ABSTRACT

We construct a stylised model of the supply side with goods and labour market imperfections to show that an economy can rationally operate at an inefficient, or 'low-effort', state in which the relationship between output and unemployment is positive. We examine data from the G7 countries over 1960-2001 and find that only German data strongly favour a persistent negative relationship between the level of output and rate of unemployment. The consequence of this is that circumstances exist in which market imperfections could pose serious obstacles to the smooth working of expansionary and/or stabilization policies and a positive demand shock might have adverse effects on employment.

Keywords: Efficiency wages, effort supply, Kalman filter, monopolistic competition, Okun's law.

JEL Classification: E62, J41, H3.
1. Introduction

In the last few decades, industrialised nations have been subjected to a variety of external and policy-induced demand shocks while simultaneously experiencing significant changes in their labour productivity and employment. Meanwhile, governments have been concerned to maintain a balance between implementing those policies which protect workers against job losses (to reduce the hardship of unemployment) and those which restrain the unemployment rate. However, as Lindbeck (1992) warns, unless we have a clear understanding of how such policies work, their implementation may produce unexpected consequences: "In the context of a nonmarket-clearing labour market, it is certainly reasonable to regard unemployment, in particular highly persistent unemployment, as a major macroeconomic distortion. There is therefore a potential case for policy actions, provided such actions do not create more problems than they solve. Experience in many countries suggests that the latter reservation is not trivial."

In this paper we focus on one such case by examining the relationship between the level of output and the rate of unemployment. The common belief regarding this relationship is dominated by Okun’s Law which predicts that a fall in output growth is normally accompanied by a significant but smaller rise in unemployment. This prediction and its policy implications are straightforward when output and unemployment exhibit a systematic negative relationship with each other beyond trend and cyclical variations. However, they are not so clear if these variables happen to be positively related. We therefore ask whether there are circumstances in which a rise in the rate of unemployment can lead to an increase the level of output, and develop a theoretical model that shows such a result can be obtained when labour and goods markets operate under certain (plausible) conditions. The model allows for a distortion in the labour market through incorporating a variant of the efficiency wage hypothesis whereby involuntary unemployment gives rise to externalities that could be exploited by economic agents; price-setting firms use high or rising unemployment as a device to deter shirking. The novelty of the variant used in this paper is that, unlike the existing models in which a worker’s effort level is discrete and can assume either a low or a high value, it allows a worker’s optimal effort supply to be a continuous function of its determinants. These determinants are: (i) the net of tax income from employment relative to

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1 Clearly, such departures from standard results are expected when models deviate from perfectly competitive conditions by allowing for some type of rigidity or distortion, e.g. efficiency wages, unionisation, wage contracts, unemployment insurance, etc. For instance, Acemoglu and Shimer (2000) focus on the effect of raising unemployment insurance within a search model and conclude that more generous welfare programmes can in fact raise output and welfare despite giving rise to a higher unemployment.
the unemployment benefit; and (ii) rate of unemployment in the economy. In such circumstances the supply side is shown to exhibit a non-linearity which is adequately captured by a humped-shape relationship between output and unemployment rate. It follows that the economy can, at any point in time, be in one of the three possible states with regard to the effort level. The standard case, in line with Okun’s Law in which output and unemployment rate are negatively related, occurs in the ‘high-effort’ state where the economy can be said to be operating ‘efficiently’. In this case, to raise the level of output in response to a rise in aggregate demand firms need to employ more workers. The opposite case occurs in the ‘low-effort’ state in which the economy may be said to be operating ‘inefficiently’. In this situation a higher level of output can be achieved at a lower level of employment since firms find it more profitable to meet the rise in demand by inducing the workers to raise their (optimal) effort supply. These two states are separated by a third, the ‘threshold effort’ state, which corresponds to the peak of the humped-shaped relationship where the combination of employment and effort yields the maximum level of output. In this sense, therefore, in the threshold effort state the economy may be said to be operating without any slack despite the existence of a positive level of involuntary unemployment

To explore the extent to which the non-linearity predicted by the model is supported by evidence, we examine the relationship between unemployment rate and level of output using data from the G7 countries. Our empirical analysis is based on estimating a state space ‘local linear trend’ model using the Kalman-filter. This approach allows us to account both for secular and cyclical variations and for changes in productivity of other factors, which do not explicitly feature in the analysis. Our evidence suggests that whilst low-effort periods have occurred significantly within the sample, periods corresponding to threshold effort seem to dominate and only German data shows a strong support for more frequent occurrence of the high-effort case.

