Who pays for job training?∗

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ABSTRACT

This paper addresses a puzzle in the UK labour market. Why is not there enough investment in job training when there is a high skill premium? We model this as a coordination game between firms and workers. Using a social planning model as a baseline, the paper demonstrates that while it is socially beneficial to invest in job training, the private sector may fail to internalize these benefits in a wide range of economies. The chance of this coordination failure is greater in economies with a higher inequality in the skill distribution and a higher rate of time preference.

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1 Introduction

There is an ongoing debate about the skill gap in the UK labour market. Vacancies remain unfilled because of this skill shortage. Haskel and Martin (2001) document that the skill shortage is greater for firms employing advanced technology. In an illuminating skill survey, Felstead, Gallie and Green (2002) document two apparently conflicting features of the UK labour market. First, there is an excess supply of intermediate skills. Second, since 1986, more jobs require advanced skill. A substantial skill premium exists at the graduate level (57% for women and 38% for men) compared with jobs which require no qualification. The recent work of Jenkins, Greenwood and Vignole (2007) also confirm this skill premium differential. Jenkins et al. confirm that the level 2 workers actually suffer a wage penalty compared to workers with no qualifications.

These two findings are puzzling. If there is such a high skill premium, why is not this exploited by the workers? If a large number of high-skilled, well paid jobs are vacant, why don’t the workers invest in job training and reap the benefits? Instead workers are content to acquire lesser intermediate qualifications and create a glut of intermediate skills in the labour market. A related question also arises: why don’t employers take advantage of this anomaly by employing low-skilled intermediate level workers and turn them into high-skilled by job training?

The issue of under-investment in job training in the context of labour market search frictions has been addressed in the literature. Acemoglu (1997) argues that search frictions create training externality and cause under-investment in training due to incomplete contract between workers and employers. Moen and Rosen (2004) argue that this under-investment arises because training firms set wages for trained workers at a too low level compared to the wage offered by the poaching firms.

In this paper, we approach this issue with a different perspective. We argue that

1The following quotation from Felstead et al. (2002) aptly summarizes this supply-demand imbalance: "...there are 6.4 million people qualified to the equivalent of NVQ level 3 in the workforce, but only 4 million jobs that demand this level of highest qualification. There are a further 5.3 million people qualified at level 2, but only 3.9 million jobs that require a highest qualification at this lower level."
this under-investment in training may arise due to strategic considerations in job training decisions. The job training decision is modelled in terms of a dynamic game between the worker and the firm instead of a competitive search equilibrium as in Moen and Rosen (2004). Both worker and firm face the choice whether to invest in job training taking into account that the other party may or may not invest in training. This strategic complementarity may give rise to a Cournot-Nash equilibrium where none may invest in training for a range of training costs. We provide a normative bend to this issue by setting up a fictitious social planning problem and asking first whether it is socially efficient to invest in such job training in a labour market environment where there is search frictions. The benevolent social planner internalizes the search frictions and dictates whether the firms or households should invest in job training and if so how much training cost each should bear. Given this social planning model as the baseline, we ask whether there is enough incentive for the private sector to invest in such a job training. If one finds that there is no such incentive in a decentralized economy while the social planner mandates investment in training, it calls for a government intervention in the form of a corrective tax/subsidy.

The paper is organized as follows. In the following section, we lay out the environment. Section 3 sets up a model of strategic training. Section 4 describes a social planning model which is used in section 5 as a baseline for comparing with the decentralized job training outcome. Section 6 concludes.

2 Environment

Two types of technologies are available: high skill (suffixed as s) and low skill (suffixed as u). There are continuum of high-skilled and low-skilled workers and firms in a unit interval. Initially there are \( \mu_0^s \) proportion of high-skilled workers and firms which means that there is no initial mismatch of skills in the economy. There is also an initial distribution of vacant high-skilled and low-skilled firms denoted as \( v_{00}^s \) and \( v_{00}^u \) respectively and an initial distribution of unemployed high-skilled and low-skilled workers denoted as \( u_{00}^s \) and \( u_{00}^u \).
respectively.

There is random matching of vacant firms and unemployed workers in each sector. The matching processes in the high-skilled and low-skilled sectors are described as follows. At each date $v_{it}^i (i = s, u)$ proportion of vacant i-type firms meet $u_{it}^i$ proportion of unemployed i-type workers. Let $\lambda^i$ be the probability that an unemployed i-type worker matches a vacant i-type firm. A match consummates when a vacant firm finds an unemployed worker and vice versa. The matching function in the $ith$ sector of such a two sided match is given by:

$$M_t^i = \lambda^i u^i_t v^i_t$$  \hspace{1cm} (2.1)

If the $ith$ worker successfully matches with the $ith$ firm ($i = s, u$), it produces fixed units of output $p^i$ where $p^s > p^u$. Let a fixed fraction $\sigma^i$ of successful matches die every period either due to exogenous retirement or layoffs or poaching.

There are two types of provisions for job training in the economy: (i) low-skilled worker undertakes self-training by joining a skill center; (ii) low-skilled firm imparts job training to a low-skilled worker. If a low-skilled worker goes through (i), he becomes high-skilled in the next period. If a low-skilled firm undertakes (ii), it turns a low-skilled worker into high-skilled in the next period and also transforms itself into a high-skilled firm. Since there is no initial mismatch between high-skilled firms and high-skilled workers, high-skilled firms do not have any incentive to train workers. The only decision problem for either the low-skilled worker or the low-skilled firm is whether to invest resources in job training. Such a decision is represented by an indicator function $\xi_t^{iw}$, $\xi_t^{if}$ taking values 0 or 1 for no training and training for worker and firm respectively. Let $\mu_t^{if}$ and $\mu_t^{iw}$ be the

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2Such a matching function is known in the literature as a quadratic matching function following Diamond and Maskin (1979). Such a matching function can be motivated by the illustrative example borrowed from Mortensen and Pissarides (1998) that both matched and unmatched firms and households have a telephone book of all matched and unmatched agents on the other end of the market. A quadratic matching function may give rise to multiple equilibria. In our context, we break such multiplicity by invoking an initial distribution of skilled and unskilled workers and firms.
number of the \( i \)th types firms and workers respectively respectively.

