1. Background: Our Most Puzzling Faculty

We use ‘imagining’ for a range of activities, from daydreaming to hallucinating and free mind-wandering. WIKI focuses on the imagination at work in suppositional thought. What was the evolutionary advantage for our ancestors, busy with very real problems – feeding, mating –, of a mental faculty allowing them to fantasize away from reality? Say you are chasing a prey that’s on the other side of a stream. You wonder: ‘What if I jumped the stream? Would I make it to the other side?’ (Williamson.2007). It may be dangerous to recklessly try. So you imagine jumping and try to guess the outcome. We often address hypothetical questions by imagining a situation where some A – call it our suppositional input – obtains (I jump the river), wondering how likely some B (I make it to the other side) would be. The use of ‘if’ in such questions links imagination to subjunctive/counterfactual conditionals (‘If A were/had been the case, then B would be/have been the case’): we often assess counterfactuals by imagining a situation where the antecedent A is true and estimating the resulting chances of the consequent B (Williamson.2020). Thus, this is called counterfactual imagination (Byrne.2005), an activity of momentous importance: we use it to track causal relations as well as abductive-explanatory connections (Douven.2022). WIKI is to address three core issues in this area.

Problem 1: the ‘Puzzle of Imaginative Use’. This is the fundamental epistemological problem concerning imagination (Kind&Kung.2016): how can imagination give us knowledge of reality, if it is free-floating deviation from reality? Various authors (e.g. Currie&Ravenscroft.2002) propose a simulationist view: imagination creates counterparts of non-imaginative states. Propositional imagination (imagining that A) is often taken as simulating belief (Arcangeli.2019). But it’s generally agreed that imagination is arbitrary, as it’s subject to voluntary control in ways belief is not (Walton.1990, Gendler.2000, Dorsch.2012, Wansing.2017): one can’t decide to believe that Oxford has been painted yellow; but one can easily imagine it. We may imagine situations known to be impossible (Jago.2014). What is the point of a simulation differing from what it simulates in such a core feature? There is consensus that that imagination may have epistemic value as constrained by a principle of minimal alteration of how we take reality to be, compatibly with the accommodation of the suppositional input (Langland-Hassan.2016, Williamson.2016, Kind.2016). Having supposed that we jump the stream, we don’t imagine flying with wings above it. We keep our physical makeup unchanged in the scenario: ‘we use our knowledge of how the world works to keep the imagination realistic.’ (Davies.2019:187) But in the philosophy of imagination there is no formally precise, normative account of how exactly such minimal alteration ought to work. So we don’t quite know under which conditions it is a rational exercise.

Problem 2: Scientific Thought Experiments. Thought experiments – from Galileo’s falling bodies to Newton’s bucket and Einstein chasing a light beam – have momentous importance in the development of science; but their workings are highly controversial (Sorensen.1992, Häggqvist.1996, Brendel.2004, Stuart.2018, Levy&Godfrey-Smith.2020). Some claim (Brown.2011) that they elicit a priori intellectual intuitions and essentially involve mental imagery (see also Kind.2001); others, that they are only arguments (Norton.1996); others, that they are tools of reconceptualization (Gendler.2000); others, that they involve the manipulation of mental models (Nersessian.1993). A promising idea on thought experiments, found in Williamson.2007, takes them precisely as exercises of counterfactual imagination. This would make of them nothing mysterious: an instance of our general capacity for counterfactual suppositional thought. But then, we lack an agreed-upon treatment of thought experiments to the extent that we lack a formally precise account linking them to that general capacity.

Problem 3: The Traps of Imagination. Reasoning fallacies like the Belief Bias (Markovits&Nantel.1989) or the Suppression Task (Byrne.1989) have variants in terms of suppositional thinking: for most categorical fallacies one can build counterpart hypothetical ones. E.g., the famous Conjunction Fallacy (judging a conjunction A&B more likely than a conjunct A: Tversky&Kahneman.1983) has a suppositional conjunctive variant (judging A&B more likely than A under the supposition that C). There is extensive literature in psychology, on how people adjust (or, fail to) their beliefs under supposition (Byrne.2005; Davies.2019). But when is an exercise of suppositional thinking irrational? To answer the question, we need to know exactly what rationality prescriptions it does not conform to. Insofar as it’s descriptive of what people generally do in reasoning tasks, the psychology of reasoning does not give straightforward answers, which would have to do, rather, with rational normativity. The descriptive answers may be compatible with people generally not living up to
the standards of optimal suppositional thought, whatever these may be. We lack a formally precise normative account to work as our yardstick.