The rest of the paper proceeds as follows. Section 2 outlines the model and shows how the non-linearity described above emerges. Section 3 explains our econometric method and reports the evidence for each of the G7 countries and Section 4 concludes the paper. The Appendix outlines the derivation of the effort function used in this paper.

2 Other recent studies which examine the link between unemployment and productivity include Malley and Moutos (2001), Daveri and Tabellini (2000), Blanchard (1998), Caballero and Hammour (1998a,b), Gordon (1997a) and Manning (1992). However, none of these studies explores the link between unemployment and output arising from both labour and product market imperfections.
2. The Relationship between Output and Unemployment: Theory

The main purpose of this paper is to throw light on the interpretation of Okun’s law with emphasis on the relationship between output and unemployment on the supply side of the economy. More precisely, we wish to focus on the structural relationship between output and unemployment implied by the supply side when there are goods and labour market imperfection, and examine how such a relationship fits in with the general observation that output and unemployment are related to each other negatively beyond trend and cyclical variations. In this section, therefore, we use the efficiency wage hypothesis to provide a simple theoretical explanation of the way output $Y$ and unemployment rate $u$ are likely to be related on the supply side. Before outlining the theoretical model, however, it is helpful highlight the problem by considering at the outset the temporal aggregate production function which may be simplified to focus on the variables of interest, namely

$$Y = Y(q, L); \quad Y_q' > 0, \quad Y_L' > 0,$$

which, at any point in time, traces the combinations of aggregate employment $L$ and output $Y$ for the corresponding level of labour productivity $q$. Now, invoking the assumption that $q$ is, ceteris paribus, determined by the level of workers’ effort and postulating that workers’ effort supply is positively related to the unemployment rate – i.e. the higher is $u$ the larger is the effort supplied and hence $dq/du > 0$; see, for example, Shapiro and Stiglitz (1984) – we can use

$$\frac{dY}{du} = Y_q' \frac{dq}{du} + Y_L' \frac{dL}{du},$$

(2)

to deduce the behaviour of sign of $dY/du$ as $u$ varies in the positive unit interval. In particular, because $dL/du < 0$ by definition, the relationship between $Y$ and $u$ on the temporal production function would resemble that depicted in Figure 1 below if, at very low levels of $u$, $Y_q'(dq/du)$ is sufficiently large and dominates $Y_L'(dL/du)$ so as to make $dY/du > 0$. In other words, it is possible that $dY/du$ changes from negative to positive as the unemployment rate falls below a certain threshold, $\bar{u}$. In such circumstances, the interpretation of Okun’s law and its consequences for macroeconomic policy differ drastically depending on the prevailing rate of actual unemployment in relation to the threshold level $\bar{u}$. That is, unless $u > \bar{u}$, the observation that a fall in $Y$ is accompanied by a rise in $u$ (and hence Okun’s law holds) could only have been caused by a shift in the temporal production function down and/or to the right. As a result, the standard macroeconomic policies are unlikely to yield the
expected results as stressed by Lindbeck (1992). More specifically, it is not certain that an exogenous stimulation of aggregate demand would lead to a reduction in unemployment.

Figure 1. The temporal relationship between output and unemployment

Consider an economy in which the product market structure is monopolistically competitive and output is a CES bundle of varieties of a horizontally differentiated product. Thus, demand for each variety $j$ is

$$y_j = Y \left( \frac{p_j}{P} \right)^s,$$

where $y_j$ and $p_j$ are quantity demanded and price of the variety, $s>1$ is the elasticity of substitution between any two varieties, and $Y$ and $P$ are the real aggregate demand and the
corresponding price level, respectively. The latter are determined by the CES aggregators below where $N$ is the mass of available varieties\(^3\),

\[
Y = \left( \int_{j \in N} y_j^{-1/(1/s)} dj \right)^{1/[1-(1/s)]}, \tag{4}
\]

\[
P = \left( \int_{j \in N} p_j^{-1/(1-s)} dj \right)^{1/(1-s)} . \tag{5}
\]

Suppose that each firm produces one variety of the good using labour as the only input with an increasing returns to scale technology whose labour requirement in efficiency units is given by

\[
e_j l_j = \lambda + y_j , \tag{6}
\]

where $l_j$ is quantity of labour input, $e_j$ is labour productivity and $\lambda$ is a constant parameter reflecting the fixed cost of production (assumed to be identical across firms). The increasing returns to scale, implied by falling average cost, therefore gives rise to the incentive for full specialisation from which a one-to-one correspondence between the mass of varieties and firms results.