There are six state variables, namely \( u^s_t, u^u_t, v^s_t, v^u_t, \mu^{sw}_t \) and \( \mu^{sf}_t \). The state transition equations facing the firms and workers are given by:

\[
\begin{align*}
\Delta u^s_t &= \sigma^s(\mu^{sw}_t - u^s_t) + \xi^w_t u^u_t v^s_t (1 - \lambda^s) - M^s_t \quad (2.2) \\
\Delta u^u_t &= \sigma^u(\mu^{uw}_t - u^u_t) - M^u_t - \xi^w_t u^u_t \quad (2.3) \\
\Delta v^s_t &= \sigma^s(\mu^{sf}_t - v^s_t) - M^s_t \quad (2.4) \\
\Delta v^u_t &= \sigma^u(\mu^{uf}_t - v^u_t) - M^u_t \quad (2.5) \\
\Delta \mu^{sw}_t &= \xi^f_t (\mu^{sw}_t - u^u_t) + \xi^w_t u^u_t \quad (2.6) \\
\Delta \mu^{sf}_t &= \xi^f_t (\mu^{sf}_t - v^u_t) \quad (2.7)
\end{align*}
\]

A few clarification of the terms in the transition equations are in order. The transition equation (2.2) shows that the number of high-skilled unemployed increases when job separations occur (first term) or unemployed low-skilled worker after investing in schooling meets a vacant high-skilled firm but the match does not consummate (second term). Likewise the number of high-skilled unemployed decreases if a successful match occurs (third term). Similar explanation applies to (2.3) except that the third term means that the number of low-skilled workers decreases when an low-skilled worker joins the skill center and thus withdraws from the pool of low-skilled unemployed and joins the pool of high-skilled unemployed. The transition equations for vacancies (2.4) and (2.5) basically mean that the number vacancies in each sector increases if job separations occur and decreases if a successful match occurs. The transition equation for the high-skilled workers (2.6) means that more high-skilled workers evolve as more low-skilled firms invest in job training (the first term) and more low-skilled worker invests in job training (the second term). Likewise more high-skilled firms evolve when firms undertake job training programme and turn themselves into high-skilled, which explains (2.7)

There are four possible steady states for this system: (i) firms invest in training while worker do not, \( \xi^f_t = 1, \xi^w_t = 0 \), (ii) firms do not invest in training but workers do, \( \xi^f_t = 0, \xi^w_t = 1 \) , (iii) both invest in training, \( \xi^f_t = 1, \xi^w_t = 1 \), (iv) none invest in training,
\( \xi^f_t = 0, \xi^w_t = 0 \). We focus on the steady state analysis only. First, we pin down the steady state configuration of the six state variables under alternative combinations of training by firms and households. Using these steady state results, in the next step we pin down the scenario where neither the employer nor the worker find it worthwhile investing in training.

In regard to the first step, following four lemmas are of interest. Since in the steady state the relevant state variables do not depend on time but only on state, the subscripts hereafter now represent the states.

**Lemma 1** Let \( u_{10}^i, v_{10}^i, \mu_{10}^{iw} \) and \( \mu_{10}^{if} \) \((i = s, u)\) be the steady state solutions of the transition equation when \( \xi^f_t = 1 \) and \( \xi^w_t = 0 \) then,

1. \( u_{10}^u = 0, v_{10}^u = 0 \),

2. \( \mu_{10}^{sw} = 1, \mu_{10}^{uw} = 0 \),

3. \( \mu_{10}^{sf} = 1, \mu_{10}^{uf} = 0 \), and

4. \( v_{10}^s = u_{10}^s = \frac{1}{2} \left( -\eta^s + \sqrt{\eta^s^2 + 4\eta^s} \right) \)

where \( \eta^s = \sigma^s/\lambda^s \).

**Proof.** Appendix. \( \blacksquare \)

**Lemma 2** Let \( u_{01}^i, v_{01}^i, \mu_{01}^{iw} \) and \( \mu_{01}^{if} \) \((i = s, u)\) be the steady state solutions of the transition equation when \( \xi^f_t = 0 \) and \( \xi^w_t = 1 \), then,

1. \( u_{01}^u = 0, v_{01}^u = \mu_{01}^u \),

2. \( \mu_{01}^{uw} = 0, \mu_{01}^{sw} = 1 \),

3. \( \mu_{01}^{sf} = \mu_{01}^s, \mu_{01}^{uf} = \mu_{01}^u \) and

4. \( v_{01}^s + \mu_{01}^u = u_{01}^s = \frac{1}{2} \left( (\mu_{01}^u - \eta^s) + \sqrt{(\mu_{01}^u - \eta^s)^2 + 4\eta^s} \right) \)
Lemma 3 Let $u_{i1}^i, v_{i1}^i, \mu_{11}^{i_1}$ and $\mu_{11}^{i_1}$ ($i = s, u$) be the steady state solutions of the transition equation when $\xi_{t}^u = 1$ and $\xi_{t}^{s} = 1$, then:

1. $v_{11}^u = u_{11}^u = 0$,
2. $\mu_{11}^{u_1} = 0, \mu_{11}^{s_1} = 1$,
3. $\mu_{11}^{u_1} = 0, \mu_{11}^{s_1} = 1$, and
4. $v_{11}^{s_1} = u_{11}^{s_1} = \frac{1}{2} (\eta^s + \sqrt{[\eta^s]^2 + 4\eta^s})$

Proof. Appendix

Lemma 4 Let $u_{00}^i, v_{00}^i, \mu_{00}^{i_1}$ and $\mu_{00}^{i_1}$ ($i = s, u$) be the steady state solutions of the transition equation when $\xi_{t}^u = 0$ and $\xi_{t}^{s} = 0$, then,

1. $v_{00}^u = u_{00}^u = \frac{1}{2} (\eta^u + \sqrt{[\eta^u]^2 + 4\eta^u\mu_{0}^u})$,
2. $\mu_{00}^{u_1} = \mu_{0}^{u}, \mu_{00}^{s_1} = \mu_{0}^{s}$
3. $\mu_{00}^{u_1} = \mu_{0}^{u}, \mu_{00}^{s_1} = \mu_{0}^{s}$ and
4. $v_{00}^{s_1} = u_{00}^{s_1} = \frac{1}{2} (\eta^s + \sqrt{[\eta^s]^2 + 4\eta^s\mu_{0}^s})$.

