Uninsurable Risk and Financial Market Puzzles*

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JULY 2007

ABSTRACT

This paper develops an integrated model, which addresses the recent Brandt, Cochrane and Santa-Clara (2006) puzzle of reconciling low international risk sharing with a high and variable equity premium. In addition, a new currency risk premium puzzle is also addressed. Following Kocherlakota and Pistaferri (2007), we examine two market structures: (i) where private risk cannot be insured and (ii) where the private risk can be partially insured by striking long term insurance contract with truth revelation constraint. Our GMM estimation based on the US-UK financial and cross-sectional household spending data lends support to the second market environment.

JEL Classification: E32; G11; G12

Keywords: Currency Premium, Equity Premium, Exchange Rate.

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*We would like to acknowledge the insightful comments of Etsuro Shioji, Tokuo Iwaisako, Fumio Hayashi, Toni Braun, Tomoyuki Nakajima and Makoto Saito. The first author gratefully acknowledges the competent research assistance by Soyeon Lee and Jiho Lee, and a British Academy grant to sponsor this project. The third author gratefully acknowledges the Grant-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, Science and Technology of Japan. The usual disclaimer applies.

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1 Introduction

Several puzzles are of concern to financial economists. First is the risk sharing puzzle. International financial markets fail to share risks in the way a textbook model suggests. In a world of full financial integration, all country consumption growth rates will move in unison. However, this hardly happens. Consumption growth rates are not correlated much across countries. There is not enough international risk sharing.

The second puzzle connects to the domestic financial markets. The equity premium which is the excess return of equity over risk-free rate is too high and volatile, while the aggregate consumption process is too smooth. Following the paper of Mehra and Prescott (1985) this has been known for two decades as the equity premium puzzle.

The third puzzle is the currency premium puzzle. In a model with forward and spot markets for currency trade, the required variability of the excess return on currency (future spot rate over forward rate) is excessively large given the small size of the consumption risk. Stylized facts suggest that there is large predictable variation in the excess return on currency (see Backus et al. (1993) and Bekaert (1996)); the excess return on currency is too volatile to be consistent with the low aggregate consumption risk.\(^1\)

The resolution of these three puzzles must come from common economic fundamentals. The challenge is to reconcile the above three anomalies in a unified theoretical framework with a common pricing kernel. This is what we precisely aim to do in this paper. It is well known that the representative agent pricing kernel fails to meet this challenge (see, for example, Brandt et al. (2006) and Kocherlakota and Pistaferri (2006, 2007)). Constantinides and Duffie (C-D for short, 1996) invent a pricing kernel which allows consumer heterogeneity in an incomplete market setting. A recent stream of literature attempts to resolve some of these aforementioned puzzles by using the C-D pricing kernel.\(^2\) However, hardly any effort is made to reconcile the above three puzzles in terms of a common pricing kernel. The recent paper by Brandt et al. (2006) points out that reconciling low risk sharing puzzle with high equity premium is difficult when the observed real exchange rate process is too smooth. We call this the Brandt et al. puzzle. Basu and Wada (2006) show that the C-D pricing kernel can resolve the Brandt et al. puzzle. However, Basu and Wada (2006) do not address the currency premium puzzle which adds another level of difficulty in reconciliation of domestic

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\(^1\)This currency premium puzzle evolved in the literature as follows. A simple version of the unbiased forward rate hypothesis tells us that the forward rate should be the unbiased predictor of the future spot rate. This is known as the unbiased forward rate hypothesis. Bekaert and Hodrick (1993) explain how the unbiasedness hypothesis fails to hold when agents maximize utility. There are also two related puzzles such as exchange rate volatility puzzle and forward rate persistence puzzle (Bekaert 1996). However, these two puzzles are connected to the more fundamental puzzle that the currency risk premium is too volatile. The representative-agent model is unable to reconcile this excessive volatility of the currency risk premium. Our scope in this paper is to reconcile the time varying currency premium with low international risk sharing and high and variable equity premium in an integrated general equilibrium framework with heterogeneous consumers.

\(^2\)See, for example, Brav et al. (2002), Vissing-Jorgensen (2002) and Kubota et al. (2006).
and international financial puzzles.\textsuperscript{3}

In this paper, we undertake a comprehensive empirical analysis to address these domestic and international financial puzzles. The specific question that we address is the following. Can we find a candidate pricing kernel which can reproduce high and volatile domestic equity premium, low cross-country consumption growth correlations, smooth real exchange rates and variable currency premium? Even if such a pricing kernel theoretically exists, we face two formidable challenges. First, the Euler equation for this pricing kernel must not be rejected by data. Second, the risk aversion parameter implied by this pricing kernel must be empirically plausible.

Our paper is motivated by the recent work of Kocherlakota and Pistaferri (2006). They invent two pricing kernels based on the following two market environments: (i) domestic financial markets are incomplete in the sense that private risk cannot be insured, but aggregate risks can be hedged in international financial markets and (ii) the private shocks can be contracted in advance in insurance markets with appropriate incentive constraints. The pricing kernels of Kocherlakota and Pistaferri are based on the cross-sectional moments of the consumption distribution. Kocherlakota and Pistaferri find that the latter market environment receives greater support based on the real exchange rate data. However, they basically address the real exchange rate puzzle and do not address the Brandt et al. and currency premium puzzles mentioned earlier.\textsuperscript{4}

Two features are novel in our framework. First, in contrast to Kocherlakota and Pistaferri (2006), we have an integrated model which includes sequential trading in stocks, bonds and currency. Such a framework is able to address both the domestic equity premium puzzle and international currency premium puzzles in terms of some common economic fundamentals. These economic fundamentals are: (i) a common risk aversion and (ii) consumption heterogeneity both between and within countries. Second, based on our integrated framework, we suggest a novel way to test the excess return equations without solving the model explicitly. Our empirical approach uses single equation calibration based on the unconditional moments of the excess return equations as well as the Generalized Method of Moments (GMM) which involves cross-equation restrictions.

The paper is organized as follows. In Section 2, we lay out the model. Section 3 addresses the empirical implementation of the model. Section 4 explains the data. Section 5 explains the calibration and estimation strategy and reports the empirical results. Section 6 concludes.

\textsuperscript{3}Both Brandt et al. (2006) and Basu and Wada (2006) leave out the currency premium puzzle. Moreover, Basu and Wada (2006) assume an exogenous real exchange rate. Thus, a related real exchange rate smoothness puzzle is not addressed by Basu and Wada (2006).

\textsuperscript{4}In a different paper, Kocherlakota and Pistaferri (2007) address the equity premium issue using these two frameworks. However, they do not address the currency premium puzzle in this paper. Our aim here is to address the issue whether the same fundamental can explain all three puzzles in an integrated framework.
2 The Model

2.1 Domestically Incomplete Markets

The market environment is similar to that in Kocherlakota and Pistaferri (2006) as well as Golosov and Tsyvinski (2007) except that we have explicit stock, bond and currency trading. There are infinitely many agents who are ex ante identical in terms of preference. They only differ in terms of skill shocks which have private and aggregate components. Domestic financial markets are incomplete in the sense that agents cannot insure their individual specific risks alone using any insurance contract. However, there are sequential asset markets in stocks, bonds and currency, where agents can partially hedge the aggregate shocks. Time is finite and indexed as \( t = 0, 1, 2, \ldots, T \).

There are two generic countries in the world: home and foreign (denoted by \(*\) ). There are infinitely many agents in the economy. Agents are not country specific in nature. They only differ in terms of private history of skill shocks. At any date \( t \), an agent experiences an idiosyncratic skill shock \( \theta_t \) which is drawn from a finite set \( \Theta \). In addition, all agents are exposed to the same aggregate shock \( z_t \) which is drawn from an uncountable set \( Z \). At date \( t \), the private shock history \( \theta^t \) and public shock history \( z^t \) are the \( t \)th component of \( \Theta^T \) and \( Z^T \) which is defined as :

\[
\theta^t = (\theta_1, \theta_2, \ldots, \theta_t), \tag{1}
\]

\[
z^t = (z_1, z_2, \ldots, z_t). \tag{2}
\]

with respective probabilities \( \pi(\theta^t) \) and \( \psi(z^t) \). We assume that these two shocks are drawn independently which means that by observing the aggregate shock one cannot infer anything about the private shocks. We invoke the law of large numbers which means that at any date \( t \), there are exactly \( \pi(\theta^t) \) agents with the private history \( \theta^t \). We look at the problem from the home country’s perspective. Home country produces two goods, tradeable \( y^t_{TR} \) and nontradeables \( y^t_{NT} \), with the following technologies:

\[
y^t_{TR}(z^t, \theta^t) = \phi^t_{TR}(z^t, \theta^t)l^t_{TR}, \tag{3}
\]

\[
y^t_{NT}(z^t, \theta^t) = \phi^t_{NT}(z^t, \theta^t)l^t_{NT}, \tag{4}
\]

where \( l^t_{TR} \) and \( l^t_{NT} \) is the labour used in the respective sectors, \( \phi^t_{TR} \) and \( \phi^t_{NT} \) are the respective marginal products of labour. Note that these labour productivities depend on the history of public and private shocks, \( z^t \) and \( \theta^t \).