We assume that labour is homogeneous and is perfectly mobile between firms. A worker employed by firm $j$ earns nominal wage $w_j$ and pays tax $t$, and the government provides an unemployment insurance scheme which pays $b$ to each unemployed worker. We also assume that workers’ productivity is determined by their attitude towards shirking. In particular, $e_j$ is assumed to represent the optimal effort supply of a typical worker which depends on: (i) the difference between net real wage and unemployment benefit, $\omega_j = (w_j - t - b)/P$; and (ii) the extent of unemployment in the economy captured by the unemployment rate $u$. We postulate the following effort supply function for a worker employed by firm $j$ (an example of this type of effort supply function, which satisfies the following properties and is obtained when workers maximise their expected utility from work, is explicitly derived in Appendix A1)

\[
e_j = e(\omega_j, u) , \tag{7}
\]

which is assumed to satisfy the following properties (subscript $j$ is dropped):
(1) \( e(\omega, u) > 0 \) as \( \omega > 0 \), \( u > 0 \); \( e(\omega, u) = 0 \) \( \forall u \in (0, 1) \) as \( \omega = 0 \); and
\( e(\omega, u) \to 0 \) \( \forall \omega > 0 \) as \( u \to 0 \).

(2) \( e \) is increasing in both \( \omega \) and \( u \): \( e_\omega \equiv \frac{\partial e}{\partial \omega} > 0 \); \( e_u \equiv \frac{\partial e}{\partial u} > 0 \); and

(3) \( e \) has plausible second and cross partial derivatives. In particular, we shall assume that
\( e_{\omega\omega} \equiv \frac{\partial^2 e}{\partial \omega^2} < 0 \) and \( e_{\omega u} \equiv \frac{\partial^2 e}{\partial \omega \partial u} > 0 \) always hold while \( e_{uu} \equiv \frac{\partial^2 e}{\partial u^2} > 0 \) when \( u \) is very close to zero and \( e \) is very low.

Each firm takes \( P, Y, N, u, t \) and \( b \) as given and chooses its ‘efficiency wage’ \( w_j \) and price \( p_j \) so as to maximise its profit
\[
\pi_j = p_j y_j - w_j l_j , \quad (8)
\]
subject to the demand function in (3) and the labour requirement function in (6), as well as taking account of its workers’ reaction to the choice of \( w_j \) which is given by the effort function in (7). The first order conditions are \( \partial \pi_j / \partial w_j = 0 \) and \( \partial \pi_j / \partial p_j = 0 \) whose solution imply the following wage and price setting rules\(^4\)
\[
w_j = \frac{Pe_j}{\partial e_j / \partial \omega} , \quad (9)
\]
\[
p_j = \frac{\sigma w_j}{e_j} , \quad (10)
\]
where \( \sigma = s/(s - 1) \). Equation (9) is a well-known result in the efficiency wage literature and implies that firm raises its wage rate up to the point where the effort function is unit elastic in real wage. Equation (10) is the usual mark-up pricing rule for a monopolistically competitive firm. In a symmetric equilibrium where all firms are identical, we drop the subscript \( j \) and write the above equations as
\[
e = (w/P)e_\omega , \quad (9')
\]
\[
e = \sigma (w/P) , \quad (10')
\]
\(^4\) The second order conditions are satisfied as long as \( s > 1 \) and \( \partial^2 e_j / \partial \omega^2 < 0 \).
Totally differentiating (7), (9′) and (10′) with respect to the endogenous variables $e$, $w/P$, $t/P$, and $u$, taking account of $\omega = (w - t - b)/P$, and solving the resulting equations we obtain (see Appendix A2 for details of derivation)

$$\frac{de}{du} = e_u - \frac{\sigma e_{\omega u}}{e_{\omega \omega}}.$$  \hspace{1cm} (11)

Thus, under our assumptions regarding the shape of the effort function, (11) implies that $de/du > 0$ always holds, which is consistent with the theoretical consensus that the net result of an increase in unemployment rate is to raise workers’ effort level. We can use this result to examine the way in which equilibrium output and unemployment are related to each other on the supply side in the aggregate. Using the definition of aggregate supply and imposing symmetry, the aggregate production function is

$$Y = \int_{j \in N} y_j \, dj = eL - N\lambda,$$  \hspace{1cm} (12)

where $L = \int_{j \in N} L_j \, dj$ is total employment. Equation (12) traces the combinations of aggregate employment and output in the short-run – i.e. $(L, Y)$ for any given number of firms $N$ – which satisfy the supply side equilibrium in which labour productivity is determined by an effort supply function and firms pay wages to induces workers to supply the effort level that maximises their profits. Or, put differently, these combinations of $L$ and $Y$ give the equilibrium locus that describes how $Y$ changes as workers respond to changes in $u$ while the firms adjust their wage and price to ensure the resulting effort supply and quantity produced maximise profits.