Proof. Appendix

Lemma 1 implies that if low-skilled firms alone invest in job training, all low-skilled workers and firms turn high-skilled in the new steady state. As a consequence, all low-skilled vacant firms disappear. Lemma 2 means that when low-skilled workers alone invest in training, they all turn high-skilled while all initial low skilled firms turn vacant in the new steady state. Lemma 3 means that if both invest in training, both vacant low-skilled firms and unemployed low-skilled workers disappear from the scene. Lemma 4 implies that if none invest, proportion of low-skilled workers and firms remain the same as the initial level. The steady state unemployment and vacancy are the same in both sectors because there is no initial mismatch of skills between workers and the firms.
3 Strategic Training

We now turn our attention to a decentralized environment where the job training decisions are made in a noncooperative, strategic environment. Let $sc$ be the cost for training a worker. Let $b$ be a common leisure value of any unemployed worker of any type, $c$ be a common cost of keeping a production unit vacant and $\omega^i$ be the wage prevailing in the $i$th sector. Low-skilled workers while deciding to incur training costs take into account that even if they do not incur this cost, there is a chance of being hired by a low-skilled firm and getting trained subsequently. A low-skilled firm while contemplating to train a low-skilled worker internalizes the fact that the same worker may leave the firm after training. The job training decisions thus appear as the equilibrium of a dynamic game between workers and firms in a search environment.

The equilibrium of this decentralized economy is formulated in three steps. First, we specify the value functions of workers and firms. Second, we describe the labour market story of wage determination. Third, we formulate the noncooperative training cost thresholds as a Nash equilibrium where neither the firm nor the worker invest in training.

3.1 Steady State Value Functions: Worker

Define the values of employed and unemployed worker in the $i$th sector as $E^i$ and $U^i$ respectively. An employed high-skilled worker can earn a wage $\omega^s$ today and face two scenarios: (i) stay employed in the next period with a probability $(1 - \sigma^s)$ or (ii) join the pool of high-skilled unemployed with probability $\sigma^s$ and faces the prospect of being matched with a vacant high-skilled firm with probability $\lambda^s v^s$. The value functions of the high-skilled workers are thus:

\[
E^s = \omega^s + \beta [\sigma^s U^s + (1 - \sigma^s) E^s] \\
U^s = b + \beta [\lambda^s v^s E^s + (1 - \lambda^s v^s) U^s]
\]
Solving these value functions one gets:

\[
E^s = \frac{\omega^s}{1 - \beta} - \frac{\beta}{1 - \beta} \left( \frac{\sigma^s (\omega^s - b)}{1 - \beta (1 - \sigma^s - \lambda^s v^s)} \right)
\]

\[
U^s = \frac{b}{1 - \beta} + \frac{\beta}{1 - \beta} \left( \frac{\lambda^s v^s (\omega^s - b)}{1 - \beta (1 - \sigma^s - \lambda^s v^s)} \right)
\] (3.9)

A low-skilled worker likewise faces two scenarios: either to stay unemployed and enjoy the unemployment benefits \( b \) or work at the going market wage \( \omega^u \). However, unlike the high-skilled worker, he faces a binary decision whether or not to incur private training cost. This training decision depends on the firm’s decision to train workers. If he works now as low-skilled, in the next period he faces the possibility of either being trained by the firm, and turn high-skilled or remain untrained. If he does not work now and join the job center by bearing the training cost, next period he becomes high-skilled and faces the same value function of a high-skilled worker. If he does not work now and also does not go for any training, he stays low-skilled and reaps the values of a low-skilled worker next period.

\[
E^u = \omega^u + \beta (1 - \sigma^u) E^u + \sigma^u U^u (1 - \xi^f) \]

\[
+ \beta [(1 - \sigma^u) E^u + \sigma^u U^u] \xi^f
\]

and

\[
U^u = b - \xi \omega^u sc + \beta \xi [\lambda^s v^s E^s + (1 - \lambda^s v^s) U^s] \]

\[
+ \beta (1 - \xi^u) [\lambda^u v^u E^u + (1 - \lambda^u v^u) U^u]
\] (3.10) (3.11)

### 3.2 Value Functions: Firm

Define next the values of a matched and vacant firm in the \( i \)th sector as \( J^i \) and \( V^i \) respectively. A high-skilled firm can either get matched with a high-skilled worker or stay vacant. If it matches, then the firm enjoys a current cash flow of \( p^s - \omega^s \) and faces the exogenous uncertainty of job separation next period. This means:
\[ J^s = p^s - \omega^s + \beta[(1 - \sigma^s)J^s + \sigma^sV^s] \] (3.12)

If it stays vacant, it incurs a vacancy cost \( c \) now and faces the prospect of being matched with an unemployed high-skilled worker next period. This means:

\[ V^s = -c + \beta(\lambda^s u^s J^s + (1 - \lambda^s u^s) V^s) \] (3.13)

Solving (3.12) and (3.13), we get:

\[
\begin{align*}
J^s &= \frac{p^s - \omega^s}{1 - \beta} - \frac{1}{1 - \beta} \frac{\beta \sigma^s (p^s - \omega^s + c)}{1 - \beta (1 - \sigma^s - \lambda^s u^s)} \\
V^s &= -\frac{c}{1 - \beta} + \frac{1}{1 - \beta} \frac{\beta \lambda^s u^s (p^s - \omega^s + c)}{1 - \beta (1 - \sigma^s - \lambda^s u^s)}
\end{align*}
\]

A low-skilled matched firm enjoys the current cash flow of \( p^u - \omega^u \), and faces the decision whether or not to invest in training the incumbent low-skilled worker. If it decides to train, the firm becomes high-skilled together with the worker and faces the same value function of a high-skilled firm in the following period. If it does not train its worker, it stays low-skilled and faces the exogenous uncertainty of separating from the low-skilled worker. The value function is thus:

\[
J^u = p^u - \omega^u \xi^u \xi^w + \left[ \xi^f \{-sc + \beta \sigma^s V^s + \beta(1 - \sigma^s)J^s\} \right. \\
\left. + (1 - \xi^f)\beta\{\sigma^u V^u + (1 - \sigma^u)J^u\} \right]
\] (3.14)

The value of the low skilled firm does not directly depend on the training strategy of the worker. The reason is that a low skilled firm can only match with a low skilled worker. Whether the low skilled worker goes for self training is not directly relevant to the firm. However, as we will see in the next section that when a low skilled worker goes for self training, it has an impact on his wage and through this channel it impacts the value of the low skilled firm. This is why the wage of the unskilled worker is state dependent.

