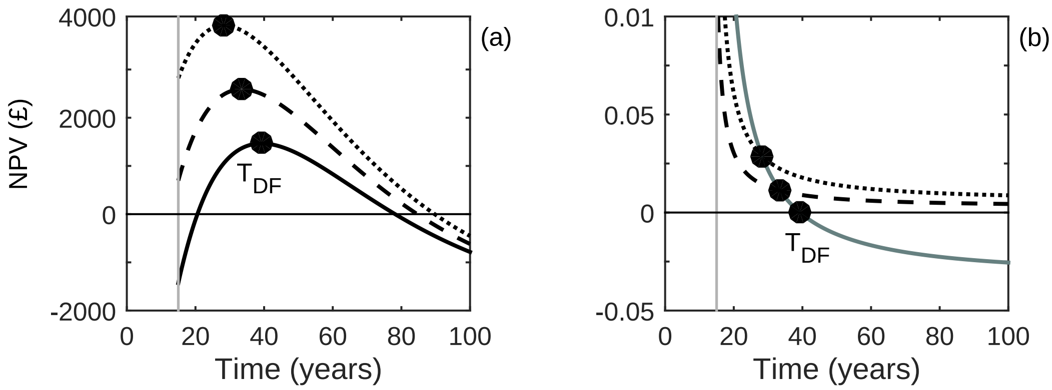

Figure 2: The effect of future land rent on the optimal rotation length. The sensitivity analysis of optimal rotation length ($T = T_{DF}$) to the land rent, a (in £ ha^{-1} year^{-1}). The optimal rotation length maximises the NPV (Equation (1)) in (a) for $a = 0$ (solid black) $a = 100$ (dashed black), and (c) $a = 200$. Alternatively plotting both sides of the first order condition in Equation (17) in (b) where the left-hand side (grey) and the right-hand side for $a = 0$ (thin black), $a = 100$ (dashed black) and $a = 200$ (dotted black) . (The left-hand side is unchanged by the land rent.) The black circle indicates the optimal rotation length for each parameter set, and T_{DF} marks the optimal rotation length when $a = 0$ (baseline parameter). In all plots the growth function is parameterised for yield class 14 of sitka spruce where T_1 is given by the vertical grey line in (a) and (b). Other parameters can be found in Table 1.

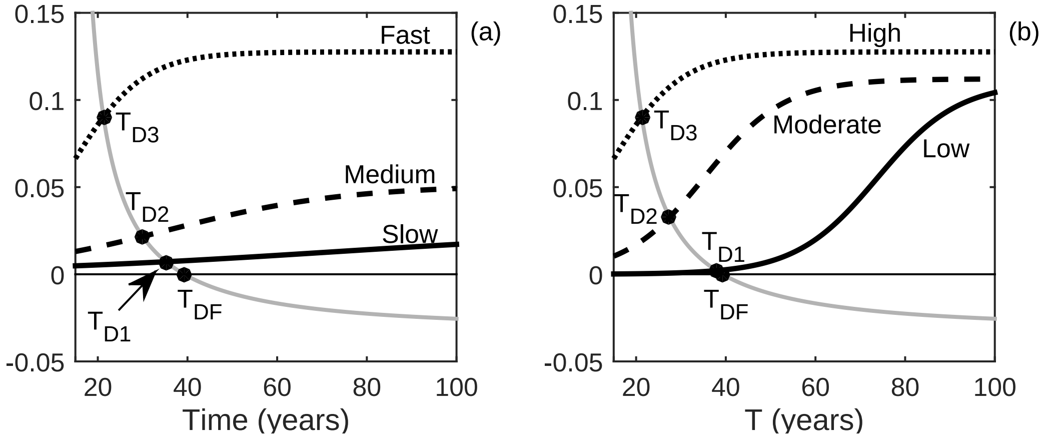

Figure 3: The effect of disease transmission rate and external disease presssure on the optimal rotation length. The first order condition in Equation (19b with the left- (grey) and right-hand side (black). In (a) the external pressure is at the baseline value and the disease transmission is slow (solid), medium (dashed) and high (dotted). In (b) the disease transmission coefficient is at the baseline value and the external pressure is low (solid), moderate (dashed) and high (dotted). The optimal rotation length is given at the intersection of these two curves: for the system without disease it is identified by T_{DF} and with disease it is identified by T_{Di} (where $i = 1$ is slow transmission/low external pressure, $i = 2$ is medium transmission/moderate external pressure and $i = 3$ is fast transmission/high external pressure in (a)/(b) respectively). Economic and ecological parameters are in Table 1 and disease parameters are in Table 2 and .