Given that $L = LF(1-u)$, where is $LF$ labour force, and treating $LF$, $N$ and $\lambda$ as exogenous, from (12) we obtain

$$\frac{dY}{du} \propto (1-u) \frac{de}{du} - e.$$  \hspace{1cm} (13)

Thus, provided that $de/du$, which is given by equation (11), is finite as $u \to 1$, we would expect the right-hand-side of (13) to be negative for sufficiently large levels of $u$. Conversely, starting from sufficiently low levels of $u$, we would expect the right-hand-side of (13) to be positive as long as $de/du$ is positive, as explained above. Given these and

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$^5$ $N$ is endogenous in the long-run whereby free entry and exit determine $N$ such that profits are eliminated. It is easy to verify that the imposition of the long-run equilibrium does not affect the shape of the relationship between aggregate supply and unemployment rate derived here.
assuming that $de/du$ in (11) is continuous in $u$, the equilibrium locus in $(u, Y)$ space will be similar to that illustrated in Figure 1 above.

The main implication of the above model that we wish to stress is that it results in a change in $dY/du$ from negative to positive as unemployment rate falls below a certain threshold, $u = \bar{u}$. This is the rate of unemployment at which output attains its highest level, $Y = \bar{Y}$. At such a point, the economy may be said to be operating without any slack despite supporting a level of unemployment. Within the region where $u > \bar{u}$, output and unemployment rate are negatively related and there is no conflict the implications of Okun’s Law. This situation corresponds to the high-effort state where the economy can be said to be operating efficiently and firms will have to employ more workers to meet a rise in aggregate demand. In contrast, the region where $u < \bar{u}$ corresponds to the low-effort state in which the economy may be said to be operating inefficiently. In this situation a higher level of output can be achieved at a lower level of employment since firms will find it more profitable to meet the rise in demand by inducing the workers to raise their (optimal) effort supply. Thus, the fact that Okun’s law holds – in that a fall in $Y$ is seen to be accompanied by a rise in $u$ – when $u \leq \bar{u}$ ought to be the result of shifts in the temporal production function, which could have adverse consequences for the effectiveness of aggregate demand policies.

3. Evidence
In this section we examine data on the level of output and the rate of unemployment from G7 countries – Canada, France, Germany, Italy, Japan, UK and US – in order to check whether evidence supports the existence of a nonlinear relationship such as that in Figure 1. More specifically, we have explored the strength of evidence to address the following questions:

(i) Does an ‘inversed U-shape’ specification adequately explain the way output is related to unemployment rate at any point in time? If so, then,

(ii) how does the actual rate of unemployment intertemporally compare with the threshold rate of unemployment which separates low-effort from high-effort states of production and corresponds to peak output?

To tackle this task, we have estimated a state space ‘local linear trend’ model using the Kalman-filter approach. The regression model consists of the measurement equation,

$$Y_t = \alpha_t + \phi_t u_t + \delta_t u_t^2 + \varepsilon_t, \quad (14)$$
which assumes that output is a quadratic function of the unemployment rate subject to an
additive stationary random disturbance term \( \epsilon_t \sim iidn(0,\sigma^2_{\epsilon}) \), while allowing the (state) parameters \( (\alpha_t, \phi_t, \delta_t) \) to evolve randomly according to appropriate transition equations
which we assume to be as follows

\[
\begin{align*}
\alpha_t &= \alpha_{t-1} + \beta_{t-1} + \zeta_t; \quad \zeta_t \sim iidn(0,\sigma^2_{\zeta}), \\
\beta_t &= \theta \beta_{t-1} + \xi_t; \quad \xi_t \sim iidn(0,\sigma^2_{\xi}), \quad 0 < \theta < 1, \\
\phi_t &= \phi_{t-1} + \eta_t; \quad \eta_t \sim iidn(0,\sigma^2_{\eta}), \\
\delta_t &= \delta_{t-1} + \psi_t; \quad \psi_t \sim iidn(0,\sigma^2_{\psi}).
\end{align*}
\]

(15) (16) (17) (18)