If the low-skilled firm remains vacant, it currently incurs the vacancy cost \( c \) and faces the prospect of being matched with a low-skilled worker in the next period. Value of a vacant low skilled firm is given by:
\[ V^u(\xi_f, \xi_w) = \max \{0, -c + \beta \lambda^u u^u_{\xi_f, \xi_w}, J^u(\xi_f, \xi_w) + (1 - \lambda^u u^u_{\xi_f, \xi_w})V^u(\xi_f, \xi_w)\} \quad (3.15) \]

Recall that \( u^u_{\xi_f, \xi_w} = 0 \) when at least one party invests in training. This means that

\[ V^u(0, 1) = V^u(1, 0) = V^u(1, 1) = 0 \]

If nobody invests in training we get:

\[ V^u(0, 0) = -c + \beta (\lambda^u u^u, J^u(0, 0) + (1 - \lambda^u u^u) V^u(0, 0)) \quad (3.16) \]

which can be explicitly solved if we know \( J^u(0, 0) \). Recall from (3.14) that

\[ J^u(0, 0) = p^u - \omega_{00}^u + \beta \{\sigma^u V^u(0, 0) + (1 - \sigma^u) J^u(0, 0)\} \quad (3.17) \]

Thus \( V^u(0, 0) \) and \( J^u(0, 0) \) can be solved from the linear system of equations (3.16) and (3.17).

It is straightforward to check from (3.14) that

\[
J^u(0, 1) = \frac{p^u - \omega_{01}^u}{1 - \beta (1 - \sigma^u)} \\
J^u(1, 0) = p^u - \omega_{10}^u - sc + \beta [\sigma^u V^u + (1 - \sigma^u) J^u] \\
J^u(1, 1) = p^u - \omega_{11}^u - sc + \beta [\sigma^u V^u + (1 - \sigma^u) J^u]
\]

### 3.3 Wage Determination

The wage in each sector is determined by a Nash bargaining:

\[
\max_{u^i} (E^i - U^i)^{\phi} (J^i - V^i)^{1-\phi}, i = s, u
\]

where \( \phi \) is a non-negative fraction representing the bargaining strength of the worker.

The solution for wages in the high-skilled sector is:
\[
\omega^s = \phi (p^s + c) + (1 - \phi) b
\]

The wage of the high-skilled sector is independent of the unemployment rate or the vacancy rate. Therefore the wage of the high-skilled sector is independent of the job training decisions of firms and workers. This is not the case in the low-skilled sector because the value functions for low-skilled workers and firms depend on the strategic job training decisions. In general, the wages in the low-skilled sector is given by

\[
\omega \xi_{\xi^w} = \phi \left( p^u + \xi^f \left[ -sc_f + J^s - p^s + \omega^s \right] \right) + (1 - \phi) \left[ \frac{(1 - (1 - \xi^f) \beta) \left( b - \xi^w (sc - U^s + b) \right)}{1 - \beta (1 - \xi^w)} - \xi^f (E^s - \omega^s) \right].
\]

Based on this, the steady state low skilled wages for the four respective states are given by:

\[
\begin{align*}
(0, 0) : & \quad \omega_{00}^u = \phi (p^u + c) + (1 - \phi) b \\
(0, 1) : & \quad \omega_{01}^u = \phi p^u + (1 - \phi) \left( (1 - \beta) (U_{01}^s - sc_w) \right) \\
(1, 0) : & \quad \omega_{10}^u = \phi (p^u - sc_f + J_{10}^s - p^s) + (1 - \phi) \left( \frac{\beta b}{1 - \beta} - E_{10}^s \right) + \omega^s \\
(1, 1) : & \quad \omega_{11}^u = \phi (p^u - sc_f + J_{11}^s - p^s) + (1 - \phi) \left( -sc_w + U_{11}^s - E_{10}^s \right) + \omega^s.
\end{align*}
\]

where \( J_{10}^s, J_{11}^s, U_{11}^s, U_{01}^s \) and \( E_{10}^s \) are obtained from the expressions \( J^s, U^s \) and \( E^s \) by replacing the unemployment and vacancy rates with the relevant expressions obtained from lemma (1, 3 and 2)

### 3.4 A Nash Equilibrium of no investment in training

Recall that the unemployed low-skilled worker’s value \( U^u \) in (3.11) is a function of both \( \xi^f \) and \( \xi^w \) via the Nash bargaining solution for wages. In the appendix we have shown that the unemployed low-skilled worker’s value \( U^u \) depends on \( \xi^f \) and \( \xi^w \) via the Nash
bargaining solution for wages as follows:

\[
U^u(\xi^f, \xi^w) = \frac{b - \xi^w (sc - [U^s - b])}{1 - \beta(1 - \xi^w)} \\
+ \frac{\beta(1 - \xi^w)\lambda^w v^u}{1 - \beta(1 - \xi^w)} \left( \frac{\omega^u + \xi^f(E^u - \omega^*)}{1 - \beta(1 - \xi^f)} - \frac{b - \xi^w (sc - [U^s - b])}{1 - \beta(1 - \xi^w)} \right).
\] (3.18)

Given that firms do not pay workers will pay if

\[
U^u(0, 1) \geq U^u(0, 0)
\] (3.19)

Given that workers do not pay firms will pay if

\[
J^u(1, 0) \geq J^u(0, 0)
\] (3.20)

Observing that \(U^u(0, 1)\) is monotonically decreasing in \(sc\), it is straightforward to verify that there exists a threshold training cost \(sc^w_n\) for which (3.19) holds as equality. The worker does not pay for training if the firm does not pay if

\[
sc > sc^w_n
\] (3.21)

Using the same line of reasoning, one can establish that there exists a threshold schooling cost \(sc^f_n\) for which (3.20) holds as equality. Given that the worker does not pay, the firm does not pay for training if

\[
sc > sc^f_n
\] (3.22)

The appendix provides an algebraic derivation of these two thresholds.

Based on the above analysis we have the following proposition.

