University of St Andrews

School of Economics and Finance Discussion Papers

On the Indifference Relation in Bewley Preferences

Georgios Gerasimou

School of Economics and Finance
Online Discussion Paper Series
issn 2055-303X
http://ideas.repec.org/s/san/wpecon.html
info: econ@st-andrews.ac.uk

School of Economics and Finance Discussion Paper No. 1609
11 Sep 2016 (revised 11 Dec 2017)
JEL Classification: D01, D03, D11
Keywords: Bewley preferences; incompleteness; degenerate indifference; ignorance.

On the Indifference Relation in Bewley Preferences

Georgios Gerasimou[*][†]

December 11, 2017

Abstract

Bewley's (2002) influential model of preferences over uncertain prospects features an incomplete preference relation and a set of priors over the states of the world such that one act is preferred to another if and only if its expected utility is higher under every prior in that set. This note shows that, under general conditions on preferences, the decision maker in the Bewley model cannot be indifferent between distinct monetary acts whenever the set of priors is fully-dimensional. In the special case of two states, in particular, such "objectively rational" preferences are incomplete if and only if the indifference relation is trivial in the above sense.

Keywords: Bewley preferences; incomplete preferences; monetary acts; trivial indifference relation.

JEL Classification: D1, D5, D7, D8, D10

[*]School of Economics & Finance, University of St Andrews. Email: gg26@st-andrews.ac.uk
[†]Acknowledgements to be added.

1 Introduction

Bewley's (2002) influential model features an incomplete strict preference relation and a set of priors over the states of the world such that the decision maker prefers one uncertain act to another if and only if its expected utility is strictly higher under every prior in the set. Since Ghirardato, Maccheroni, Marinacci, and Siniscalchi (2003), a natural alternative approach towards modelling this robustness/unanimity preference rule is to take a possibly incomplete *weak* preference relation as the primitive and let it be represented instead by weak dominance with respect to every prior.

Bewley preferences of both types have been employed in general equilibrium theory (e.g. Rigotti and Shannon 2005; Kajii and Ui 2009; Danan and Riedel 2013; Ma 2015), financial economics (e.g. Easley and O'Hara 2010), contract theory (Lopomo et al 2011), mechanism design (e.g. Chiesa et al 2015; Lopomo et al 2014) and social choice theory (e.g. Danan et al 2016). In addition, they have motivated a number of decision-theoretic extensions (e.g. Gilboa, Maccheroni, Marinacci and Schmeidler, 2010; Ok, Ortoleva and Riella, 2012; Galaabaatar and Karni, 2013; Faro, 2015). Since Gilboa et al (2010), in particular, these preferences are commonly referred to as *objectively rational*.

This note shows that when strict or weak Bewley preferences are defined on the Euclidean space of purely uncertain monetary acts and satisfy standard regularity assumptions such as strict monotonicity and, in addition, the set of priors is of full dimension, then the indifference relation associated with them is trivial or degenerate in the sense that there exist no distinct choice alternatives between which the decision maker is indifferent. This implies that, in the special case of two states (capturing, for example, situations where an investment is either profitable or not profitable, or where a contracting agent exerts either high or low effort), weak Bewley preferences are incomplete if and only if indifference is trivial in the above sense.

2 Fully Dimensional Priors and Trivial Indifference

There is a finite state space $S := \{1, \dots, n\}$ and the choice domain is the set of all acts $f : S \to \mathbb{R}$. This domain coincides with the Euclidean space $\mathbb{R}^n$. The zero vector in $\mathbb{R}^n$ is identified with $\mathbf{0}$. The set of all strictly positive probability measures on S is $\operatorname{rint} \Delta(S)$. A set $P \subset \operatorname{rint} \Delta(S)$ is of *full dimension* if it contains n linearly independent elements.

Following Ghirardato, Maccheroni, Marinacci, and Siniscalchi (2003), it will be said that a reflexive binary relation $\succsim$ on $\mathbb{R}^n$ admits a *weak Bewley representation* if there exists a continuous, strictly increasing function $u : \mathbb{R} \to \mathbb{R}$ and a compact, convex set $P \subset \operatorname{rint} \Delta(S)$ such that, for all $x, y \in \mathbb{R}^n$,

$$x \succsim y \iff \sum_{i=1}^{n} p_i u(x_i) \geq \sum_{i=1}^{n} p_i u(y_i) \text{ for all } p \in P \tag{1}$$

and where the indifference relation $\sim$ that is derived from such a representation is defined by

$$x \sim y \iff \sum_{i=1}^{n} p_i u(x_i) = \sum_{i=1}^{n} p_i u(y_i) \text{ for all } p \in P. \tag{2}$$

Following Bewley (2002), it will be said that an irreflexive binary relation $\succ$ on $\mathbb{R}^n$ admits a *strict Bewley representation* if there exists a continuous, strictly increasing function $u : \mathbb{R} \to \mathbb{R}$ and a compact, convex set $P \subset \operatorname{rint} \Delta(S)$ such that, for all $x, y \in \mathbb{R}^n$,

$$x \succ y \iff \sum_{i=1}^{n} p_i u(x_i) > \sum_{i=1}^{n} p_i u(y_i) \text{ for all } p \in P \tag{3}$$

and where the indifference relation $\sim$ that is derived from such a representation also defined in Bewley (2002) as in (2).