The generality allowed by this set up is particularly useful when it is applied to bivariate
relationships which both: (a) involve variables that have strong secular pattern and/or are
subject to cyclical fluctuations; and (b) are, by construction, restricted and fail to condition
explicitly on a host of other potentially relevant variables. The state-space representations,
in this context, are very flexible since the non-stationary processes generating \( \phi \) and \( \delta \) are
allowed to evolve in a manner capable of capturing any fundamental changes, which may
have occurred in the historical relationship between \( Y_t \) and \( u_t \). Moreover, to account for
trends in output growth and the unemployment rate over our estimation period (1960-2001),
we have allowed for local linear trends where both the level, \( \alpha_{t-1} \) and the slope, \( \beta_{t-1} \) vary
over time. To estimate (14) allowing for (15)-(18), we require starting values for the state
vector and its variance-covariance matrix, \( (\alpha_0, \beta_0, \phi_0, \delta_0) \) and \( \Sigma_0 \). In the absence of any

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6 While there are a wide variety of alternative non-linear functions capable of capturing the non-monotonic
link between \( Y_t \) and \( u_t \) predicted by our theory, we have opted for the simplest and most parsimonious of
these.

7 Both output and unemployment have these properties and the estimation method adopted here is a superior
alternative to isolating the secular and cyclical components by filtering the series before checking how they
relate to each other over time.

8 In the absence of any explicit dynamics, we employ contemporaneous values of both output and the
unemployment rate. This approach might reasonably be expected to yield biased parameter estimates, given
the joint endogeneity of the variables. To assess the extent of this bias we also experimented with IV and
GMM estimation and found any biases to be quantitatively negligible. To preserve space, these latter results
are not reported here but will be made available on request.

9 Note that in contrast to the other parameters which follow random walks, \( \beta \) is assumed to follow a
stationary AR(1) process. This assumption is employed since a non-stationary process for this parameter
would imply \( Y_t \sim I(2) \). This, however, is against the widely acknowledged stylised fact that the growth rate of
output is stationary, which is also supported by our data set. For example, univariate evidence based on
ADF, weighted-symmetric and Phillips-Perron tests suggest that \( y_t \) has only one unit root (this evidence is not
presented here but will be made available on request).
prior information on the initial distribution\textsuperscript{10}, we have employed a diffuse prior which involves setting the starting values of the coefficients equal to zero and letting $\Sigma_0 = \kappa I$ where $I$ is the conformable unit matrix and $\kappa$ is a very large number (see Harvey, 1989, for detail).

Empirical support for our theory, within the context of the questions (i) and (ii) posed above, at the beginning of this section, requires that:

(i)$'$ \(\phi_t\) must be significantly greater than zero, \(\delta_t\) must be significantly less than zero, the estimated residuals, \(\hat{\epsilon}_t\), must be stationary, and the threshold rate of unemployment, denoted by \(\bar{u}_t\) and given by \(\bar{u}_t = \phi_t / (2\delta_t)\) from the quadratic function in (14), should be significantly greater than zero.

(ii)$'$ Evidence should indicate that in addition to \(u^*_t > \bar{u}_t\), \(u^*_t = \bar{u}_t\) and \(u^*_t < \bar{u}_t\) have also occurred significantly over the sample period.

To examine these, we obtained filtered estimates of the state vector for each of the G7 countries. Data are quarterly over the period 1960:Q1-2001:Q1 and the results are reported in Table 1 below. Columns (I), (III) and (V) give, respectively, the filtered estimates of \(\phi_t\), \(\delta_t\) and the implied threshold rate of unemployment \(\bar{u}_t = -\phi_t / 2\delta_t\), for the final observation \((t = T)\). Columns (II), (IV) and (VI) report, respectively, the proportion of observations over the estimation period for which the null hypotheses \(\phi_t > 0\), \(\delta_t < 0\) and \(\bar{u}_t > 0\) cannot be rejected at the 5% critical level. These results, together with the satisfactory behaviour of the estimated residuals \(\hat{\epsilon}_t\) (standard tests not reported here but available on request), suggest that the quadratic specification in which the peak output occurs at a plausible level of unemployment is consistent with data, beyond any co- and/or counter-movements due to secular and/or cyclical patterns in the underlying series. Moreover, since for each \(t\) one of the three cases \(u^*_t > \bar{u}_t\), \(u^*_t = \bar{u}_t\) or \(u^*_t < \bar{u}_t\) will have to hold, it is helpful to compare the actual and the estimated threshold levels of unemployment. Table 2 below reports the percentage of significant occurrences of these cases at 5% and at 10% critical levels. According to the results only German data provides a strong support for \(u^*_t > \bar{u}_t\); US, Canada, Italy and Japan fully reject \(u^*_t > \bar{u}_t\) while UK and to a much lesser extent France show a mild tendency towards exhibiting \(u^*_t > \bar{u}_t\).