2. Core idea: A Formal Epistemology of Imagination

WIKI will for the first time apply systematically in the philosophy of imagination tools from formal epistemology, in particular epistemic and probabilistic logic. If we start by taking counterfactual imagination as a kind of simulated belief revision, we can then attempt to analyze it using formal theories of belief revision. In qualitative (focusing on full, all-or-nothing belief) and quantitative (focusing on credences/degrees of belief), these have been successful from computer science to decision theory and economics (Alchourrón et al.1986, Baltag&Smeets.2008). They are increasingly popular in philosophy (Pettigrew&Weisberg.2019) but have not yet been applied extensively in the philosophy of imagination. WIKI will give a mathematically precise model of the workings of counterfactual imagination and use it to address the three aforementioned problems.

WIKI will need, however, to venture beyond mainstream theories of belief revision. That's because, as spotted by some (Joyce.1999, Eva et al.2022), counterfactual imagination is not apply modelled by Bayesian conditionalization (in which we fix our subjective probabilities for B under the hypothesis that A as P(B|A): the probability of B conditional on A). Such a procedure may be suitable to model indicative supposition (imagining what is the case if A is the case). However, counterfactual supposition (imagining what would have been the case if A were/had been the case) differs sharply from it. In the former case, we imagine receiving new information about the actual world; in the latter, we imagine that the world differs from how we know or believe it to be. Take the famous indicative/counterfactual pair from Lewis.1973: (I) 'If Oswald didn't kill Kennedy, then someone else did.' (C) 'If Oswald hadn't killed Kennedy, someone else would have.' In the former case, we imagine that we learn new information on the actual world: Oswald didn't really kill Kennedy. We retain our belief that Kennedy was killed. So it must have been someone else. Hence we accept (I). But in the latter, we imagine that things have gone otherwise than they actually have. In the counterfactual scenario, we may find it plausible that nobody else kills Kennedy. Hence we reject (C). As argued by eminent AI scientists (Pearl.2013) as well as philosophers (Williamson.2020), it's the latter kind of imagination that we use in abductive reasoning and to track causal connections: we assess what would happen if we hypothetically intervened in reality and made something happen, rather than merely recording that it happens. We thus need a procedure different from conditionalization to model it.

WIKI will adopt one due to Lewis.1976 and called imaging. Let's use the notation ‘P^A(B)’ for the probability of B under imaging on A. Imaging is minimal revision in a different sense from conditionalization.1 Technically, it works thus: take a finite set of possible worlds W (ways things could or have been), ordered by closeness or similarity, as in the Lewis.1973 standard semantics for counterfactuals. (world w_i is closer to w_j than w_k when w_i is more similar in the relevant respects to w_j than w_k.) Subjective probabilities are distributed among the worlds so as to add up to 1. That, e.g., the probability of a sentence is the sum of the probabilities of the worlds where it is true. Imaging on A switches probabilities from each world w where A is not true to the worlds closest to w, where A is. Each world where A is already true retains its probability and may gain probability from the worlds where it's not. Probabilities are only moved around, so if P is a probability distribution, P^A is, too. Imaging models exactly the effect of the counterfactual supposition of A on our degrees of belief. It captures in a precise way the idea of minimal, reality-oriented simulated revision of beliefs under a counterfactual supposition: when we image on A, we hypothetically revise by looking at the likelihood of situations which are closest or most similar to how we take reality to be, but where A is true.