Proposition 1.
Suppose $\sim$ is the indifference relation that is derived from a weak or strict Bewley representation in $\mathbb{R}^n$ by means of a continuous and strictly increasing utility function $u : \mathbb{R} \to \mathbb{R}$ and a compact, convex set of priors $P \subset \operatorname{rint} \Delta(S)$. If P is of full dimension, then, for any $x, y \in \mathbb{R}^n$,

$$x \sim y \iff x = y. \tag{4}$$

Proof.

Let $\{q^1, \dots, q^n\} \subset P$ be a set of n linearly independent strictly positive probability measures on S. These exist by assumption, and form a basis of $\mathbb{R}^n$. Suppose $x, y \in \mathbb{R}^n$ are such that $x \sim y$ in the sense of (2) and define $\mathbf{z} := \mathbf{u}(x) - \mathbf{u}(y)$, where, for any $v \in \mathbb{R}^n$, $\mathbf{u}(v) := \big(u(v_1), \dots, u(v_n)\big)$. To establish that $x = y$ it suffices to show that $p\mathbf{z} = 0$ for all $p \in P$ implies $\mathbf{z} = \mathbf{0}$. (Indeed, since u is continuous and strictly increasing in $\mathbb{R}$, it readily follows that $\mathbf{u}(x) = \mathbf{u}(y) \Leftrightarrow x = y$.) To prove the assertion, suppose $p\mathbf{z} = 0$ for all $p \in P$. It holds that

$$q^i \mathbf{z} = 0, \quad \text{for all } i = 1, \dots, n \tag{5}$$

$$\sum_{i=1}^{n} \lambda_i q^i \neq \mathbf{0}, \quad \text{for all } (\lambda_1, \dots, \lambda_n) \in \mathbb{R}^n \setminus \{\mathbf{0}\} \tag{6}$$

$$\left(\sum_{i=1}^{n} \lambda_i q^i \right) \mathbf{z} = 0, \quad \text{for all } (\lambda_1, \dots, \lambda_n) \in \mathbb{R}^n \setminus \{\mathbf{0}\} \tag{7}$$

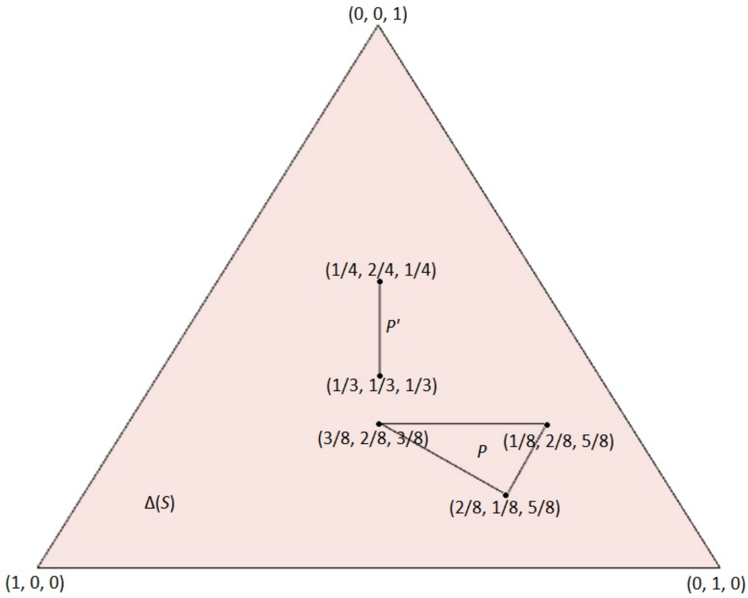
where (5) is true by assumption, (6) follows from the fact that $\{q^1, \dots, q^n\}$ is a basis of $\mathbb{R}^n$, and (7) is implied by (5). In particular, it follows from (6) and (7) that $\mathbf{z} = \mathbf{0}$, which, in view of the above, is equivalent to $x = y$. ■

In the special case of two states, Proposition 1 implies that the indifference relation in the Bewley model is trivial except when P consists of a single prior (in which case the decision maker is a subjective expected utility maximizer). The special case of two states is relevant for both pedagogical and illustrative purposes. In view of the present analysis, for example, Bewley indifference curves for monetary acts over two states are literally non-existent under weak regularity assumptions on preferences (see also below).