The aggregate outputs of traded and nontraded goods for the home country are:

\[
Y^t_{TR}(z^t) = \sum_{\theta^t} y^t_{TR}(z^t, \theta^t)\pi(\theta^t), \tag{5}
\]

\[
Y^t_{NT}(z^t) = \sum_{\theta^t} y^t_{NT}(z^t, \theta^t)\pi(\theta^t). \tag{6}
\]
There are five assets: (i) two home nominal stocks which are claims to the nominal proceeds from traded and non-traded sectors, (ii) a riskfree one period nominal bond which pays a nominal interest rate of $i_t$, (iii) home currency traded in the international spot and forward markets. We assume that the only assets which are traded abroad are spot and forward contracts in currency, while stocks and bonds are not internationally traded. The currency performs two roles here: (i) role as a medium of exchange which is specified as a cash-in-advance constraint; (ii) store of value which means that the same currency can be invested in international spot ($s$) and forward ($f$) markets. The spot transaction means converting home currency into foreign currency using a spot market, and convert this back into home money using a spot market next period. Forward transaction means that the home money is converted into foreign money now using a spot market and then convert this back into home money next period using a forward market which is contracted now. There are several other trading strategies in which we are not interested because our focus is on the one-period currency premium which is defined conventionally as the forward minus the expected spot rate.

### 2.2 Timeline

The sequencing of markets is crucial here. Financial markets open before the goods market. In each period at the start of the day agents trade in stocks, bonds and currency. Once the financial transactions are completed, the household takes the left over cash to transact in goods. Each household has two distinct entities: a shopper and a producer. As a producer the household produces traded and nontraded goods while as a shopper they purchase the same goods. Since in the market place there are infinitely many shoppers and producers and shopper meets a producer randomly, a cash-in-advance constraint is necessitated.

The optimization problem facing an agent in the home country is:

$$\text{Max} \quad E_1 \sum_{t=1}^{T} \beta^{t-1} \left[ \frac{\{u(c_t^{TR}, c_t^{NT})\}^{1-\gamma}}{1-\gamma} - \nu(l_t^{TR}, l_t^{NT}) \right]$$  \hfill (7)

s.t.

$$m_t^c + m_t^s + m_t^f + Q_t^{TR} \xi_t^{TR} + Q_t^{NT} \xi_t^{NT} + b_t \leq S_t^{m_{t-1}} \frac{F_{t-1}m_{t-1}^f}{S_{t-1}} + (1 + i_{t-1})b_{t-1},$$  \hfill (8)

---

5In order to keep equity premium puzzle a purely home financial puzzle, we assume here that stock and bond are nontraded assets – an extreme form of home bias. As a result, we rule out the possibility of earning a risk-free interest on the currency held from one period to another. Since there is significant home bias in asset holding (Tesar and Werner (1995)), home bias in stock and bond holding is not an unreasonable assumption.
\[ P_t^{TR}c_t^{TR} + P_t^{NT}(z^t)c_t^{NT} \leq m_t^c \]  

where \( P_t^{TR} \) is the nominal price of the traded good, \( P_t^{NT} \) is the nominal price of nontraded good, \( Q_i^j \) is the nominal price of new equity purchase, \((\xi^t - \xi^t_{t-1})\) where i=TR, NT. We suppress the history dependence of prices to economize notations. The momentary utility function \( u(c_t^{TR}, c_t^{NT}) \) is assumed to be linearly homogenous as in Backus and Smith (1993), and \( v(\cdot) \) is monotonically increasing in its argument. The expectation \( E_0 \) is computed with respect to the probability measure of all possible evolution of private and aggregate states.

Other symbols are: \( m_t^s \) is the home money invested at date \( t \) in the spot market, \( m_t^f \) is the home money invested at date \( t \) in the forward market, \( m_t^c \) is the currency used for goods purchase, \( F_t \) stands for date \( t \) forward exchange rate, and \( S_t \) represents the date \( t \) spot exchange rate. All prices are denominated in home money. All prices, interest rate and exchange rates are functions of the aggregate shock history \( z^t \) because private shocks are not contractible.

Certain facts should be kept in mind. Since agents cannot insure themselves against private skill shocks, all prices, interest rate and exchange rates are functions of public history of shocks \( z^t \) only. The crucial assumption here is that stocks and bonds do not hedge the private shocks. In this respect, the markets are domestically incomplete.

### 2.3 Setting up the lagrange

The lagrange of (7) can be written compactly as:

\[ L = E_t \left[ \sum_{j=t}^{\infty} \beta^{j-t} \frac{u(c_j^{TR}, c_j^{NT})^{1-\gamma}}{1-\gamma} - v(i_j^{TR}, i_j^{NT}) \right] + \]

\[ E_t \sum_{j=t}^{\infty} \lambda_j \left\{ \sum_{i=TR,NT} D_{ij}^{TR} \xi_{j-1} + Q_{ij}^{TR} + \frac{S_{t} m_t^s}{S_{t-1}} + \frac{F_t m_t^f}{S_{t-1}} + \right. \]

\[ + \left. E_t \sum_{j=t}^{\infty} \mu_j \left[ m_j^c - P_j^{TR} c_j^{TR} + P_j^{NT} c_j^{NT} \right] \]

where \( E_t \) is the date \( t \) mathematical expectation computed with respect to the probability measures of \( z^{t+1} \) and \( \theta^{t+1} \) and the \( D_t^{TR} \) and \( D_t^{NT} \) are dividends from traded and nontraded sectors given by:

\[ D_t^{TR} = P_t^{TR} Y_t^{TR} \]
\[ D_t^{NT} = P_t^{NT} Y_t^{NT} \]
which is independent of private shocks because they are all integrated out while aggregating with respect to individuals.

2.4 First order conditions

The following intratemporal first-order conditions are relevant, where the subscript of $u$ means the partial derivative with respect to the relevant argument:

$$ c_{TR}^T : u_t^\gamma c_{TR}^T = \lambda_t P_{TR}^t $$  

(10)

$$ c_{NT}^T : u_t^\gamma c_{NT}^T = \lambda_t P_{NT}^t $$  

(11)

$$ l_{TR}^T : v_l^T \pi(\theta^t) \psi(z^t) = \lambda_t \phi_{TR}^t P_{TR}^t $$  

(12)

$$ l_{NT}^T : \beta^t v_{lNT} \pi(\theta^t) \psi(z^t) = \lambda_t \phi_{NT}^t P_{NT}^t $$  

(13)

$$ b_t : -\lambda_t + E_t((1 + \iota_t)\lambda_{t+1} $$  

(14)

$$ \zeta^i_t : -\lambda_t Q^i_t + E_t(\lambda_{t+1}(Q^i_{t+1} + D^i_{t+1})) = 0 \ \text{for} \ \ i = TR, NT $$  

(15)

$$ m^s_t : -\lambda_t + E_t \lambda_{t+1} \frac{S_{t+1}}{S_t} = 0 $$  

(16)

$$ m^f_t : -\lambda_t + E_t \lambda_{t+1} \frac{F_t}{S_t} = 0 $$  

(17)

$$ m^c_t : -\lambda_t + \mu_t = 0 $$  

(18)

The last F.O.C (18) basically means that the agents allocate money for transaction purpose so as to equate the marginal benefit of transaction to the marginal opportunity cost of the foregone earnings from currency trading.

Based on the intratemporal first order conditions it is straightforward to verify the following static efficiency condition for labour supply decision:

$$ \frac{u_{cTR}^* \phi_{TR}^*}{u_{cNT}^* \phi_{NT}^*} = \frac{v_{lTR}^*}{v_{lNT}^*} $$  

(19)

which basically shows the equivalence between the ratio of marginal disutilities of labour and the corresponding marginal utilities from consumption in each sector.
2.5 Reduction to a Composite Good Problem

It is convenient to reduce the two good setting to a composite good problem following the same procedures as in Kocherlakota and Pistaferri (2006). Exploiting the linear homogeneity of the momentary utility function and the duality property, we can write:

\[ P_t c_t = P_T R_t c_{TR} + P_{NT} c_{NT} \]  

where \( P_t \) is the minimum expenditure required to attain one unit of utility. In other words,

\[ P_t = \min_{c_{TR}, c_{NT}} [P_T R_t c_{TR} + P_{NT} c_{NT}] \]

s.t.

\[ u(c_{TR}, c_{NT}) = 1 \]

which means that the momentary utility \( u(c_{TR}, c_{NT}) \) is nothing, but the real consumption expenditure or a composite consumption good which we label \( c_t \) hereafter.