Once rephrased in precise imaging-probabilistic terms, the Puzzle of Imaginative Use for counterfactual imagination becomes: Under which conditions are we rationally justified in believing B to the extent that we deem it likely under the counterfactual supposition, that is, the image, of A? WIKI will answer by

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1 ‘Imaging [probability distribution] P on A gives a minimal revision in this sense: unlike all other revisions of P to make A certain, it involves no gratuitous movement of probability from worlds to dissimilar worlds. Conditionalizing P on A gives a minimal revision in this different sense: unlike all other revisions of P to make A certain, it does not distort the profile or probability ratios, equalities, and inequalities among sentences that imply A.’ (Lewis.1976: 311)
exploiting a result from 'accuracy-first' epistemology (Pettigrew.2016): the view that accuracy, i.e., closeness to the truth, is the key epistemic virtue of representational mental states. The result, originally due to Hannes Leitgeb with Richard Pettigrew (Leitgeb&Pettigrew.2010), but never applied to a formal treatment of counterfactual imagination, says: belief revision via imaging\(^2\) maximizes the expected accuracy of our degrees of belief, when this is measured by the Brier or quadratic loss function score, the most used mathematical measure of the (in)accuracy of a probability distribution. This was a mouthful, but it means: when we suppose counterfactually that A, and hypothetically adjust our degrees of belief by imaging on A, our belief system is hypothetically updated so that it’s expected to be as accurate as it is mathematically possible, by the lights of our current probability distribution. This gives an answer to the Puzzle of Imaginative Use for counterfactual imagination: qua governed by imaging, it can be rationally justified as it’s expected to give us hypothetical beliefs that are as accurate as possible.

3. Objectives, Sub-Projects

**Subproject 1: Building a Formal Logic of Imaging. [PA, Postdoc 1]** The key idea of Subproject 1, carried out with the help of WIKI’s Postdoc 1, a mathematical logician, is that we can build a formal logic representing how imagination, understood via imaging, works: a probabilistic-epistemic logic of imaging for counterfactuals, ‘If it were/had been the case that A, then it would be/have been the case that B’, and corresponding counterfactual suppositional expressions, ‘Counterfactually supposing A, one imagines that B would be the case’, analyzed in terms of an assignment of probability to B under the image of A. I have expertise in epistemic logics for suppositional belief operators (Berto.2022) and in probability logics for conditionals (Berto&Özgün.2022), including completeness proofs. However, the logic envisaged as the main outcome of Subproject 1 will require innovative logical work – hence the need for a postdoc in mathematical logic, enrolled from the start of WIKI: while there’s a rich literature on probabilistic logics (Halpern.2005) and combinations of epistemic logic operators with probabilities (Kooi.2003, Van Benthem.2011), such accounts only use plain or conditional probabilities, not imaging-probabilities. An imaging-based probabilistic-epistemic logic will take us into uncharted logical territory.\(^3\)

**Subproject 2: The Logic of Scientific Thought Experiments. [PA, Postdoc 1]** The new logic of imagination, which will be the outcome of Subproject 1, can then be used to answer questions on the nature of thought experiments. The philosophical assumption here, taken from Williamson.2007, is that thought experiments are, in a way, nothing mysterious: they are a special application of our general capacity to engage in counterfactual imagination. If great scientists are better at them than the rest of us, it can be because they come up with better suppositions (more original; of better explanatory value; etc.); or because, given proper scientific training, they have better beliefs to hypothetically revise (Stuart.2022a-b). Our logic will then give natural and often conciliatory replies to open questions under the rubric of Problem 2. E.g., yes: thought experiments are arguments; but they are arguments couched in a probabilistic non-monotonic epistemic logic. Intuitively: supposing that A (that Tweety was a bird) we may conclude that B (that Tweety would fly) or increase the likelihood of B under the supposition. But these will be defeasible conclusions: supposing more, namely that A&C (that Tweety was a penguin bird), we may retract B, or revise its probability downwards. I conjecture that our logic will be able to model abductive arguments (Douven.2022) going beyond the rigid dichotomy of the old Brown-Norton debate on thought experiments, which mostly considered just deductive and inductive arguments. And yes, thought experiments can involve the Nersessian idea of manipulation of mental models. But we can say how exactly they are manipulated by looking at model-change operations encoded in our logics – imaging-probability changes, dynamic model-transformers: ‘After changing our model by imaging on A, B is the case’. I have expertise with model-changing operators in epistemic logic

\(^2\) Specifically, imaging of a certain kind: imaging is in fact a scheme for a group of revision procedures, differing from each other in how probabilities under the counterfactual supposition of A are switched to the most similar worlds where A is true. The specific kind to which the Leitgeb-Pettigrew result applies is called Laplacian imaging. In it, the switched probabilities are distributed uniformly among all the closest A-worlds.