\textsuperscript{10} Given that three of the four transition equations are non-stationary, the unconditional distribution of the state vector is not defined.
Table 1. Selected results from estimation of equation (14) based on quarterly 1964-2001 data for G7 countries

<table>
<thead>
<tr>
<th></th>
<th>(I) $\hat{\phi}_r$</th>
<th>(II) $\phi_r &gt; 0$</th>
<th>(III) $\hat{\delta}_r$</th>
<th>(IV) $\delta_r &gt; 0$</th>
<th>(V) $\hat{\mu}_r$</th>
<th>(VI) $\mu_r &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>US</strong></td>
<td>2.49 (0.232)</td>
<td>100%</td>
<td>-0.250 (0.036)</td>
<td>100%</td>
<td>5.62 (1.21)</td>
<td>100%</td>
</tr>
<tr>
<td><strong>Canada</strong></td>
<td>2.26 (0.223)</td>
<td>100%</td>
<td>-0.124 (0.022)</td>
<td>100%</td>
<td>9.14 (2.26)</td>
<td>92%</td>
</tr>
<tr>
<td><strong>UK</strong></td>
<td>2.47 (0.640)</td>
<td>100%</td>
<td>-0.158* (0.178)</td>
<td>100%</td>
<td>8.02* (5.03)</td>
<td>95%</td>
</tr>
<tr>
<td><strong>France</strong></td>
<td>4.73 (0.776)</td>
<td>100%</td>
<td>-0.258 (0.124)</td>
<td>100%</td>
<td>9.13 (2.13)</td>
<td>80%</td>
</tr>
<tr>
<td><strong>Germany</strong></td>
<td>21.06 (4.60)</td>
<td>100%</td>
<td>-2.47 (0.593)</td>
<td>100%</td>
<td>4.26 (0.07)</td>
<td>100%</td>
</tr>
<tr>
<td><strong>Italy</strong></td>
<td>4.75 (0.560)</td>
<td>100%</td>
<td>-0.224 (0.082)</td>
<td>100%</td>
<td>10.58 (2.77)</td>
<td>98%</td>
</tr>
<tr>
<td><strong>Japan</strong></td>
<td>4.26 (1.04)</td>
<td>100%</td>
<td>-0.585 (0.231)</td>
<td>96%</td>
<td>3.65 (0.59)</td>
<td>85%</td>
</tr>
</tbody>
</table>

(1) French data do not start until 1964:Q4. The initial 4 years (16 observations) were used to allow the filtered estimates sufficient time to stabilise and were excluded in obtaining estimates in this table. The local linear trend components were not significant for German data and hence were excluded in final estimation for that country.

(2) The statistical significances of $\hat{\phi}_r$ and $\hat{\delta}_r$ in columns (I) and (III) are based on their asymptotic standard errors (the numbers in parentheses). An asterisk indicates not significant at the 5% level. To assess the statistical significance of $\hat{\mu}_r$ on a period-by-period basis we have conducted a parametric bootstrap using 2000 replications for each quarter. The numbers in parentheses are the bootstrapped standard errors for the final period. An asterisk indicates not significant at the 5% level.

Table 2. Comparison between the actual and threshold levels of unemployment

<table>
<thead>
<tr>
<th></th>
<th>proportion of $u_r &lt; \bar{u}_r$</th>
<th>proportion of $u_r = \bar{u}_r$</th>
<th>proportion of $u_r &gt; \bar{u}_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sig. at 5%</td>
<td>sig. at 10%</td>
<td>sig. at 5%</td>
</tr>
<tr>
<td><strong>US</strong></td>
<td>0.09</td>
<td>0.20</td>
<td>0.91</td>
</tr>
<tr>
<td><strong>Canada</strong></td>
<td>0.05</td>
<td>0.09</td>
<td>0.95</td>
</tr>
<tr>
<td><strong>UK</strong></td>
<td>0.01</td>
<td>0.03</td>
<td>0.77</td>
</tr>
<tr>
<td><strong>France</strong></td>
<td>0.00</td>
<td>0.00</td>
<td>0.94</td>
</tr>
<tr>
<td><strong>Germany</strong></td>
<td>0.19</td>
<td>0.19</td>
<td>0.01</td>
</tr>
<tr>
<td><strong>Italy</strong></td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Japan</strong></td>
<td>0.00</td>
<td>0.01</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The above evidence is also in line with the findings reported by studies that have examined the behaviour of labour productivity in connection with employment and output in the industrialised countries and provide evidence on the way in which labour productivity has changed over the last few decades. Recent examples include Disney, et al. (2000), Barnes
and Haskel (2000), Marini and Scaramozzino (2000), van Ark et al. (2000) and Sala-i-Martin (1996). The evidence provided in these studies is usually interpreted using either direct causes – which are the standard reasons for productivity gains, i.e. i) improved skill due to training; ii) increased efficiency due to progress in management and restructuring; and iii) rising physical productivity of other factors of production due to R&D, etc. – or the indirect causes whereby market forces induce a rise in efficiency that is needed in order for the firms to survive competition and market selection. The separating line between these two accounts, however, is not very clear in the sense that the latter will have to be achieved through the former when the economy is operating efficiently. But if the economy happens to be in an inefficient phase, market forces can act directly without having to induce any of the factors in the first category. The efficiency wage hypothesis argument used in this paper is a typical example of this case. Moreover, given our definition of the threshold rate of unemployment – that separates the efficient and inefficient phases of production – and the evidence in Table 2 above that in a number of countries the actual unemployment rate has a tendency to coincide with a time varying estimate of such a threshold rate, exploring the links between this concept and the time-varying NAIRU – for example as that studied by Gordon (1997b) – can throw light on the determination of the natural rate of output and hence provides an interesting direction for future research.