2.6 Intertemporal First Order Conditions

Based on this composite consumption, (10), (11), (18) can be combined to obtain:

\[ c_t : \beta^t \xi_t^{-\gamma} \pi(\theta^t(z_t)) - \lambda_t P_t = 0 \]  

3 Monetary Policy

Monetary policy in this model represents an initial cross-country distribution of money stocks namely home money, \( M_0 \), and foreign money, \( M_0^* \), to fix the date 0 spot rate such that

\[ M_0 = S_0 M_0^* \]  

In other words, central banks in both home and foreign countries coordinate monetary policies in such a way that the initial spot rate \( S_0 \) is pinned down. After this, the central banks keep hands off and let the nominal exchange rate float according to currency trading among countries.

3.1 Initial Distribution of Assets

Initial distribution of stocks and bonds are such that

\[ \sum_{\theta^0} \xi_0(\theta^0, z^0) \pi(\theta^0) = 1, \]  

\[ \sum_{\theta^0} b_0(\theta^0, z^0) \pi(\theta^0) = 0, \]  

\[ \sum_{\theta^0} m_0(\theta^0, z^0) \pi(\theta^0) = M_0. \]
3.2 Market Equilibrium

In an equilibrium, the home country solves (7).

1. Its stock market clears at each date which means \[ \sum_{\theta^t} \xi_t(\theta^t, z^t)\pi(\theta^t) = 1. \]

2. Its bond market clears at each date which means \[ \sum_{\theta^t} b_t(\theta^t, z^t)\pi(\theta^t) = 0. \]

3. The spot market clears which means \[ X_t^{cT}(z^t, \theta^t) + X_t^{cF}(z^t, \theta^t) = S_t \sum_{\theta^t} (m_t^s(\theta^t, z^t) + m_t^f(\theta^t, z^t))\pi(\theta^t) \text{ for all } t. \]

4. The forward market clears which means that \[ X_{t+1}^{f}(z^{t+1}, \theta^{t+1}) = 0 \text{ for all realizations } z^{t+1}. \]

5. Traded and Nontraded markets clear at each date meaning \[ X_t^{cTR}(z^t, \theta^t) + X_t^{cTR}(z^t, \theta^t) = \sum_{\theta^t} (y_t^{TR}(z^t, \theta^t) + y_t^{TR}(z^t, \theta^t))\pi(\theta^t), \text{ for all } \theta^t. \]

3.3 Equity Premium Equation

Plugging (23) into (15) and rearranging terms, we get:

\[ q_i^t(z^t)c_t^i(\theta^t, z^t)^{-\gamma} = \beta \sum_{z^{t+1}} \{ q_i^{t+1}(z^{t+1}) + d_i^{t+1}(z^{t+1}) \} \psi(z^{t+1}|z^t) \sum_{\theta^{t+1}} c_{t+1}(\theta^{t+1}, z^{t+1})^{-\gamma}\pi(\theta^{t+1}|\theta^t), \]

where \( q_i^t(z^t) = Q_i^t(z^t)/P_t \) is the real \( i \)th equity price and \( d_i^{t+1}(z^{t+1}) = D_i^{t+1}(z^{t+1})/P_{t+1} \) is the real dividend from the \( i \)th share.\(^6\)

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\(^6\)This is the analogous to equity premium equation of Kocherlakota and Pistaferri (2007). The only difference is that the consumption is defined in terms of composite consumption units.
Define
\[
E(c_{t+1}^{-\gamma}|z^{t+1}, \theta^t) = \sum_{\theta^{t+1}} c_{t+1}(\theta^{t+1}, z^{t+1})^{-\gamma} \pi(\theta^{t+1}|\theta^t)
\] (33)
as the $\gamma$th cross-sectional raw moment of composite good consumption conditional on the private history $\theta^t$ and public history $z^{t+1}$. Thus, (33) can be rewritten as:
\[
q_t(z^t)c_t(\theta^t, z^t)^{-\gamma} = \beta \sum_{z^{t+1}} \{q_{t+1}^i(z^{t+1}) + d_{t+1}^i(z^{t+1})\} \psi(z^{t+1}|z^t) E(c_{t+1}^{-\gamma}|z^{t+1}, \theta^t).
\] (34)

Next, integrating both sides over $\theta^t$ and using the law of iterated expectations, we get:
\[
q_t(z^t)E(c_t^{-\gamma}|z^t) = \beta \sum_{z^{t+1}} \{q_{t+1}^i(z^{t+1}) + d_{t+1}^i(z^{t+1})\} \psi(z^{t+1}|z^t) E(c_{t+1}^{-\gamma}|z^{t+1}),
\] (35)
which can be rewritten in terms of real equity return ($R_{im}^i$) as:
\[
1 = \beta E_t \left\{ R_{mt+1}^i \frac{E(c_{t+1}^{-\gamma}|z^{t+1})}{E(c_t^{-\gamma}|z^t)} \right\},
\] (36)
where
\[
R_{mt+1}^i = \frac{q_{t+1}^i + d_{t+1}^i}{q_t^i}.
\] (37)

**Defining a Market Portfolio** Based on the traded and nontraded stock returns one can now define a market portfolio return as:
\[
R_{mt+1} = \sum_{i=TR,NT} \xi_{t+1}^i R_{mt+1}^i
\] (38)

Since in equilibrium $\xi_{t+1}^i = 1$ for all $i$, the equilibrium market portfolio return is the sum of the returns on traded and nontraded stocks. In other words:
\[
R_{mt+1} = \sum_{i=TR,NT} R_{mt+1}^i
\] (39)

Using a similar procedure, we can obtain the real risk-free rate ($R_f$) equation based on the nominal bond:
\[
1 = \beta E_t \left\{ R_{ft+1} \frac{E(c_{t+1}^{-\gamma}|z^{t+1})}{E(c_t^{-\gamma}|z^t)} \right\},
\] (40)
where
\[
R_{ft+1} = \frac{(1 + i_{t+1})P_t}{P_{t+1}}.
\] (41)

Define the incomplete market stochastic discount factor as
\[
sdf_{t+1} = \frac{\beta E(c_{t+1}^{-\gamma}|z^{t+1})}{E(c_t^{-\gamma}|z^t)}. 
\] (42)
Subtracting (41) from (37), we get the equity premium equation:\(^7\)

\[
E_t \left[ \{R_{mt+1} - R_{ft+1}\}sd_{t+1}\right] = 0.
\] (43)

### 3.4 Currency Premium Equation

The first-order conditions for \((s)\) and \((f)\) give the spot and forward rate equations as follows:

\[
m_s^t : E_t \left\{ \frac{\lambda_{t+1} S_{t+1}}{\lambda_t S_t} \right\} = 1,
\] (44)

\[
m_f^t : E_t \left\{ \frac{\lambda_{t+1} F_t}{\lambda_t S_t} \right\} = 1.
\] (45)

Mimicking the same steps as in equity premium equation (43), we obtain

\[
\beta E_t \left\{ \frac{S_{t+1}}{S_t} \frac{P_t}{P_{t+1}} \frac{E(c^t_{t+1}|z^{t+1})}{E(c^t_{t+1}|z^t)} \right\} = 1,
\] (46)

\[
\beta E_t \left\{ \frac{F_t}{S_t} \frac{P_t}{P_{t+1}} \frac{E(c^t_{t+1}|z^{t+1})}{E(c^t_{t+1}|z^t)} \right\} = 1.
\] (47)

Subtracting (46) from (47), we obtain the currency premium equation:

\[
E_t \left\{ \left( \frac{F_t - S_{t+1}}{S_t} \right) \frac{P_t}{P_{t+1}} \right\} \frac{sd_{t+1}INC}{INC_{t+1}} = 0.
\] (48)

### 3.5 Real Exchange Rate Equation

The real exchange rate (denoted as \(rx_t\)) story is the same as in Kocherlakota and Pistaferri (2006). It is the ratio of composite prices of each country (defined in utils). Taking this ratio and eliminating the Lagrange multipliers, we get:

\[
r_{INC}^t = \frac{E(c^t_{t+1}|z^{t+1})}{E(c^t_{t+1}|z^t)}\kappa,
\] (49)

where \(\kappa\) is a constant that depends on the Lagrange multipliers of the flow resource constrains of home and foreign countries at date 0.