**Proposition 1** If the training cost per pupil is such that

\[
sc > \max(sc^w_n, sc^f_n)
\] (3.23)

neither firm nor the worker finds it worthwhile investing in training.
4 Pareto Optimal Training: A Social Planning Problem

We next turn into a normative question: is it socially optimal to invest in training? If so, who should really take charge of job training, firm or worker or both? In order to answer this question, we set up a fictitious social planning problem. The social planner internalizes the benefits and costs of keeping a worker unemployed and positions vacant and faces the following per period net benefits for the economy:

\[
r(\xi^f_t, \xi^w_t) = \sum_{i=s,u} [p^i(\mu^i_t - u^i_t) + bu^i_t - cv^i_t] - \xi^f_t \cdot sc.(\mu^w_t - u^w_t) - \xi^w_t \cdot sc.u^w_t
\]

where \(\xi^f_t\) is an indicator function which takes the value unity if the social planner dictates the firm to spend on job training and zero otherwise, \(\xi^w_t\) is such an indicator function if the planner asks the worker to invest in job training. The social planner takes the evolution of the state variables, \(u^s_t, u^u_t, v^s_t, v^u_t, \mu^w_t, \mu^s_f\) as given and only chooses \(\xi^w_t\) and \(\xi^f_t\) to maximize the discounted stream of societal benefits. In other words, the planner’s problem is to decide whether it is socially beneficial to invest in schooling and if so, who should undertake this investment and how much he should pay.

Formally the social planner solves the following dynamic optimization problem:

\[
\max_{\xi^f, \xi^w} \sum_{t=0}^{\infty} \beta^t r(\xi^f_t, \xi^w_t)
\]

st. (2.2) through (2.7).
5 Setting up steady state value function for the social planner

Given the above four lemmas, it is evident that the state of the economy facing the planner in the steady state is entirely dependent on the state of training, \((\xi^f_t, \xi^w_t)\).\(^3\) Starting from this state, the planner can undertake four possible actions by commanding the firms and the workers as follows: (i) none invest in training (ii) only firms invest in training, (iii) only workers invest in training, (iv) both invest in training. Given the decision of the planner, there will be a transition to a new steady state in the next period while the society will incur a one period transition cost which is the training cost.

Define the value function of the social planner as \(\Omega(\xi^f_t, \xi^w_t)\). Formally we can write:

\[
\Omega(\xi^f_t, \xi^w_t) = \max_{\xi^f_{t+1}, \xi^w_{t+1}} \left[ r(\xi^f_t, \xi^w_t) - \xi^f_{t+1}(1 - \xi^w_{t+1})sc.(\mu^w_t - u^w_t) - \xi^w_{t+1}(1 - \xi^f_{t+1})sc.u^w_t - \xi^w_{t+1}\xi^f_{t+1}sc.\mu^w_t + \beta\Omega(\xi^f_{t+1}, \xi^w_{t+1}) \right] 
\]

\(^3\)In principle, the entire history of training should comprise the current state facing the planner. However, given the absorbing nature of the state (meaning when either the worker or the firm invests in training, an unskilled worker or firm turns permanently skilled next period), the current state is thus summarised only by the current state if job training.
From the previous lemmas the steady state values of the returns to the social planner are given as:

**Lemma 5**  
1. \( r(1, 0) = p^s(1 - u^*_10) + (b - c) u^*_10 \)
2. \( r(0, 1) = p^s(1 - u^*_01) + bu^*_01 - cv^*_01, \)
3. \( r(1, 1) = p^s(1 - u^*_11) + (b - c) u^*_11, \)
4. \( r(0, 0) = \sum_{i=u,s} [p^i (\mu^i_0 - u^i_0) + (b - c) u^i_0] \)

The following lemma is self-evident.

**Lemma 6** Define \( r^{\text{max}} = \max(r(0, 0), r(0, 1), r(1, 0), r(1, 1)) \) and the total schooling cost as \( \tilde{SC} \). The planner initiates a change from no training to positive training if

\[
\frac{r(0, 0)}{1 - \beta} \leq r(0, 0) - \tilde{SC} + \beta \frac{r^{\text{max}}}{1 - \beta} \tag{5.25}
\]

Based on (5.25) it follows that for the following range of schooling costs the planner initiates a change:

\[
\tilde{SC} \leq \frac{\beta}{1 - \beta} [r^{\text{max}} - r(0, 0)] \tag{5.26}
\]

The right hand side of (5.26) is the annuity value of the return differential when the planner initiates a change from no training to positive training. However, this inequality is not too informative because we do not yet know the precise restrictions on the structural parameters for which the planner aspires such a change and we also do not know the value of \( r^{\text{max}} \). The following lemmas answer these questions.

**Lemma 7** : There exists a range of \( \eta^u \) such that \( r(0, 0) < \min(r(1, 0), (1, 1), r(0, 1)) \).
**Proof:** To prove this observe from Lemma 5 that none of $r(1,0), (1,1), r(0,1)$ depends on $\eta^{u}$ and based on (4) $r(0,0)$ depends on $\eta^{u}$ via $u^{u}_{00}$. Thus there exists at least one $\eta^{u}$ for which $r(0,0) < \min(r(1,0), (1,1), r(0,1))$.

Given that the condition for social planner’s intervention holds, the next question is: who pays for job training. The following lemma describes a range of initial distribution of skill for which the social planner dictates that the worker should pay for job training. This is formalized in terms of the following lemma.

**Lemma 8**: There exists a range of $\mu^{*}_{0}$ such that $r^{\text{max}} = r(0,1)$.

Proof: Observe from lemma (5) and lemmas (1) through (3) that only $r(0,1)$ depends on $\mu^{*}_{0}$. Thus one can always find a set of $\mu^{*}_{0}$ for which $r(0,1) = r^{\text{max}}$. //

**Lemma 9**: For $r(0,1) \neq r^{\text{max}}$, the planner is indifferent whether worker or firm pays.

Proof: Observe from Lemma (5) and Lemma (1) and Lemma (3) that $r(1,0) = r(1,1)$. Thus if $r(0,1) \neq r^{\text{max}}$, then $r^{\text{max}} = r(1,0) = r(1,1)$. The planner is thus indifferent who pays. //

Whether it is socially optimal for either the worker or the firm to invest in job training thus critically depends on the degree of labour market friction ($\eta^{u}$), and the initial distribution of skill ($\mu^{*}_{0}$).

### 5.1 Market Failure in Training: Some Quantitative Analysis

We now turn to the key question. Is it privately optimal for either the worker or the firm to pay for schooling when it is deemed socially optimal that at least one of them should pay? We do the following conceptual experiment. First, construct sample economies characterized by the initial distribution of skills and the labour market frictions such that the social planner intervenes. In other words, we restrict ourselves to sample economies
for which it is socially optimal to invest in job training. We then ask the question whether it is privately optimal to invest in training. In order to accomplish this task, we highlight the following two scenarios:

(i) where the social planner mandates that at least one of the two parties should pay for training but in a Nash equilibrium as depicted in Proposition (1) nobody finds it privately optimal to pay because \((3.23)\) holds.