\(^3\) Of particular interest is proofs that such a logic is free from trivialization: Lewis.1976 proved that if one equates the probability of an indicative ‘If A, then B’ with P(B|A), the equality can only be satisfied by trivial probability distributions. But he showed that the equation of the probability of a counterfactual ‘If it were the case that A, it would be the case that B’ (understood as true iff the closest A-world makes B true) with the corresponding imaging-probability, P_A(B), is free from triviality. However, the Laplacian imaging to which the aforementioned Leitgeb-Pettigrew result applies is a bit different from the one used by Lewis and the issue of to what extent Laplacian imaging is triviality-free is to be settled.
(Ozgün&Berto.2020), having been trained at Amsterdam's ILLC school of Dynamic Epistemic logic. But again, how to come up with ones in the context of our logic of imaging is uncharted territory.

**Subproject 3: Improving Mental Simulation. [PA, Postdoc 2]** Once we know that counterfactual imagination as modelled in Subproject 1 maximizes expected accuracy, we have a rational normativity criterion as a benchmark for our fallacies in hypothetical thinking: they count as fallacious reasoning *insosfar* as they depart from simulated belief revision, so understood. WIKI's Postdoc 2, a psychologist of reasoning, will help here. They will be enrolled starting from WIKI's Year 2, i.e., after the formal results mentioned above will start to be in place. We will then produce a number of papers on how to become better mental simulators, by comparing our logic with empirical results from psychology, concerning common fallacies of suppositional reasoning. We are not planning to carry out new experiments. But I will need the skills of Postdoc 2 to navigate the empirical literature, select the relevant results, interpret them, and make precise how our normative model can make for a yardstick, from which people can be seen as departing in fallacious hypothetical reasoning (see Section 5 for the methodology of this).

**Subproject 4: Synthesis. [PA]** In the last year I will work on a book, drawing on the interim results of WIKI. Although the problems tackled in Sub-Projects 1-3 are related, the emerging full picture cannot be adequately brought up only in separate publications scattered over journals. WIKI will end with what will be, as far as I am aware, the first monograph on the formal logic of counterfactual imagination.

4. **Significance**

I think WIKI will significantly advance the state-of-the-art in the philosophy of imagination: it will address a number of foundational epistemological issues by applying formal logical techniques to them. The techniques will be logically innovative as well, insosfar as the logic of imaging-probability is under-explored. And the spin-offs will be momentous, ranging from a better understanding of the workings of scientific thought experiments to a plethora of substantiated recommendations on how to improve our counterfactual reasoning skills in everyday life.

5. **Methodology: From the Formal to the Experimental, and Back**

The WIKI project is grounded in the *methodology of cooperative philosophical research*, championed by St Andrews' Arché research centre. It is based on the project researchers meeting intensively to test research. A WIKI Seminar will run weekly, with two kinds of seminar slots:

1. **Testing slots:** the researchers responsible for the output present their paper in intensive Q&A sessions. The purpose is to gain feedback via the raising of suggestions and objections. It is vital that the feedback be given by a group continuously attending the seminar, so that members share common knowledge on the research topics. The Testing slots will be attended by Arché members: epistemologists (Jessica Brown); logicians (Greg Restall, Gillian Russell, Aaron Cotnoir, Stephen Read, Catarina Dutilh Novaes); philosophers of language (Simon Prosser, Derek Ball).

2. **Minuting slots:** the testing slots are followed by minuting meetings, attended only by the WIKI researchers (PA and postdocs) who revisit the issues, identify strands, and discuss ways of turning the research to publishable outputs. After three-four rounds of testing and minutes, a draft paper has improved to the point that it can be submitted with good chances to a top journal.

WIKI will also employ an innovative combination of *formal* and *empirical* methodology championed by eminent philosophers of logic and cognition like Igor Douven.2015-2022. It will systematically apply to the philosophy of imagination techniques from formal epistemology, epistemic and probabilistic mathematical logic. Subproject 1 involves rigorous research in mathematical logic: we build a formal logic with a semantics and we deliver non-triviality, completeness, axiomatization, computational complexity, and (un)decidability results for it. For reasons explained above, this will require innovative mathematical techniques for which I’ll need the help of Postdoc 1, a mathematical logician.