4. Summary and Conclusions
The main motivating factor underlying our study has been the fact that in some circumstances a positive policy shock might give rise to adverse employment effects. We have develop a model which shows that if firms can use the threat of unemployment to induce workers to supply more effort, the supply side relationship between aggregate output and unemployment rate will be non-monotonic. In particular, these variables can be positively related if the gain in productivity is sufficiently large to outweigh the negative effect of the reduction in employment. In such circumstances, an expansionary policy will have an adverse effect on unemployment. Our evidence, based on data from G7 countries over the period 1960-2001, shows strong support for the non-monotonicity implied by our model. Using an estimation method which allows for trends, cyclical changes and breaks, we find that only German data strongly favour a persistent negative relationship between the level of output and rate of unemployment.
Clearly, our results – which complement those of the literature on the effects of contractionary fiscal policy (Barry and Devereux, 1995) and on the positive effects of unemployment insurance (Acemoglu and Shimer, 2000) – suggest that plausible circumstances do exist in which market imperfections pose serious obstacles to the smooth working of expansionary and/or stabilization policies.
5. References


Manning, A. (1992), Productivity growth, wage setting and the equilibrium rate of unemployment, Centre for Economic Performance, Discussion paper No 63.

Marini G. and P. Scaramozzino (2000), Endogenous growth and social security, CeFiMS DP 02/00/02, Centre for Financial and Management Studies, SOAS, University of London


6. Appendix

A1. Derivation of the Effort Supply Function, $e(\omega, u)$

This appendix explains how a specific effort supply function such as that in equation (7) can be derived within the framework of the efficiency wage hypothesis where, following common practice, the agent (consumer/worker) is assumed to maximise the expected utility of remaining in employment.

We assume that all agents participate in the labour market and at any point in time an individual agent can be in one of the following states: (i) employed (working); (ii) being fired (when caught shirking at work); (iii) unemployed (being without a job); or (iv) being hired (finding a job). Let the utility indices corresponding to the above states be denoted as follows:

(i) employed (working): $V^E$

(ii) being fired (losing one’s job): $V^F$

(iii) unemployed (being without a job): $V^U$

(iv) being hired (finding a job): $V^H$

It is straightforward to derive $V^U$ and $V^E$. For simplicity, here we approximate these by the indirect utility of a typical agent at any point in time, which can be written as $V = m - \lambda \cdot f(e)$. $m$ is the real disposable income of the agent from work; normalising the price level $P$ to unity, $m = w - t$ (net of tax real wage) and $m = b$ (real benefit) for employed and unemployed agents, respectively. The function $f(e) \geq 0$ captures the disutility of effort $e$; $\lambda = 1$ and $\lambda = 0$ for employed and unemployed agents, respectively, and we assume that $f' > 0$ and $f'' \geq 0$ which imply that the disutility of effort rises with a non-decreasing rate. For simplicity, and without loss of generality, we shall use the explicit form $f(e) = ke^2$ where $k > 0$ is a scaling factor. Thus,

$V^U = b$, \hspace{1cm} (A1.1)

$V^E = (w - t) - ke^2$. \hspace{1cm} (A1.2)

We assume that $V^H$, which is the satisfaction a consumer attaches to finding a job or being hired is in principle not distinguishable from $V^E$ and for simplicity we let

$V^H = V^E$. \hspace{1cm} (A1.3)

The probabilities associated with moving from one state to another are assumed to be determined as follows:
(a) Probability associated with being fired when shirking, $F$.