(ii) where the social planner mandates that the worker should pay for training but the worker does not find it privately optimal because \((3.21)\) holds.

There are 11 structural parameters, namely \(p^s_0, p^s, p^u, b, c, \beta, \lambda^s, \lambda^u, \sigma^s, \sigma^u, \phi\) characterizing the economy. For brevity, denote this by a \((11 \times 1)\) parameter vector \(\theta\). Since many of these structural parameters are quite non-standard, it is difficult to obtain estimates of these from micro studies. Calibration of the model parameters is, therefore, not a viable option. Instead of calibration, we resort to simulation to assess the comparative statics and probabilistic importance of market failure in light of our model. One of the main criticisms of calibration is that it does not adequately model uncertainty since we do not include the distributional properties of the micro parameters. Hansen and Heckman (1996) provides a useful review of the comparison between calibration and simulation of complex models.

We perform the following design of experiment. Fix \(\beta = 0.96\), and make random draws for the proportion parameters \((\mu^s_0, \lambda^s, \lambda^u, \sigma^s, \sigma^u, \phi)\) independently from a hyper-rectangular uniform distribution with a \([0, 1]^6\) support. The parameters \(p^s\) and \(p^u\) are drawn from a beta distributions with the means set at \(\bar{p}^s/(\bar{p}^s + \bar{p}^u)\) and \((\bar{p}^u/(\bar{p}^s + \bar{p}^u))\) respectively. Likewise \(b\) and \(c\) are drawn from beta distributions with means \(\bar{b}/(\bar{b} + \bar{c})\) and \(\bar{c}/(\bar{b} + \bar{c})\). We generate 10,000 such samples. Denote \(\mathcal{E}(\theta)\) as the set of all such 10,000 economies. We then create a subset of these sample economies (call it \(\mathcal{E}(\theta_p)\)) such that \(p^s > p^u - b\) and it is socially optimal for the planner to intervene and mandate that at least one party should bear the cost of job training. In view of Lemma 7, such a set of economies is given by:
\[ \mathcal{E} (\theta_p) = \{ \mathcal{E} (\theta) : r(0, 0) < \min(r(1, 0), (1, 1), r(0, 1)) \} \]

Given the set of economies where the planner finds it optimal to intervene in job training, we next construct two subsets economies as follows:

\[ \mathcal{E}(\theta_1) = \{ \mathcal{E} (\theta) : r(0, 0) < \min(r(1, 0), (1, 1), r(0, 1)) , sc > \max(sc_{w}^{n*}, sc_{f}^{n*}) \} \]

\[ \mathcal{E}(\theta_2) = \{ \mathcal{E} (\theta) : r(0, 0) < \min(r(1, 0), (1, 1), r(0, 1)) , sc > sc_{w}^{n*} \} \]

We next compute three probabilities: (i) the probability that the planner will intervene in the training market, (ii) Given that the social planner asks at least party to invest in job training, none invest, (iii) Given that the social planner asks worker to pay, the worker does not find it privately optimal to pay. These three probabilities are respectively given by:

\[ \text{Prob}_1 = \frac{\# \mathcal{E}(\theta_p)}{\# \mathcal{E}(\theta)} \]  \hfill (5.27)

\[ \text{Prob}_2 = \frac{\# \mathcal{E}(\theta_1)}{\# \mathcal{E}(\theta_p)} \]  \hfill (5.28)

\[ \text{Prob}_3 = \frac{\# \mathcal{E}(\theta_2)}{\# \mathcal{E}(\theta_p)} \]  \hfill (5.29)

where the \# stands for the number of elements in the relevant set.

Table 1 plots each of these three probabilities against \( \beta \) values fixing \( \bar{p} = 10 \) and \( \bar{p}^u = 1.1 \). This basically means that on the average the high skilled sector produces about 5 times more output than low skilled sector. In economies with a lower rate of time preference the probability of market failure is higher although the social planner’s
intervention is nearly the same. Shortsighted private agents will care less for job training which boosts their future earning.

Table 1

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>.96</th>
<th>.90</th>
<th>.86</th>
<th>.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob$_1$</td>
<td>.96</td>
<td>.96</td>
<td>.95</td>
<td>.94</td>
</tr>
<tr>
<td>Prob$_2$</td>
<td>.35</td>
<td>.49</td>
<td>.57</td>
<td>.65</td>
</tr>
<tr>
<td>Prob$_3$</td>
<td>.37</td>
<td>.51</td>
<td>.60</td>
<td>.69</td>
</tr>
</tbody>
</table>

In Table 2, we plot the same probabilities for different values of $\bar{p}^s$ fixing $\beta = .96$. The probability of market failure is higher if the average output differential between the high skilled and low skilled sector is less.

Table 2

<table>
<thead>
<tr>
<th>$\bar{p}^s$</th>
<th>10</th>
<th>8</th>
<th>6</th>
<th>4</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob$_1$</td>
<td>.96</td>
<td>.96</td>
<td>.95</td>
<td>.94</td>
<td>.92</td>
</tr>
<tr>
<td>Prob$_2$</td>
<td>.34</td>
<td>.36</td>
<td>.38</td>
<td>.41</td>
<td>.44</td>
</tr>
<tr>
<td>Prob$_3$</td>
<td>.36</td>
<td>.39</td>
<td>.40</td>
<td>.44</td>
<td>.46</td>
</tr>
</tbody>
</table>

Figure 1 presents the class of economies in terms of initial distribution of skill where the social planner intervenes. The social planner appears to intervene for all ranges of initial distribution of skill which suggests that it is socially beneficial to invest in job training regardless of the initial configuration of the economy. Figure 2 presents the distribution of economies where the planner does not intervene for a range of economies classified in terms of the degree of labour market friction in the low-skilled sector. Notice that the social planner wishes not to intervene in economies mostly in economies with very low degree of labour market frictions ($\eta^u$). Figure 3 depicts the first scenario of market failure where the social planner mandates that at least one party should pay for job training but neither likes to pay. Figure 4 plots the second scenario of market failure where the planner mandates that the worker should pay but the worker refuses. Both these figures suggest that the likelihood of such market failures is less in economies which are populated initially by a larger number of skilled workers and firms. In other words, the chance of
market failure dramatically increases in economies with high initial inequality in the skill distribution.

