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Velocity in the Long Run: Money and Structural Transformation

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Velocity in the Long Run: Money and Structural Transformation¹

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Abstract

Monetary velocity declines as economies grow. We argue that this is due to the process of structural transformation - the shift of workers from agricultural to non-agricultural production associated with rising income. A calibrated, two-sector model of structural transformation with monetary and non-monetary trade accurately generates the long run monetary velocity of the US between 1869 and 2013 as well as the velocity of a panel of 102 countries between 1980 and 2010. Three lessons arise from our analysis: 1) Developments in *agriculture*, rather than non-agriculture, are key in driving monetary velocity; 2) Inflationary policies are disproportionately more costly in richer than in poorer countries; and 3) Nominal prices and inflation rates are not ‘always and everywhere a monetary phenomenon’: the composition of output influences money demand and hence the secular trends of price levels.

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1 Introduction

How does a country's long-run money demand change with its economic development? An extensive literature² finds that for broad enough measures of the money stock and over long periods of time, increases in income per capita tend to be associated with increases in the money-to-GDP ratio or, equivalently, a falling monetary velocity.³ The possible sources of this stylized fact have been widely debated and include institutional changes, financial innovations, improvements in communication and information-gathering technologies as well as changes in the composition of output.⁴ Perhaps surprisingly, this research has been almost entirely empirical in nature, which has made it challenging to quantify the role played by individual channels.⁵ In this paper we address this gap by constructing and calibrating a model of long-run monetary demand, and we use the model to quantify the role played by a single mechanism potentially driving the income-velocity relationship: *structural transformation*. This process, also known as industrialization, is the change in the composition of an economy's employment and output, from agricultural towards non-agricultural goods, associated with economic growth. Whilst structural transformation is certainly known to influence money demand (see for example Jonung (1983), Friedman (1959) or Chandavarkar (1977)), no theoretical models of the process exist, and the quantitative importance of this channel is unclear. We use a calibrated model to show how structural transformation influences money demand over the development process and that quantitatively this channel is the main driver of within and across country differences in long-run monetary velocity.

The suggested mechanism is simple. Agriculture - especially traditional agriculture - is largely a non-monetary sector. As is argued by Chandavarkar (1977), this is true for at least three reasons. First, agricultural workers are often compensated in kind, either through share-cropping or other informal credit arrangements. Second, households in poorer countries tend to home-produce their agricultural consumption. Third, the agricultural setting is conducive to barter transactions.⁶

² See for example Wicksell (1936), Warburton (1949), Friedman (1959), Bordo and Jonung (1981, 1987), Jonung (1983) or Ireland (1991).

³ Income velocity of money, V , is defined as the ratio of gross domestic product, Y (measured in current prices, P) to the nominal money stock, M : $V \equiv \frac{P \times Y}{M}$. This implies that the inverse of velocity is simply the money-share: $V^{-1} = \frac{M}{P \times Y}$. Velocity is thus a convenient measure of how money demand compares to income, with lower money demand relative to income translating into higher velocity and vice-versa. This also means that a rising money share is equivalent to a falling velocity.

⁴ Wicksell (1936), for example, argues that this trend stems from an increase in the prominence of organized markets, a shift from home to market production as well as the increase in both worker specialization and the complexity of goods. Friedman and Schwartz (1963) suggest that real money balances are luxury goods and hence have an income elasticity larger than unity. Bordo and Jonung (1987, 1990) emphasize institutional changes and financial innovations as systematically influencing velocity over long periods, whilst Townsend (1987) and Goodfriend (1991) highlight that improvements in communications and information-gathering technologies can contribute to a falling velocity. Finally, Friedman (1959), Chandavarkar (1977) and Jonung (1983), assert that the composition of an economy can be important in driving money demand.

⁵ One notable exception is Ireland (1994), who constructs a theoretical model of changes in the composition of monetary aggregates. Our paper instead focuses on the changes in the total share of money-stock-to-GDP and abstracts from compositional effects.

⁶ Since most individuals wish to consume as varied a bundle of food as possible, most household-producers will wish to consume the agricultural goods of other household-producers in addition to their own. Thus, double coincidence of wants is easy to achieve when traders meet and thus the probability of barter is likely to be relatively high.