### 3.6 Summing up the INC equations

Based on this analysis, we end up with the following three equations which are of empirical interest.

The equity premium equation:

\[
E_t \left[ \{R_{mt+1} - R_{ft+1}\}sd_{t+1}INC\right] = 0.
\] (50)

---

\(^7\)This is analogous to equity premium equation of Kocherlakota and Pistaferri (2007). The only difference is that the consumption is defined in terms of composite consumption units.
The currency premium equation:

\[ E_t \left\{ \left( \frac{F_t - S_{t+1}}{S_t} \right) \frac{\overline{P}_t}{\overline{P}_{t+1}} sd t^{INC} \right\} = 0. \]  

(51)

The real exchange rate equation:

\[ r x_t^{INC} = \frac{E(c_t^{\gamma})}{E(c_t^{\gamma})} \cdot \text{const} \]  

(52)

4 Private Information Pareto Optimum Setting

In this alternative setting, agents are able to insure against private specific skill shocks. The model is a dynamic extension of Mirrlees (1971) type private information setting. Trading convention is similar to Golosov and Tsyvinski (2006) and Kocherlakota and Pistaferri (2006). Agents are all ex ante identical with the same preferences. There is a continuum of insurance firms which act on behalf of the households and play the following roles: (i) produce the traded and nontraded goods by hiring workers, (ii) sell these goods in national and international markets, (iii) trade among themselves in stock, bond and currency in sequential markets and, finally, (iv) with the resulting profits from this trade insure households against private skill shocks. Timing of goods and financial markets is exactly same as in INC setting. The same cash-in-advance constraint (?) applies to the insurance companies when they trade in goods.

These insurance firms are owned equally by all agents. At date 0 before the realization of private and aggregate shocks, the contract market opens only once. In this market, these competitive insurance firms offer contracts to the households about consumption bundles of traded and nontraded goods \( \{c_t^{TR}, c_t^{NT}\} \) which provide maximum ex ante utility to the households. Since the insurance company does not observe the private shock history and labour supply, it stipulates contract about the observed output sequence of traded and nontraded goods \( \{y_t^{TR}, y_t^{NT}\} \) such that it is incentive compatible for the agents to reveal the truth about the history of private shocks. These contracts are long-term contracts, which means full commitment on both sides. After the contract market closes, from date 1 onward the insurance firms start trading in goods and financial markets in the same sequential manner as in the INC setting.

A typical insurance company located in the home country maximizes the present value of the nominal payoffs to its owners:

\[ \max_{\{c_t^{TR}, c_t^{NT}, y_t^{TR}, y_t^{NT}, b_t, m_t, m_t'\}} \sum_{t=0}^{T} \prod_{i=1}^{t} (1 + \rho_i(z^i))^{-1} \Pi_i(z^i) \psi(z^i) \]  

(53)
\[
\Pi_t(z^t) + m_t^a(z^t) + m_t^b(z^t) + m_t^c(z^t) + Q_t(z^t)\xi_t(z^t) + b_t(z^t) \\
\leq \sum_{\theta_t} \left[ \left\{ P_t^{TR}(z^t)Y_t^{TR}(\theta^t, z^t) + P_t^{NT}(z^t)Y_t^{NT}(\theta^t, z^t) \right\} \pi(\theta^t) \right] \xi_{t-1}(z^t) \\
+ Q_t(z^t)\xi_{t-1}(z^{t-1}) \cdot \frac{S_t(z^t)}{S_{t-1}(z^{t-1})} m_{t-1}^a(z^{t-1}) \\
+ F_{t-1}(z^{t-1}) \cdot \frac{S_{t-1}(z^{t-1})}{S_{t-1}(z^{t-1})} m_{t-1}^f(z^{t-1}) + (1 + \iota(z^{t-1}))b_{t-1}(z^{t-1}).
\]

Cash-in-advance constraint:
\[
\sum_{\theta_t} \left\{ P_t^{TR}(z^t)c_t^{TR}(\theta^t, z^t) + P_t^{NT}(z^t)c_t^{NT}(\theta^t, z^t) \right\} \pi(\theta^t) \leq m_t^c(z^t)
\]

Participation constraint:
\[
\sum_{t=0}^{T} \beta^t \sum_{\theta^t} \sum_{z^t} \left[ \frac{u(c_t^{TR}(\theta^t, z^t), c_t^{NT}(\theta^t, z^t))}{1 - \gamma} - v \left( \frac{y_t^{TR}(\theta^t, z^t)}{\phi_t^{TR}(\theta^t, z^t)}, \frac{y_t^{NT}(\theta^t, z^t)}{\phi_t^{NT}(\theta^t, z^t)} \right) \right] \pi(\theta^t) \psi(z^t) \geq u
\]
and the incentive constraint:
\[
\sum_{t=0}^{T} \beta^t \sum_{\theta^t} \sum_{z^t} \left[ \frac{u(c_t^{TR}(\theta^t, z^t), c_t^{NT}(\theta^t, z^t))}{1 - \gamma} - v \left( \frac{y_t^{TR}(\theta^t, z^t)}{\phi_t^{TR}(\theta^t, z^t)}, \frac{y_t^{NT}(\theta^t, z^t)}{\phi_t^{NT}(\theta^t, z^t)} \right) \right] \pi(\theta^t) \psi(z^t) \geq \rho(z^t)
\]
where \(\Pi_t(z^t)\) is the date \(t\) cash flow of the insurance firm in the \(j\)th country contingent on the shock history \(z^t\), \(\rho(z^t)\) is the \(z^t\) contingent discount rate and \(\theta^t_R\) is the history of shocks that the household reports to the financial intermediaries. Since the insurance firm does not observe private shock history, all its relevant choices are dependent on the aggregate shock history \(z^t\).

Using the same expenditure minimization problem as in \(INC\) one can reduce the problem to the same composite good problem as in (20).

### 4.1 Market Equilibrium

Given the initial distribution of money, stock and bond holdings as in (24) through (27), equilibrium represents an allocation \(\{c_t^{TR}, c_t^{NT}, y_t^{TR}, y_t^{NT}\}\) such that

1. Insurance firms solve (53) taking \(\{\rho_i(z^i)\}\) and \(u\) as given.
2. Stock market clears at each date which means \( \sum_{\theta^t} \xi_t(\theta^t, z^t) \pi(\theta^t) = 1. \)

3. Bond market clears at each date which means \( \sum_{\theta^t} b_t(\theta^t, z^t) \pi(\theta^t) = 0. \)

4. The spot market clears which means

\[
\sum_{\theta^t} (m^s_t(\theta^t, z^t) + m^f_t(\theta^t, z^t)) \pi(\theta^t) = S_t \sum_{\theta^t} (m^s_t(\theta^t, z^t) + m^f_t(\theta^t, z^t)) \pi(\theta^t) \text{ for all } t. \tag{57}
\]

5. The forward market clears which means

\[
\sum_{\theta^{t+1}} m^s_{t+1}(\theta^{t+1}, z^{t+1}) \pi(\theta^{t+1}) = 0 \text{ for all realizations } z^{t+1}. \tag{58}
\]

6. Traded and Nontraded markets clear at each date meaning

\[
\sum_{\theta^t} (c^T_t(z^t, \theta^t) + c^TR_t(z^t, \theta^t)) \pi(\theta^t) = \sum_{\theta^t} (y^T_t(z^t, \theta^t) + y^TR_t(z^t, \theta^t)) \pi(\theta^t), \tag{59}
\]

\[
\sum_{\theta^t} c^NT_t(z^t, \theta^t) \pi(\theta^t) = \sum_{\theta^t} y^NT_t(z^t, \theta^t) \pi(\theta^t). \tag{60}
\]

### 4.2 Summing up the PIPO equations

Following Kocherlakota (2005), one can show that the equilibrium allocation for this decentralized economy solves a constrained social planning problem, where the constraints involve the truth revelation incentive constraint. Because of this optimality, Kocherlakota and Pistaferri call this allocation Private Information Pareto Optimum (PIPO for short).