We assume that shirking is the only reason for being fired (we do not explicitly model the monitoring technology). Therefore, \textit{ceteris paribus}, $F$ is a monotonic function of the effort level, $e$. Thus,

$$F = F(e);\ F(0) = 1;\ F(1) = 0;\ F' < 0.$$  

For simplicity, normalise the maximum possible effort to unity and let

$$F = 1 - e.$$  

(A1.4)

(b) Probability associated with finding a job, or being hired, when unemployed, $H$.

We assume that the labour force is homogeneous and, \textit{ceteris paribus}, $H$ is a monotonic function of the unemployment rate, $u$ (we do not explicitly model the search technology). Thus,

$$H = H(u);\ H(0) = 1;\ H(1) = 0;\ H' < 0.$$  

For simplicity we let

$$H = 1 - u.$$  

(A1.5)

We define the optimal level of effort as that which maximises a household’s expected utility of remaining in employment. The latter is denoted by $R(e)$ and is, by definition, given by

$$R(e) = (1 - F)V^E + FV^F.$$  

(A1.6)

Also, given that a ‘fired’ worker can either be hired or remain unemployed, we let $V^F$ be a weighted average of $V^{hl}$ and $V^U$. Thus,

$$V^F = HV^{hl} + (1 - H)V^U.$$  

(A1.7)

Equations (A1)-(A7) yield

$$R(e) = -(w - t - b)ue^3 + u(w - t - b)e + (1 - u)(w - t) + ub.$$  

(A1.8)

The agent takes $(w, t, b, u)$ as given and chooses $e$ to maximise $R(e)$. The first order condition for this is $-e^2 - ((2/3)(1 - u)/u)e + (1/3(k)(w - t - b) = 0$. This has two roots of which only one is positive, which also satisfies the second order for a maximum and can, after some normalisation, be written as

$$e = e(\omega, u) = \left[\omega + \frac{(1 - u)}{u}\right]^{1/2} - \frac{1 - u}{u},$$  

(A1.9)
where \( \omega = (w - t - b) \) and \( \gamma = 3/k \). It is clear that equation (A1.9) satisfies our specified conditions since \( e(\omega, u) = 0 \ \forall u \in (0,1) \) as \( \omega = 0 \); \( e(\omega, u) > 0 \) as \( \omega > 0 \); \( e_\omega > 0 \); \( e_u > 0 \); \( e_{\omega\omega} < 0 \); \( e_{uu} > 0 \); and \( e_{uu} > 0 \) for small values of \( u \), as required.

**A2. Derivation of Equations (11).**

Equation (11) is derived from equations (7), (9') and (10') and the definition \( \omega = (w_j - t - b) / P \), which are reproduced below as (A2.1)-(A2.4), respectively, where we have normalised \( P = 1 \) and dropped subscript \( j \).

\[
e = e(\omega, u), \quad (A2.1)
\]
\[
e = w e_\omega, \quad (A2.2)
\]
\[
e = \sigma w, \quad (A2.3)
\]
\[
\omega = w - t - b. \quad (A2.4)
\]

Totally differentiating the above, treating \( e, \omega, u, w \) and \( t \) as endogenous (note that \( t \) ought to be treated as endogenous when the government fixes \( b \) since variations in \( u \) can cause a budget deficit or surplus), we obtain

\[
d e = e_\omega (dw - dt) + e_u du, \quad (A2.5)
\]
\[
d e = e_\omega dw + w(e_{\omega\omega}(dw - dt) + e_{\omega u} du), \quad (A2.6)
\]
\[
d e = \sigma dw, \quad (A2.7)
\]
\[
d \omega = dw - dt. \quad (A2.8)
\]

Given that (A2.2) and (A2.3) imply \( e_\omega = \sigma \), (A2.7) can be used to write (A2.5) and (A2.6) as

\[
-\sigma dt + e_u du = 0, \quad (A2.9)
\]

and

\[
e_{\omega\omega} (dw - dt) + e_{\omega u} du = 0. \quad (A2.10)
\]

Solving these yields

\[
\frac{dt}{du} = \frac{e_u}{\sigma} > 0, \quad (A2.11)
\]

and

\[
\frac{dw}{du} = \frac{e_u}{\sigma} - \frac{e_{\omega u}}{e_{\omega\omega}} > 0. \quad (A2.12)
\]

Finally, using (A2.7) and (A2.12) we obtain

\[
\frac{de}{du} = e_u - \frac{\sigma e_{\omega u}}{e_{\omega\omega}} > 0.
\]
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