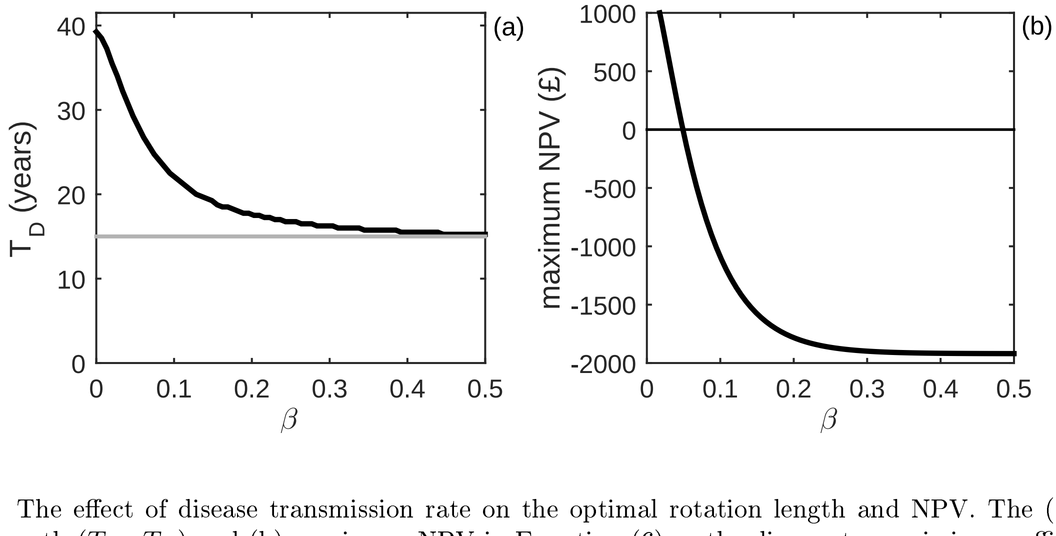

Figure 4: The effect of disease transmission rate on the optimal rotation length and NPV. The (a) optimal rotation length ($T = T_D$) and (b) maximum NPV in Equation (6) as the disease transmission coefficient, β, is varied (with $\rho = 0$ and $a = 0$). The lower harvesting boundary (T_1) is in grey and the external pressure is at the baseline value (Table 2. Economic and ecological parameters can be found in Table 1.

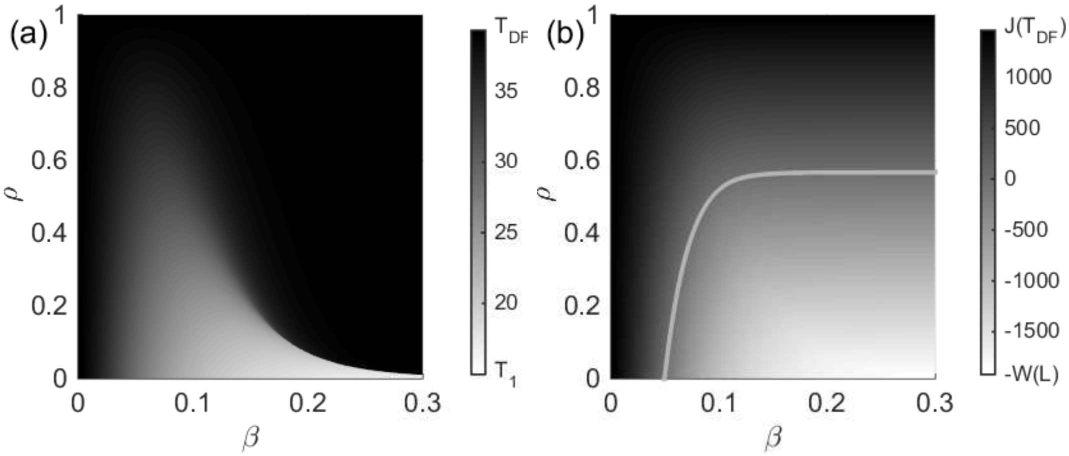

Figure 5: The effect of varying the value of timber that is infected on the optimal rotation length. The (a) optimal rotation length and (b) maximum NPV are plotted against the disease transmission coefficient, β, and relative revenue from timber that is infected, ρ. The colourmap on the right-hand side of both plots indicate the optimal rotation length (in years) and maximum NPV (in £) respectively. The grey curve in (b) highlights where the NPV is zero. The external pressure is at the baseline value (Table 2) and economic and ecological parameters can be found in Table **??**.