<Figures 1 through 4 come here>

6 Conclusion

In this paper, we attempt to explain two apparently conflicting stylized facts in the UK labour market. First, there is an acute skill shortage in the UK economy. High skilled positions remain vacant for a long time while there is an excess supply of intermediate skills. Second, there exists a substantial high to low skill premium. There is unexploited profit opportunity in the high skilled sector while neither the worker nor the firm appear to take advantage of these through job training. We propose an explanation of this anomaly in terms of a coordination failure of firm’s and worker’s decisions regarding job training. Our model demonstrates that while it is socially optimal to invest in job training, the private sector may not internalize this benefit. There could be underinvestment or possibly no investment in training. Such a coordination failure in job training is a distinct possibility in a range of economies which have high inequality in the initial skill distribution. This gives rise to the question whether the government should step in with an agenda of active public policy to correct this market failure. In order to come to a definitive answer to this policy question, one needs to model public policy taking into consideration of the government budget constraint.
7 Appendix 1

Proof of Lemma 1: The steady state solutions using (2.5) and (2.3) are given by

\[ \sigma^u (\mu^{uw} - u^\mu) = \lambda^u u^u v^u \quad (7.30) \]
\[ \sigma^u (\mu^{uf} - v^u) = \lambda^u u^u v^u \quad (7.31) \]

\((\mu^{uw} - u^\mu) = (\mu^{uf} - v^u)\) or \(v^u = u^\mu - (\mu^{uw} - \mu^{uf}) = u^\mu - \delta^u\), where \(\delta^u\) is the equilibrium mismatch between low-skilled workers and low-skilled firms. Use (7.30) and (7.31) to get:

\[ \sigma^u (\mu^{uw} - u^\mu) - \lambda^u u^u [u^\mu - \delta^u] = 0 \]
\[ u^\mu [u^\mu - \delta^u] - \eta^u (\mu^{uw} - u^\mu) = 0 \]
\[ [u^\mu]^2 - (\delta^u - \eta^u) u^\mu - \eta^u \mu^{uw} = 0 \]

where \(\eta^u = \frac{\sigma^u}{\lambda^u}\). Then the solutions for \(u^\mu\) and \(v^u\) are given by

\[ u^\mu = \frac{1}{2} \left( [\delta^u - \eta^u] + \sqrt{[\delta^u - \eta^u]^2 + 4\eta^u \mu^{uw}} \right) \]
\[ v^u = u^\mu - \delta^u \]

Our next task is to solve \(\mu^{uw}\) and \(\mu^{uf}\). Using (2.6) and (2.7) we get:

\[ \Delta \mu^{uw} = (\mu^{uw} - u^\mu) \quad (7.32) \]
\[ \Delta \mu^{uf} = (\mu^{uf} - v^u) \quad (7.33) \]

Conjecture a solution \(\mu^{uw} = \mu^{uf} = 0\). Since \(0 \leq u^\mu \leq \mu^{uw}\) and \(0 \leq v^u \leq \mu^{uf}\) this means \(u^\mu = v^u = 0\) as well and

\[ v^u = u^\mu = \frac{1}{2} \left( -\eta^u + \sqrt{[\eta^u]^2} \right) = 0 \]

are satisfied as well.

We can then solve for \(u^s\) from equation (2.2) and (2.4) as before

\[ u^s = \frac{1}{2} \left( [\delta^s - \eta^s] + \sqrt{[\delta^s - \eta^s]^2 + 4\eta^s \mu^{ew}} \right) \]
\[ v^s = u^s - \delta^s \]
where $\delta^s = (\mu^{sw} - \mu^{sf})$ is the equilibrium mismatch between high-skilled workers and high-skilled firms. Plugging in the values of $\mu^{sw} = \mu^{sf} = 1$ we have

$$v^s = u^s = \frac{1}{2} \left( -\eta^s + \sqrt{\eta^s + 4\eta^s} \right).$$

Proof of Lemma 2: As before the steady state solutions using (2.5) and (2.3) is given by

$$u^u = \frac{1}{2} \left( [\delta^u - \eta^u] + \sqrt{[\delta^u - \eta^u]^2 + 4\eta^u u^{uw}} \right) \quad (7.34)$$

$$v^u = u^u - \delta^u$$

In the steady state, from (2.6)

$$\Delta \mu_t^{sf} = 0 \quad (7.35)$$

which implies that $\mu^{sf} = \mu^0$ and $\mu^{uf} = \mu^0$

which after plugging into (2.7) we have

$$v^u = \mu^0$$

Next from (2.6) we get:

$$\Delta \mu_t^{sw} = u^u$$

which means $\Delta \mu_t^{sw} \geq 0$. If the equality holds $u^u = 0$. If the strict inequality holds, $\mu_t^{sw}$ approaches the upper limit which means $\mu_t^{uw}$ approaches zero. Since $u_t^u \leq \mu_t^{uw}$, this means that $u_t^u$ also approaches zero. Thus the solution is $u^u = 0$.

Now using (2.4) and (2.2) since $u^u = 0$ the steady state solutions are

$$\sigma^s(\mu^{sw} - u^s) = \lambda^s u^s v^s$$

$$\sigma^s(\mu^{sf} - v^s) = \lambda^s u^s v^s$$

again as before

$$u^s = \frac{1}{2} \left( [\delta^s - \eta^s] + \sqrt{[\delta^s - \eta^s]^2 + 4\eta^s u^{uw}} \right)$$

$$v^s = u^s - \delta^s$$
where \( \delta^s = (\mu^sw - \mu^sf) \) is the equilibrium mismatch between high-skilled workers and high-skilled firms. Plugging in the values of \( \mu^sw = 1 \) and \( \mu^sf = \mu_0^s \) we have

\[
\begin{align*}
    u^s &= \frac{1}{2} \left( (\mu_0^u - \eta^s) + \sqrt{(\mu_0^u - \eta^s)^2 + 4\eta^s} \right) \\
    v^s &= u^s - \mu_0^u.
\end{align*}
\]

**Proof of Lemma 3:** As before the steady state solutions using (2.5) and (2.3) is given by

\[
\begin{align*}
    u^u &= \frac{1}{2} \left( [\delta^u - \eta^u] + \sqrt{[\delta^u - \eta^u]^2 + 4\eta^u\mu^uw} \right) \quad (7.36) \\
    v^u &= u^u - \delta^u \quad (7.37)
\end{align*}
\]

