Sketching Graphs

To sketch a graph properly we must ensure that all the important features of the graph are shown. The usual information to include is: where the graph crosses the x-axis, where the graph crosses the y-axis,the behaviour at infinity, and any turning points.

Suppose we want to sketch the graph of a function $y = f(x)$. How do we find the relevant information?

1. To find where our graph crosses the x-axis we set $f(x) = 0$ and solve for x.

2. To find where our graph crosses the x-axis we substitute $x = 0$ into our function.

3. To find the behaviour as x tends to plus (respectively minus) infinity, we consider how our function behaves when we substitute in very large positive (respectively negative) values of x. We must also consider if there are any asymptotes, where y tends to plus or minus infinity.

4. To find the turning points we first must differentiate our function to get $f'(x)$. We then solve $f'(x) = 0$ to get the x-coordinate of the turning point. Then we may substitute this value of x back into our original function to get the y coordinate. We can differentiate again to determine whether this is a maximum or a minimum (though often this can be infered from the rest of the information we already have). We find $f''(x)$ and substitute in the x-coordinate of the turning point. If this is positive we have a minimum, if it is negative we have a maximum, and if it is 0 we have a point of inflection.

An Example

Let $y = (x + 1)(x - 2)$, so $f(x) = (x + 1)(x - 2)$. Then:

1. Setting $f(x) = 0$ gives us $(x + 1)(x - 2) = 0$, and so $x = -1$ or $x = 2$. These are the two points where we cross the x-axis.

2. Setting $x = 0$ gives us $y = (0 + 1)(0 - 2) = -2$. This gives us the point where we cross the y-axis.

3. As x gets large and positive, the 1 and -2 become negligable and so the behaviour is like that of x^2. Thus y also gets very large and positive. Similarly,as x gets large and negative, the 1 and -2 become negligable

and so the behaviour is like that of x^2. Thus y also gets very large and negative.

4. We first differentate to get that $f'(x) = 2x - 1$. Setting this equal to 0 gives that $x = 1/2$. Now substituting this back into $y = (x+1)(x-2)$ gives us that $y = -9/4$. Thus our turning point is at $(1/2, -9/2)$. As this is the only turning point, and we know that we get very large as x tends to infinity, this must be a minimum

This gives us enough information to be able to sketch the graph.