The WIKI formal framework will then be compared with literature from empirical and experimental psychology of reasoning, in particular on the fallacies of mental simulation (Roese&Olson.1993,
Taylor et al. (1998, Markman et al. 2009). Again, we are not planning to carry out new experiments, but – being a theoretical logician & philosopher with no expertise in experimental research – I will need here the help of project Postdoc 2, a psychologist of reasoning, to scan through, select, and interpret, the relevant literature on the biases and fallacies of hypothetical and counterfactual thinking. After determining exactly how such thinking deviates from the normative criterion of simulated belief update by imaging, we will indicate where there is room for working towards correcting our fallacies in imaginative thinking and become better mental simulators.

An embryonic application of this methodology can be found in a co-authored paper of mine (Solaki-Berto-Smets 2021): here we modelled fallacious reasoning in a formal epistemic logic assigning cognitive costs to reasoning steps. That the cost assigned to inference X in a model is low means that it is predicted by the model that ordinary reasoners mostly get inference X right. If that turns out not to be the case on the basis of experiments on reasoning tasks, the cost for X can be raised in the model until it matches the data. For inferences like modus ponens/tollens and fallacies of Affirming the Consequent/Denying the Antecedent, the data were fairly clear due to the amount of research on these four classical figures of conditional reasoning. But for other inferences the data were scarce, and/or needed a more nuanced interpretation than we logicians could come up with. Predicting a similar scenario for the fallacies of hypothetical thinking that WIKI will consider, Postdoc 2 will (i) find the relevant empirical/experimental results from psychology, and (ii) fine-tune how the data can be related to the WIKI logical theory, as explained by or as evidence against it. There will be, thus, a virtuous back-and-forth between the formal models and the empirical data: the former can help to interpret and shed light on the latter, while the latter can help to fine-tune the explanations of the former.

6. Details of how results will be published

All the project papers are to appear Open Access in the best peer-reviewed research journals. While we expect authorship of the outputs of the subprojects to be chiefly tied to the researchers affiliated to them, we set no limit to the possibilities of cooperation between all members of the research team:

(1) Subproject 1 will deliver at least three papers in top journals. At least one will be on the philosophical foundations of WIKI and will appear in a mainstream philosophy/philosophical logic journal (targeting Mind, Phil Studies, Journal of Philosophical Logic). At least two papers will be on the formal setting of our logic of imaging and will appear in philosophical or mathematical logic journals (Review of Symbolic Logic, Studia Logica, Notre Dame Journal of Formal Logic). They will present axiomatization, non-triviality, complexity, and (un-)decidability results.

(2) Subproject 2 will deliver at least three papers in top journals, developing a systematic account of scientific thought experiments by the lights of our logic of imagination. We are targeting both philosophy of science journals (British Journal for the Philosophy of Science, European Journal for Philosophy of Science) and generalist ones friendly to topics in the philosophy of thought experiments (Noûs, Synthese, Erkenntnis, etc.).

(3) Subproject 3 will deliver at least four papers in top journals of philosophy and/or theoretical cognitive psychology (Review of Philosophy and Psychology, Cognition, Cognitive Psychology). These will present our normative theory of rational counterfactual supposition, analyze fallacies of counterfactual reasoning qua deviations from the norm, and propose ideas on how we can get closer to the norm and improve our hypothetical reasoning skills.

(4) Subproject 4 will deliver a book gathering the project’s results for a major scientific publisher (targeting OUP, CUP, MIT Press). We here follow best practice in systematic epistemology and logic, where a monograph is preceded by papers building up to the larger work. Three cases of such best practice: Williamson’s Knowledge and Its Limits (OUP 2000); Priest’s Doubt Truth to Be a Liar (OUP 2006); Van Benthem’s Logical Dynamics of Information and Interaction (CUP 2011) – all monographs relying on previously published, often co-authored papers. Funding is asked in order to buy Open Access rights for the book, to make it publicly available and maximize its diffusion.
References


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