From (2.6) and (2.7) we have

\[
\begin{align*}
    \Delta \mu^sf_t &= \mu^uw_t \\
    \Delta \mu^sf_t &= (\mu^uf_t - v^u_t)
\end{align*}
\]

Again as in Lemma 1, conjecture that \( \mu^uw = (1 - \mu^sw) = \mu^uf = 1 - \mu^sf = 0 \). Then since \( 0 \leq u^u \leq \mu^uw \) and \( 0 \leq v^u \leq \mu^uf \) then \( u^u = v^u = 0 \). Then

\[
\begin{align*}
    \Delta \mu^sw &= (\mu^uw - u^u) \quad (7.38) \\
    \Delta \mu^sf &= (\mu^uf - v^u) \quad (7.39)
\end{align*}
\]

and solutions (7.36) and (7.37) also gives

\[
\begin{align*}
    u^u &= \frac{1}{2} \left( \eta^u + \sqrt{[\eta^u]^2} \right) = 0 \\
    v^u &= u^u = 0.
\end{align*}
\]

We can then solve for \( u^s \) from equation (2.2) and (2.4) as before

\[
\begin{align*}
    u^s &= \frac{1}{2} \left( [\delta^s - \eta^s] + \sqrt{[\delta^s - \eta^s]^2 + 4\eta^s\mu^sw} \right) \\
    v^s &= u^s - \delta^s
\end{align*}
\]

where \( \delta^s = (\mu^sw - \mu^sf) \) is the equilibrium mismatch between high-skilled workers and high-skilled firms. Plugging in the values of \( \mu^sw = \mu^sf = 1 \) we have

\[
v^s = u^s = \frac{1}{2} \left( -\eta^s + \sqrt{(\eta^s + 4) \eta^s} \right).
\]
Proof of Lemma 4: Using (2.7) and (2.6) \( \mu^s f = \mu^s 0 \) and \( \mu^s w = \mu^s 0 \) do not change and are given by initial conditions. Using (2.4) and (2.2) we have \( (\mu^s 0 - u^s) = (\mu^s 0 - v^s) \) or \( v^s = u^s \). We can then solve for \( u^s \) from equation (2.2) as before,

\[
\begin{align*}
\frac{1}{2} \left( -\eta^s + \sqrt{[\eta^s]^2 + 4\eta^s \mu^s 0} \right) \\
\end{align*}
\]

Similarly using (2.5) and (2.3) we have \( (\mu^u 0 - u^u) = (\mu^u 0 - v^u) \) or \( v^u = u^u \).

\[
\begin{align*}
\frac{1}{2} \left( -\eta^u + \sqrt{[\eta^u]^2 + 4\eta^u \mu^u 0} \right) \\
\end{align*}
\]

Derivation of \( sc^w \):

Suppose the firm is not paying then \( \xi^f = 0 \) and the worker is not paying as well \( \xi^w = 0 \) then

\[
U^u (0, 0) = \frac{b}{1 - \beta} + \frac{\beta}{1 - \beta} \left( \lambda^u v^u_{00} (\omega^u_{00} - b) \right)
\]

where \( \omega^u_{00} = \phi (p^u + c^u) + (1 - \phi) b \). If the worker starts paying i.e. \( \xi^w = 1 \) we have

\[
U^u (0, 1) = -sc + \frac{b}{1 - \beta} + \frac{\beta}{1 - \beta} \left( \frac{\lambda^u v^u_{01} (\omega^u - b) \lambda^u v^u_{01} (\omega^u - b)}{1 - \beta (1 - \sigma^u - \lambda^u v^u_{01})} \right)
\]

therefore the threshold cost for worker to pay is given by equating \( U^u (0, 0) \) with \( U^u (0, 1) \):

\[
sc^w = \frac{\beta}{1 - \beta} \left( \frac{\lambda^u v^u_{01} (\omega^u - b) \lambda^u v^u_{01} (\omega^u - b)}{1 - \beta (1 - \sigma^u - \lambda^u v^u_{01})} \right) > sc.
\]

Derivation of \( sc^f \):

Note that \( sc^f \) solves

\[
J^u (1, 0) = J^u (0, 0)
\]

Based on (3.16) and (3.17) we can write the solution for \( J^u (0, 0) \) and \( V^u (0, 0) \) as:

\[
\begin{bmatrix}
J^u (0, 0) \\
V^u (0, 0)
\end{bmatrix} = \begin{bmatrix}
1 - \beta (1 - \sigma^u) & -\beta \sigma^u \\
-\beta \lambda^u v^u_{00} & 1 - \beta (1 - \lambda^u v^u_{00})
\end{bmatrix}^{-1} \begin{bmatrix}
p^u - \omega^u_{00} \\
c
\end{bmatrix} \tag{7.40}
\]
Next recall that

\[ J^u(1, 0) = p^u - \omega_{10}^u - sc + \beta[\sigma^sV^s + (1 - \sigma^s)J^s] \quad (7.41) \]

where

\[ \omega_{10}^u = \phi(p^u - sc + J_{10}^s - p^s) + (1 - \phi) \left( \frac{\beta b}{1 - \beta} - E_{10}^s \right) + \omega^s \]

and

\[ E_{10}^s = \frac{\omega^s}{1 - \beta} - \phi \left( \frac{\beta}{1 - \beta} \frac{\sigma^s (p^s + c - b)}{1 - \beta (1 - \sigma^s - \lambda^s u_{10}^s)} \right) \]

\[ J_{10}^s = \frac{p^s - \omega^s}{1 - \beta} - (1 - \phi) \left( \frac{\beta}{1 - \beta} \frac{\sigma^s (p^s + c - b)}{1 - \beta (1 - \sigma^s - \lambda^s u_{10}^s)} \right) \]

Equating \( J^u(0, 0) \) in (7.40) to (7.41) one gets the following expression for the threshold schooling cost:

\[ sc^*_n = \frac{p^u(1 - \phi) + \phi(J_{10}^s - p^s) + \beta\{\sigma^sV_{10}^s + (1 - \sigma^s)J_{10}^s\} + (1 - \phi) \left( \frac{\beta b}{1 - \beta} - E_{10}^s \right) - J^u(0, 0)}{1 - \phi} \]
8 References


Diamond and Maskin (1979),


Figure 1: Distribution of economies where planner intervenes

Figure 2: Distribution of economies where planner does not intervene
Figure 3: Distribution of economies where nobody paying when at least one should pay

Figure 4: Distribution of economies where worker not paying when the worker should pay
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