Assuming three states and using the Marschak-Machina triangle, Fig. 1 presents two examples of compact and convex sets of priors P and P' that satisfy and violate the full dimensionality condition of Proposition 1, respectively. Indeed, the two-dimensional triangular set P is the convex hull of the three linearly independent probability measures $p = (\frac{1}{8}, \frac{2}{8}, \frac{5}{8})$, $p' = (\frac{2}{8}, \frac{1}{8}, \frac{5}{8})$, $p'' = (\frac{3}{8}, \frac{2}{8}, \frac{3}{8})$ and, in light of Proposition 1, is therefore associated with trivial indifference under both types of Bewley preferences.

On the other hand, the one-dimensional linear segment P' is the convex hull of $q = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, $q' = (\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$. Since P' does not contain three linearly independent probability measures, it does not imply trivial indifference. Indeed, $px = py = 0$ holds for all $p \in P'$ whenever $x = (a, 0, -a)$ and $y = (b, 0, -b)$ are such that $a \neq b$.

Figure 1: Sets of priors with/without full dimensionality



As far as the underlying preferences' structure is concerned, the strict-preference version of Bewley's model asserted in Proposition 1 is characterized by axioms requiring $\succ$ to be a strict partial order that also satisfies independence, strict monotonicity ($x \geq y$ and $x \neq y$ implies $x \succ y$) and open-graph continuity.[1] The weak-preference analogue of that model on the other hand is characterized by axioms requiring $\succsim$ to be a preorder that also satisfies independence (as applied to such a relation), strict monotonicity and closed-graph continuity.[2]

It is presently unclear if intuitive axioms on preferences exist to characterize full dimensionality of the set of priors in the two versions of the Bewley model that are studied in this paper, and if the indifference-triviality conclusion carries over to more general domains of choice under uncertainty. It is also unclear if analogous results are obtainable in the extensions of the Bewley model that were proposed in Galaabaatar and Karni (2013) and Faro (2015). Answering these questions is left for future work.

References

Bewley, T. F. (2002): "Knightian Decision Theory. Part I," *Decisions in Economics and Finance*, 25, 79–110.

Chiesa, A., S. Micali, and Z. A. Zhu (2015): "Knightian Analysis of the Vickrey Mechanism," *Econometrica*, 83, 1727–1754.

[1]This follows from Theorem 1 in Bewley (2002).
[2]This follows from Proposition A.1 in Ghirardato, Maccheroni, and Marinacci (2004).

Dana, R.-A., and F. Riedel (2013): "Intertemporal Equilibria with Knightian Uncertainty," *Journal of Economic Theory*, 148, 1582–1605.

Danan, E., T. Gajdos, B. Hill, and J.-M. Tallon (2016): "Robust Social Decisions," *American Economic Review*, 106, 2407–2425.

Easley, D., and M. O'Hara (2010): "Liquidity and Valuation in an Uncertain World," *Journal of Financial Economics*, 97, 1–12.

Faro, J. H. (2015): "Variational Bewley Preferences," *Journal of Economic Theory*, 157, 699–729.

Galaabaatar, T., and E. Karni (2013): "Subjective Expected Utility with Incomplete Preferences," *Econometrica*, 81, 255–284.

Ghirardato, P., F. Maccheroni, and M. Marinacci (2004): "Differentiating Ambiguity and Ambiguity Attitude," *Journal of Economic Theory*, 118, 133–173.

Ghirardato, P., F. Maccheroni, M. Marinacci, and M. Siniscalchi (2003): "A Subjective Spin on Roulette Wheels," *Econometrica*, 71, 1897–1908.

Gilboa, I., F. Maccheroni, M. Marinacci, and D. Schmeidler (2010): "Objective and Subjective Rationality in a Multiple Prior Model," *Econometrica*, 78, 755–770.

Kajii, A., and T. Ui (2009): "Interim Efficient Allocations Under Uncertainty," *Journal of Economic Theory*, 144, 337–353.

Lopomo, G., L. Rigotti, and C. Shannon (2011): "Knightian Uncertainty and Moral Hazard," *Journal of Economic Theory*, 146, 1148–1172.

——— (2014): "Uncertainty in Mechanism Design," *working paper*.

Ma, W. (2015): "The Existence and Efficiency of General Equilibrium with Incomplete Markets under Knightian Uncertainty," *Economics Letters*, 134, 78–81.

Ok, E., P. Ortoleva, and G. Riella (2012): "Incomplete Preferences under Uncertainty: Indecisiveness in Beliefs Versus Tastes," *Econometrica*, 80, 1791–1808.

Rigotti, L., and C. Shannon (2005): "Uncertainty and Risk in Financial Markets," *Econometrica*, 73, 203–243.