Using this setting, we can show (proof is relegated to the appendix) that the stochastic discount factor or the pricing kernel (call it \( sdf^{PIPO}_{t+1} \)) is given by:

\[
sdf^{PIPO}_{t+1} = \frac{\psi(z^{t+1}|z^t) \, \overline{t}_{t+1}(z^{t+1}) \, \overline{p}_{t}(z^t)}{(1 + \rho_{t+1}(z^{t+1})) \, \overline{p}_{t}(z^t)} = \frac{\beta E(c_t^z \mid z^t)}{E(c_{t+1}^z \mid z^{t+1})}. \tag{61}
\]

The equity premium and the currency premium equations are as follows:

\[
E[R_{mt+1} - R_{ft+1}] sdf^{PIPO}_{t+1} | z^t] = 0, \tag{62}
\]

\[
E \left\{ \left( \frac{F_t - S_{t+1}}{S_t} \right) \frac{\overline{P}_t}{\overline{P}_{t+1}} sdf^{PIPO}_{t+1} \mid z^t \right\} = 0. \tag{63}
\]

The real exchange rate equation is the same as in Kocherlakota and Pistaferri (2006) and given by:

\[
q^{PIPO}_t = \frac{E(c_t^z)}{E(c_{t+1}^z)} \nu, \tag{64}
\]

where \( \nu \) is a constant that depends on the Lagrange multipliers of the flow resource constrains of home and foreign countries at date 0.
5 Empirical Formulation

In order to bring these two scenarios closer to the data, we consider a specific parameterization of the post trade world composite consumption process. For the sake of empirical application, we need to take some liberty in notation here. Define $i$ as an investor and $j$ as a country. Call the date $t$ consumption of the $i$th investor in the $j$th country as $c_{ij,t}$. We view the $i$th investor as an individual with a draw $(\theta^i, z^i)$ from the space $(\theta^T, z^T)$. In other words, define:

$$c_{ij,t} = c_{jt}(\theta^i, z^i)$$

In a similar spirit as in Sarkissian (2003), we represent the post-trade allocation of consumption as follows.$^8$

$$c_{ij,t} = \delta_{ij,t}\delta_{jt,t}C_t$$

where $\delta_{ij,t}$ is the $i$th investor’s share in country $j$’s consumption and $\delta_{jt,t}$ is the country $j$’s share in world consumption $C_t$. We assume the following processes for $\delta_{ij,t}$ and $\delta_{jt,t}$:

$$\delta_{ij,t} = \exp(u_{ij,t}\sqrt{x_{jt,t}} - \frac{x_{j,t}}{2})$$

and

$$\delta_{jt,t} = \exp(u_{j,t}\sqrt{x_t} - \frac{x_t}{2}),$$

where $u_{ij,t}$ and $u_{j,t}$ are standard normal shocks which are i.i.d. across countries, individuals and time, $x_{j,t}$ is the within-country variance of country $j$’s log consumption level and $x_t$ is the between-country variance of log consumption level of country $j$ and the rest of the world.

The $s$th raw moment of the cross-sectional distribution of consumption is given by:

$$E_t(c_{ij,t}^s) = C_t^s \exp \left( \frac{s^2 - s}{2}(x_{j,t} + x_t) \right).$$

Note that by construction the aggregate consumption is the sum of individual consumption which can be checked by setting $s = 1$. Thus, this lognormal process satisfies the feasibility condition. The next issue is: Does it satisfy the optimality conditions? We follow a reverse engineering approach here. If we can find a pricing kernel for each market environment that supports this allocation of world consumption and is also independent of agent’s private history, then it must be satisfying individual optimality conditions.

$^8$Sarkissian (2003) writes the post trade allocation in terms of consumption growth rate, while we write here in terms of the level of consumption. The motivation for doing this is to apply this post-trade allocation to the Kocherlakota-Pistaferri (2006, 2007) discounting methodology. The Kocherlakota-Pistaferri incomplete market discount factor is based on the growth rates of the cross-sectional moments of consumption in level while Sarkissian (2003) and also Semenov (2004) use the Constantinides and Duffie (1996) discount factor which is based on the cross-sectional average of the intertemporal marginal rates of substitution. See Kocherlakota and Pistaferri (2007) for a discussion of the difference in methodology.
5.1 Pricing Kernels

Plugging (68) into (42) and evaluating at \(s = -\gamma\), we obtain the following pricing kernel for the INC environment from the \(j\)th country’s perspective:

\[
sd_f^{j,INC}_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \exp \left( \frac{(\gamma^2 + \gamma)}{2} (x_{jt+1} - x_{jt} + x_{t+1} - x_t) \right). \tag{69}
\]

We call this the INC discount factor \(K_1\).

Likewise plugging (68) into (61) and evaluating at \(s = \gamma\), we obtain

\[
sd_f^{j,PIPO}_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \exp \left( \frac{(\gamma^2 - \gamma)}{2} (x_{jt} - x_{jt+1} + x_t - x_{t+1}) \right). \tag{70}
\]

We call this the PIPO discount factor \(K_2\).

5.2 Real Exchange Rates in two Discounting Environments

Using (49), (64) and (68), we get the following expressions for the proportionate change in real exchange rates between the \(j\)th and \(k\)th country for these two environments:

\[
\ln \frac{rx_{t+1}^{INC}}{rx_t} = \frac{\gamma^2 + \gamma}{2} [x_{kt+1} - x_{kt} - x_{jt+1} + x_{jt}], \tag{71}
\]

\[
\ln \frac{rx_{t+1}^{PIPO}}{rx_t} = \frac{\gamma^2 - \gamma}{2} [x_{kt} - x_{kt+1} + x_{jt+1} - x_{jt}]. \tag{72}
\]

The immediate implication is that the real exchange rate is independent of the cross-country variance of consumption. Real exchange rate depreciates (appreciates) in response to increase in foreign (home) within-country variance in an INC environment. The implication is exactly reverse in a PIPO model.

The real exchange rate story is the same as in Kocherlakota and Pistaferri (2006). Let \(j\) be the home country. In an INC market environment, higher uninsurable risk triggers a precautionary demand for both traded and nontraded goods which makes real exchange rate appreciate. In a PIPO economy, on the other hand, agents buy contracts from insurance firms to insure against individual shocks subject to incentive constraints. In this setting, there is a conflict between precautionary saving (prudence) and incentive cost. Higher consumption inequality (meaning large \(x_{jt+1}\) in our context) means a lot of poor people. If leisure is a normal good, more poor people mean they shirk less which means that incentive costs are less if the risk aversion coefficient is higher.\(^9\) On the other hand, prudence is more if the risk aversion coefficient is large. In a PIPO setting the former incentive effect dominates the latter prudence effect. This means that there is less precautionary demand for goods when consumption inequality is higher. The real exchange rate thus depreciates.

\(^9\)Basu and Renstrom (2006) show that leisure is normal if the proportional risk aversion coefficient exceeds unity.
6 Addressing Three Puzzles

We follow the same principle as in Basu and Wada (2006) to address all three puzzles. Both these discount factors K1 and K2 incorporate incomplete consumption risk sharing by default. As described earlier, in the K1 discounting environment, agents cannot insure consumption at all using the domestic financial market. In the K2 discounting environment, agents can insure consumption using the domestic financial markets, but, due to hidden work effort, financial intermediaries strike incentive compatible constraint which prevents full risk sharing. Which of these two discounting environments reconciles the observed fluctuations of the real exchange rate, equity premium and currency premium better? If we get a decisive answer to this question, we have made some progress in understanding these extant financial puzzles.

6.1 Estimation Strategy

We estimate three equations jointly for alternative discounting environments INC and PIPO. These three equations are (i) the real exchange rate equation (71) and (72), (ii) the equity premium equations (43) and (62) and (iii) the currency risk premium equations (48) and (63). Hereafter, we will consider the US as the home country, and the UK as the foreign country. Thus, the pricing kernel refers to US. The estimable equations for the two discounting environments are summarized as follows:

**INC**

\[
\ln \frac{r_{xt+1}^{INC}}{r_{xt}} = \gamma^2 + \gamma [x_{UK,t+1} - x_{UK,t} - x_{US,t+1} + x_{US,t}], \tag{73}
\]

\[E_t [sdf_{t+1}^{INC} (P_{mt+1}^{US} - P_{ft+1}^{US})] = 0, \tag{74}\]

\[E_t [sdf_{t+1}^{INC} ECR_{t+1}] = 0. \tag{75}\]

**PIPO**

\[
\ln \frac{r_{xt+1}^{PIPO}}{r_{xt}} = \gamma^2 - \gamma [x_{US,t+1} - x_{US,t} - x_{UK,t+1} + x_{UK,t}], \tag{76}\]

\[E_t [sdf_{t+1}^{PIPO} (P_{mt+1}^{US} - P_{ft+1}^{US})] = 0, \tag{77}\]

\[E_t [sdf_{t+1}^{PIPO} ECR_{t+1}] = 0, \tag{78}\]

where \( ECR_{t+1} = (F_t - S_{t+1})/S_t \) represents the excess real currency return and the formulas for the pricing kernels \( sdf_{t+1}^{INC} \) and \( sdf_{t+1}^{PIPO} \) are given by (69) and (70).

6.2 Data

We treat the US as the home country and the UK as the foreign country. The US and UK data are from the following sources respectively. First, the quarterly seasonally adjusted, dollar real per capita consumption of non-durable goods is from the Bureau of Labor Statistics (BLS) for the U.S. The same for the UK (in pounds) are available from the Office for National
Statistics (ONS) and the UK Data Archive (UKDA). Second, the nominal spot and 3-month forward exchange rate at the start of each quarter are from Datastream. Third, the US and UK Consumer Price Indexes (CPIs) are from BLS and OECD main economic indicators. Fourth, the cross-sectional variance of log real per capita consumption level is calculated using data from the Consumer Expenditure Survey (CEX) for the US and from the Food and Expenditure Survey (FES) for the UK. Finally, the real return on the market portfolio and the risk-free rate are from CRSP. The sample period is from the first quarter of 1982 to the fourth quarter of 2004 because of the poor quality of the CEX data before 1982.

In order to estimate the three-equation system for each discounting scenario, we need to measure: (i) aggregate consumption $C_t$, (ii) within-country variances $x_{US,t}$ and $x_{UK,t}$, (iii) between-country variance $x_t$ and (iv) the real exchange rate $q_t$

The aggregate consumption $C_t$ is defined as

$$C_t = c_t^{US} + q_t c_t^{UK}, \tag{79}$$

where $c_t^{US}$ is constant dollar US GDP and $c_t^{UK}$ is constant pound UK GDP for the same base year and the observed real exchange rate (which is US good per unit of UK good) is defined as:

$$r_x t = \frac{S_t CPI_t^{UK}}{CPI_t^{US}}. \tag{80}$$

For each quarter, the between-country consumption variance $x_t$ is the cross-sectional variance of the log level of real per capita consumption in local currency for the US and the UK using 2005 as the base year. For within-country consumption variance, we need to use the information about the cross-sectional distribution of consumption for each country. We now explain the details of the procedure for arriving at the summary measure of within-country quarterly cross-sectional variance of real consumption for the US and the UK respectively.

### 6.3 The Within-Country Consumption Variance for the US

For the US, the measure of consumption is consumption of nondurable and services. For each household, we calculate quarterly consumption expenditures for all the disaggregate consumption categories offered by the Consumer Expenditure Survey (CEX). Then, we deflate obtained values in 2005 US dollars by the CPI’s (not seasonally adjusted, urban consumers) (the CPI series are obtained from the BLS) for appropriate consumption categories. Aggregating the household’s quarterly consumption across these categories is made according to the National Income and Product Account definition of consumption of nondurables and services.

Following Brav et al. (2002), in each quarter we drop households that do not report or report a zero value of consumption of food, consumption of nondurable and services, or

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10In April 2001, the Family Expenditure Survey (FES) was replaced by the Expenditure and Food Survey (EFS) which also covered the National Food Survey (NFS).
total consumption. We also delete from the sample the nonurban households, the households residing in student housing, the households with incomplete income responses, the households that do not have a fifth interview, and the households whose head is under 19 or over 75 years of age.

To calculate the household’s quarterly per capita consumption, we divide the quarterly consumption expenditure of each household by the number of people in the household in that quarter. The within-country consumption variance for each quarter is then calculated as the cross-sectional variance of the log household’s real per capita consumption.

### 6.4 The Within-Country Variance for the UK

For the UK, we use the Family Expenditure Survey (FES), a voluntary survey of a random sample of private households in the UK, conducted by the ONS. The data of approximately 6,500 households are collected throughout the year to cover seasonal variations in expenditures, with either the week or month in which the fieldwork is carried out being randomly assigned to each individual household. Of the data available in the FES, we utilize the diary records of daily expenditure, kept for two weeks by each individual aged 16 or over in the household survey. Using these diary data, the cross-sectional variances of logarithms of quarterly real, per capita consumption level are computed as follows. First, we calculate the household-wide consumption of nondurable and services, by adding the consumption only of nondurable and services (measured in UK pounds) for each individual in the household. The definition of nondurable and services follows that of Attanasio and Weber (1995). Second, given that the household consumption data are for the two week durations only, we multiply them by 6.5 so that the data are converted into quarterly frequency. Third, we divide the quarterly consumption of household by the total number of people in each household to derive quarterly per capita consumption of nondurables and services. Fourth, we categorize the household consumption data into four quarterly groups, based on the quarter or month the survey was conducted for the household. By dividing the data by the quarterly CPI for all items from the OECD MEI (not seasonally adjusted) with the basis of the first quarter of 2005, the real quarterly per capita consumptions are calculated. Finally, we take the natural logarithms on the data derived in the previous step, followed by the calculation of the cross-sectional variance of the log real per capita consumptions.

### 6.5 Sample Statistics

Table 1 reports the sample statistics for quarterly observations. The mean for nominal spot exchange rate, nominal 3-month forward rate, the currency premium and the growth rate of real spot exchange rate are 1.590, 1.599, -0.004 and 1.001, respectively. The mean real world consumption growth rate, the real total return on value-weighted NYSE/AMEX/NASDAQ,

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the equal-weighted NYSE/AMEX/NASDAQ, the 3-month risk-free rate and the equity premium of the equal-weighted return over the risk-free rate are 1.007, 0.026, 0.031, 0.007 and 0.024, respectively. The mean for between the country, foreign within-country and home within-country variance of log consumption levels are 0.396, 0.455 and 0.421, respectively. The quarterly mean US equity premium is 2.4% (which means about 9.6% annually). The standard deviation of the equity premium (11.6%) exemplified by the minimum (-30.8%) and maximum (30.7%). The mean currency premium is a tiny -0.4%. The standard deviation is 5% and the range of variation is from -15.6% to 15.4%.

Table 1 comes here

Table 2 reports the correlation matrix. The correlation between the spot rate and the forward rate is quite high at 0.998, and that between the spot and the currency premium is negative at -0.326 and statistically significant. The correlation between world consumption growth and the real rate of return on the value-weighted NYSE/AMEX/NASDAQ is negative at -0.099, while that between the world consumption growth and the real return on the 3-month risk-free rate is negative at -0.140. The correlation between the value-weighted NYSE/AMEX/NASDAQ return and the risk-free rate is positive at 0.069, while the correlation between currency premium and the home within-country variance of log consumption is positive at 0.033. The correlation between the equity premium and the between-country variance of log consumption level is negative at -0.07, while that between the equity premium and the foreign within-country variance of log consumption level is negative at -0.018. Finally, the correlation between the equity premium and the home within-country variance of log consumption is negative at -0.022.

Except for the correlation between forward and spot rates and one between spot and currency premium, all other correlations reported here are statistically insignificant.

Table 2 comes here

Table 3 reports the autocorrelation. There is a high autocorrelation for nominal spot exchange rate and the forward rate for the first and even the second lag, but there is a small positive autocorrelation for the currency premium, growth rate of real exchange rate, world consumption growth for the 1st lag. There is a large autocorrelation for the risk-free rate for the first and even the second lag. There is a negative autocorrelation for the return on NYSE/AMEX/NASDAQ for the first lag.

Table 3 comes here

7 Calibration and Estimation Results

The summary statistics presented in Tables 1 through 3 give us the preliminary description of the data. There are two popular methodologies of bringing the data closer to the model.
The first is the calibration, where we attempt to match some of these summary statistics to a calibrated dynamic stochastic general equilibrium (DSGE) model. The second methodology is GMM based on the Euler equations from a dynamic stochastic general equilibrium model. The GMM approach takes into account the joint restrictions on the excess return and the pricing kernel based on all the moments of the forcing process. We follow a comprehensive approach which has elements of both these methodologies. Basically, we first check the model performance based on a single equation calibration. In the next step, we perform a full blown GMM estimation. The idea was to start with the simplest possible test (a single equation calibration) in order to check for the potential of PIPO to solve the puzzles and then to apply a more general test (GMM) to estimate the model parameters. When calibrating, we can not estimate the robust standard errors of the parameters, e.g., and hence to perform standard tests, whereas it is possible to do so if GMM is used.

7.1 Single Equation Calibrations

We compute the sample counterparts of the excess return Euler equations (74), (75) and (77), (78) for a range of risk aversion coefficients from 0 to 9. The sample counterparts of the excess return equations for \( i = \text{INC} \) and PIPO are defined as:

\[
\sum_{t=1}^{T} \left[ \frac{sdf_{i}^{t}(R_{m}^{US} - R_{f}^{US})}{T} \right] = \xi_{1},
\]

\[
\sum_{t=1}^{T} \left[ \frac{sdf_{i}^{t}ECR_{t}}{T} \right] = \xi_{2},
\]

where \( \xi_{1} \) and \( \xi_{2} \) are sample residuals.

The population unconditional counterparts of (81) and (82) are basically

\[
E(\xi_{1}) = 0, \quad E(\xi_{2}) = 0.
\]

The deviations of \( \xi_{1} \) and \( \xi_{2} \) from zero gives us some preliminary idea about the model performance. Table 4 reports point estimates of \( \xi_{1} \) and \( \xi_{2} \) (using both value-weighted and equal-weighted returns). Based on this criterion, we find that the PIPO model consistently does better than INC in predicting the excess equity return and excess currency return for all plausible values of the risk aversion coefficient.

We obtain a similar picture if adopt a different criterion of checking for model adequacy. Rewrite the excess return equations in an unconditional form as:

\[
E(R_{m}^{US} - R_{f}^{US}) = -\frac{\text{cov}(sdf^{i}, R_{m}^{US} - R_{f}^{US})}{E(sdf^{i})},
\]

\[
E(ECR) = -\frac{\text{cov}(sdf^{i}, ECR)}{E(sdf^{i})}.
\]
We have calibrated the sample mean equity premium and mean currency premium by plugging in the value for the risk aversion coefficients from 1 to 9 into the sample counterparts of (84) and (85). Table 5 reports the results.

First, in the context of the with value-weighted return (columns 2 and 3), PIPO performs slightly worse than INC in the sense that the calibrated excess return from INC is close to the observed value of 0.019 when the risk aversion coefficient is between 7 and 8, while that from PIPO is still smaller than 0.019 when the risk aversion coefficient is equal to 9. Second, in the case of the equal-weighted return (columns 4 and 5), PIPO does worse than INC in the sense that the calibrated excess return from INC is close to the observed value of 0.024 when the risk aversion coefficient is between 6 and 7, while that from PIPO is always negative except for the case of risk aversion coefficient being equal to 1. For excess currency return (last two columns), PIPO performs better than INC in the sense that the calibrated excess return from PIPO is close to the observed value of -0.004 when the risk aversion coefficient is between 6 and 7, while that from INC is always positive.

Note the difference between these approaches to check for model performances. The former approach is close to what Kocherlakota and Pistaferri (2006) considers the overall performance of the Euler equation models of INC and PIPO. The latter approach which is used more in finance (see for example, page 12 of Duffie (1996)) considers the quantitative performance of the Euler equation model of INC and PIPO in terms of the first moment of excess return only. Table 5 reports the estimates of the equity premium and currency premium for different values of the risk aversion coefficient.

Finally, we turn to the real exchange rate equations (73) and (76). These real exchange rate equations are exact equations and do not involve any expectation operator. We check for the model adequacy of these two equations by performing a straightforward mean squared error (MSE) comparison based on the observed and calibrated values of the real exchange rate series. The difference between the observed and the theoretical series is attributed to measurement errors. Tables 6 presents the results for MSE for INC and PIPO models for the same range of γ values. PIPO outperforms INC in terms of the mean squared error criterion. Thus, based on the single equation calibration, we find mixed performance of PIPO in predicting the relevant financial aggregates vis-a-vis INC model.

7.2 GMM Estimation

The single-equation calibration reported so far do not tell us precisely whether a model is rejected or not rejected by the data. Nor does it provide an estimate of the fundamental risk aversion parameter γ which satisfies the joint restrictions on the excess return and the pricing kernel based on all the moments of the forcing process. In addition, the single equation calibration is really based on unconditional moments of the excess returns, while the GMM approach is based on conditional moments of excess returns. To test which model performs better in explaining the excess equity return and the excess currency return jointly,
we employ GMM estimation involving the two excess return equations and the real exchange rate equation for each model described in (73) through (78). Following sets of instruments are used: (i) a constant and a twice lagged domestic within the country variance of log consumption, (ii) a constant and twice lagged forward premium, (iii) a constant and a twice lagged real risk-free return and (iv) a constant and a twice lagged foreign within the country variance of log consumption. In order to avoid the time aggregation problem, we used lagged instruments (see Heaton (1995), for instance). Table 7 reports the parameter estimates for the risk aversion coefficients with Newey and West (1987) type HAC standard errors (Bartlett kernel with the number of autocorrelation equal to 3) below and the values of Hansen’s $J$ statistics with $p$-values below.

Based on the first set of instruments, the risk aversion coefficient from the $PIPO$ model is 4.23 for the value-weighted return and is statistically significant. The model is not rejected at 5%. The risk aversion coefficient from the $INC$ model on the right is negative at –0.786, it is not statistically significant and the model is not rejected at the 5% significance level. The $PIPO$ model performs better than $INC$ model in the sense that the magnitude of the risk aversion coefficient is plausible in the former, but it is not in the latter. Similar picture emerges when we employ the other three sets of instruments. For the second set of instruments, the risk aversion coefficient $\gamma$ from the $PIPO$ model is 3.25, it is statistically significant and the model is not rejected at 5%. For $INC$, $\gamma$ is significantly negative at -0.98, and the model is not rejected at 5%. For the third set of instruments, the risk aversion coefficient from the $PIPO$ model is 4.89, it is statistically significant and the model is not rejected at 5%. The risk aversion coefficient from the $INC$ model is –1.15, it is statistically significant and the model is not rejected at 5%. Finally for the fourth set of instruments, for the $PIPO$ model $\gamma$ is 1.34, it is statistically significant and the model is rejected at 5%. On the other hand, $\gamma$ from the $INC$ model is negative again, although it is statistically significant at the 5% level and the model is not rejected. The apparent poor performance of the fourth model is due to the weak correlation of instruments used in this model with the variables in the Euler equations. In fact, none of the correlations between the instrument (twice lagged foreign within the country variance of log consumption) and the variables in the Euler equations are statistically significant at the 5% level.

For results based on equal-weighted stock returns, none of the models are rejected. However, the risk aversion coefficient from $PIPO$ model is statistically significant and positive, while that from the $INC$ model is statistically significant and negative. In this sense, $PIPO$ model performs better than $INC$ model.

To summarize the results, the $INC$ model although being accepted in a few cases, produces an implausible negative estimate of the risk aversion coefficient. The $PIPO$ model, on the other hand, is accepted for 7 out of 8 sets of instruments at the 5% level of significance as evident from the $J$ statistic. The risk aversion coefficient ranges from 1.34 to 5.15 and it is always statistically significant. These estimates are plausible based on various microeconomic
studies. Given that the GMM estimation imposes stringent restrictions on the data, we deem our model to reconcile various financial anomalies because it yields plausible range of the risk aversion estimate.

<Table 7 comes here>

Our estimation results are consistent with Kocherlakota and Pistaferri (2006) who also find that PIPO model is supported by the data. The redeeming feature of our study is that we have an integrated model which is capable of reconciling various puzzles on the domestic and international fronts. What is especially noteworthy is that we are able to reconcile the equity premium and currency premium puzzles with a plausible degree of risk aversion within the PIPO framework.

8 Conclusion

This paper addresses a few extant anomalies on the domestic and international financial fronts. The existing literature points to the difficulty in reconciling low international risk sharing (manifested by home bias in asset portfolio) with a high and variable equity premium, variable currency risk premium and real exchange rates. We propose a way to resolve these anomalies by using the pricing kernels developed by Kocherlakota and Pistaferri (2006, 2007) which allow incomplete risk sharing in economies with consumer heterogeneity. Two types of market structures are explored: (i) domestically incomplete financial markets, where idiosyncratic privately observed shocks are uninsured, while sequential trade in assets enables agents to partially hedge publicly observed shocks and (ii) a market environment, where both private and public shocks are insured subject to truth revelation constraint by agents. Using the cross-sectional data on household consumption and between-country variances of consumption, we are able to construct two pricing kernels for these market environments. For a plausible magnitude of risk aversion, the latter market pricing kernel receives greater empirical support. This suggests that the observed behavior of equity premium, currency premium and the real exchange rates are consistent with a world economy whose real allocation mimics a dynamic Mirrlees economy’s social planning optimum.

The immediate question arises about the practical implementation of this Mirrlees type allocation in a world economy, where agents trade in assets, while they are exposed to private skill shocks. In our model, this is implemented by fictitious insurance firms striking incentive compatible contracts with full commitment which is a stretch from the real world. About the issue of practical implementation of the PIPO allocation, one may speculate a bit and leave it for future research. Perhaps a global fiscal policy coordination among countries with nonlinear taxes as in Kocherlakota (2005) could be a way to solve this mechanism design problem for a world economy.
Appendix

Let $\lambda_t(z^t)$, $\mu_t(z^t)$, $\omega_t(z^t)$ and $\eta_t(z^t)$ be the Lagrange multipliers associated with the flow budget constraint (54), participation constraint (55) and the incentive constraint (56), respectively. First-order conditions for (53) are as follows:

$$\Pi_t(z^t) : \prod_{i=1}^{t} (1 + \rho_i(z^i))^{-1} \psi(z^t) - \lambda_t(z^t) = 0,$$

$$\xi_t^i : -Q_t^i(z^t) \lambda_t(z^t) + \sum_{z^{t+1}} (Q_{t+1}(z^{t+1}) + D_{t+1}(z^{t+1})) \lambda_{t+1}(z^{t+1}) = 0, \text{ for } i = TR,NT \quad (86)$$

$$b_t : -\mu_t(z^t) + \sum_{z^{t+1}} \mu_{t+1}(z^{t+1})(1 + i_{t+1}(z^t)) = 0,$$

$$m_t^i : -\mu_t(z^t) + \sum_{z^{t+1}} \mu_{t+1}(z^{t+1}) \frac{S_{t+1}(z^{t+1})}{S_t(z^t)},$$

$$m_t^f : -\lambda_t(z^t) + \mu_t(z^t) = 0 \quad (A.6)$$

$$c_t^{TR} : \beta^t(\omega_t(z^t) + \eta_t(z^t))u_t^{-\gamma}u_{i_t}^{\gamma}n\pi(\theta^t)\psi(z^t) = \mu_t(z^t)P_t^{TR},$$

$$c_t^{NT} : \beta^t(\omega_t(z^t) + \eta_t(z^t))u_t^{-\gamma}u_{i_t}^{\gamma}n\pi(\theta^t)\psi(z^t) = \mu_t(z^t)P_t^{NT},$$

$$l_t^{TR} : \beta^t(\omega_t(z^t) + \eta_t(z^t))v_t^{i_t}n\pi(\theta^t)\psi(z^t) = \mu_t(\theta^t, z^t)\phi^{TR}(\theta^t, \theta^t)P_t^{TR},$$

$$l_t^{NT} : \beta^t(\omega_t(z^t) + \eta_t(z^t))v_t^{i_t}n\pi(\theta^t)\psi(z^t) = \mu_t(\theta^t, z^t)\phi^{NT}(\theta^t, \theta^t)P_t^{NT}. \quad (A.9)$$

The use of (A.7) through (A.10) yields the same static efficiency condition as (19)

Plugging (A.1) into (??) and rearranging terms, we get:

$$1 = \sum_{z^{t+1}} \left\{ \frac{Q_t^i(z^{t+1}) + D_t^i(z^{t+1})}{Q_t^i(z^t)} \right\} \frac{1}{(1 + \rho_{t+1}(z^{t+1}))} \psi(z^{t+1}|z^t). \quad (A.11)$$

Likewise plugging (A.1) into (A.3), (A.4) and (A.5), we get the respective equations:

$$1 = \sum_{z^{t+1}} \frac{S_{t+1}(z^{t+1})}{S_t(z^t)} \frac{1}{(1 + \rho_{t+1}(z^{t+1}))} \psi(z^{t+1}|z^t),$$

$$1 = \sum_{z^{t+1}} \frac{F_t(z^t)}{S_t(z^t)(1 + \rho_{t+1}(z^{t+1}))} \psi(z^{t+1}|z^t). \quad (A.14)$$
In the next step, note that the household still solves the same composite good problem as in section 2.4 which means that the two goods can be reduced to a composite good $c_t$ with an associate composite price $\bar{P}_t$ and we get the following equality:

$$\bar{P}_t c_t = P_t^{TR}(z^t) c_t^{TR}(\theta^t, z^t) + P_t^{NT}(z^t) c_t^{NT}(\theta^t, z^t).$$  \hfill (A.15)

Next, we need to characterize the discount rates $\rho(z^t)$. We use the same line of reasoning as in Golosov et al. (2006) and Kocherlakota (2005). Fix the date $t$ history $\theta^t$ and $z^t$. Decrease the composite good at date $t$ for this history group by a small amount $\beta \Delta t$ and increase across the board the date $t + 1$ composite good by $\Delta t$. This compensating variation leaves the objective function and the incentive and participation constraints unaffected. It only impacts the resource constraints. The insurance companies now make sure that this perturbation minimizes the resource cost at $\Delta t = 0$.

To solve this problem define

$$\frac{\bar{c}_t(\theta^t, z^t)^{1-\gamma}}{1-\gamma} = \frac{c_t(\theta^t, z^t)^{1-\gamma}}{1-\gamma} - \beta \Delta t$$  \hfill (A.16)

and

$$\frac{\bar{c}_{t+1}(\theta^{t+1}, z^{t+1})^{1-\gamma}}{1-\gamma} = \frac{c_{t+1}(\theta^{t+1}, z^{t+1})^{1-\gamma}}{1-\gamma} + \Delta_t.$$  \hfill (A.17)

The insurance company thus chooses $\Delta t$ such that the cost of resources at date $t$ and $t + 1$ evaluated at the respective shadow prices $\mu_t(z^t), \mu_{t+1}(z^{t+1})$ is minimized at $\Delta t = 0$. Using the flow resource constraint (54) and (A.15) this cost minimization problem can be rewritten as:

$$\min_{\Delta t} \mu_t(z^t) P_t(z^t) \left\{ c_t(\theta^t, z^t)^{1-\gamma} - \beta(1 - \gamma) \Delta t \right\}^{1/(1-\gamma)}$$

$$+ \mu_{t+1}(z^{t+1}) P_{t+1}(z^{t+1}) \sum_{\theta^{t+1}} \left\{ c_{t+1}(\theta^{t+1}, z^{t+1})^{1-\gamma} + (1 - \gamma) \Delta_t \right\}^{1/(1-\gamma)} \pi(\theta^{t+1} | \theta^t).$$  \hfill (A.18)

The first-order condition with respect to $\Delta t$ evaluated at $\Delta t = 0$ and the use of (A.1) and (A.6) yields the following inverse Euler equation:

$$\beta \bar{P}_t(z^t) c_t^{\gamma}(\theta^t, z^t) = (1 + \rho_{t+1}(z^{t+1}))^{-1} \bar{P}_{t+1}(z^{t+1}) \sum_{\theta^{t+1}} c_{t+1}^{\gamma}(\theta^{t+1}, z^{t+1}) \pi(\theta^{t+1} | \theta^t) \psi(z^{t+1} | z^t).$$  \hfill (A.19)

which can be rewritten as:

$$\beta \bar{P}_t(z^t) c_t^{\gamma}(\theta^t, z^t) = (1 + \rho_{t+1}(z^{t+1}))^{-1} \bar{P}_{t+1}(z^{t+1}) E(c_{t+1}^{\gamma} | z^{t+1}, \theta^t) \psi(z^{t+1} | z^t).$$  \hfill (A.20)

Finally, integrate with respect to $\theta^t$

$$\beta \bar{P}_t(z^t) E(c_t^{\gamma} | z^t) = (1 + \rho_{t+1}(z^{t+1}))^{-1} \bar{P}_{t+1}(z^{t+1}) E(c_{t+1}^{\gamma} | z^{t+1}) \psi(z^{t+1} | z^t).$$  \hfill (A.21)
which means that

\[
\frac{\psi(z^{t+1}|z^t)}{(1 + \rho_{t+1}(z^{t+1}))} = \frac{\beta E(c_t^\gamma \mid z^t)}{E(c_{t+1}^{\gamma} \mid z^{t+1})} \frac{P_t^j(z^t)}{P_{t+1}^j(z^{t+1})}.
\]  

(A.22)

Next, plug (A.22) into (A.11) through (A.14) to complete the proof.
References


Table 1: Sample Statistics

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<th>sample statistics</th>
<th>nominal spot exchange rate</th>
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The sample period 1982:Q1–2004:Q4
Table 2: Correlation Matrix

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Note: same as table 1

Table 4: Calibrated Euler Equations
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Note: same as table 1
Table 6: Mean Squared Errors

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Note: same as table 1
Table 7: GMM results for three equations

Results from value weighted return

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Results from equal weighted return

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Note: same as table 1

The set of instruments:
1: a constant and a twice lagged, 1st difference of home within the country variance
2: a constant and a twice lagged currency premium
3: a constant and a twice lagged real risk free rate
4: a constant and a twice lagged, 1st difference of foreign within

For parameter estimates, the standard errors are below
For J values, p-values are below.
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<th>Author(s) (presenter(s) in bold)</th>
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