The non-agriculture sector, however, tends to be far more complex and hence is more likely to need money to enable exchange.⁷ As an economy grows, its structure will change from one dominated by a (predominantly non-monetary) agricultural sector towards one dominated by a (predominantly monetary) non-agricultural sector. This changing composition of the economy results in a rising demand for money, a rising money-to-GDP ratio and consequently a falling monetary velocity.

To capture the above mechanisms, we build a simple model with two sectors: agriculture, which produces goods traded without money, and non-agriculture, in which an endogenous share of goods is exchanged using money. The demand for money in the non-agricultural sector is introduced through a cash-in-advance constraint on consumption goods following Cole and Kocherlakota (1998). Building on a long-tradition of models exemplified by Herrendorf et al. (2014), Buera and Kaboski (2012), Gollin et al. (2002), Restuccia et al. (2008), Yang and Zhu (2013) and Stefanski (2014a,b), structural transformation is generated by non-homothetic preferences: consumers have a subsistence level of agricultural consumption. Intuitively, households need to eat a minimum quantity of food to survive. In poor countries where agricultural productivity tends to be low, more workers need to be employed in the agricultural sector in order to produce enough food to satisfy subsistence needs. A poor economy is thus dominated by non-monetary agriculture and most transactions take place without money. As agricultural productivity increases, less workers are needed to satisfy subsistence consumption, prompting workers to migrate into the non-agricultural sector and thus increasing that sector's share in total employment and output. Given that a part of non-agricultural goods are traded with money, the shift in the composition of the economy towards the non-agricultural sector will result in an increase in monetary transactions, an increase in the money-to-GDP ratio and hence in a lower velocity.

The model is calibrated to match the 1869-2007 patterns of US growth, structural transformation, and monetary supply. Our simple framework replicates several features of the long run data including agricultural labor shares, GDP per worker, sectoral prices, nominal interest rates, and aggregate inflation. Most importantly, the model reproduces the evolution of the US long run money-to-GDP ratio over 140 years, capturing 76% of the increase in the data. A similarly calibrated one-sector model fails to replicate observed money-share, as it is unable to match the observed price dynamics - and in particular the so-called 'Great-Deflation' of the late 19th century. The model also accurately predicts the variability in money-shares *across* countries. Keeping preference parameters of the US, we recalibrate the model to a panel of 102 countries between 1980 and 2010. The model accurately captures cross-country differences in incomes, employment shares, interest rates, and inflation rates. The baseline model captures 80% of the increase in money-share between the top and bottom deciles of cross-country data and does exceptionally well in replicating the income-velocity relationship.⁸

⁷ Crucially, there are more varieties of goods in the non-agricultural sector. Thus, the probability that an economics professor - for example - can barter his output for the services of a car mechanic is far lower than the probability that two farmers producing different types of vegetables may wish to trade. For an in-depth discussion of this transactional role of money see Ostroy and Starr (1990).

⁸ In particular, a one percent increase in GDP per worker is found to increase the money-to-GDP ratio by 0.139 percentage points in the data and 0.143 percentage points in the model.

The multi-sector framework, and in particular the reallocation of workers from agriculture to non-agriculture, is the key driver of our results. To illustrate this point, we perform a number of exercises that isolate individual channels affecting monetary velocity, like variation in monetary growth rates, productivity growth rates and productivity levels. We also extend the baseline model to allow for heterogeneous endowments of capital across countries,⁹ to include monetary and non-monetary agricultural goods, and we perform a series of alternative calibration exercises and tests. Quantitatively, we find that differences in agricultural productivity levels are the key source of cross-country variation in velocity between rich and poor countries since they influence the size of the predominantly-monetary, non-agricultural sector. A one-sector version of our model without structural transformation fails entirely in capturing across and within country variation in monetary velocity.

Finally, we examine how the costs of suboptimal monetary policies vary with income. Inflation is more costly in richer countries than in poorer countries - where the monetary part of the economy is smaller and therefore distortions from the inflation tax are less damaging. For example, a hyperinflation of approximately 400% a year in a poor country like Zimbabwe (where GDP per worker is 2% of that of the US) will have negligible welfare costs. By contrast, the same hyperinflation in a country like Argentina (30% of US GDP per worker) will have very large welfare costs. Argentinean incomes would have to rise by approximately 25% to deliver the same expected flow utility as without the hyperinflation. The low cost of inflation in poorer countries may help explain why poorer countries tend to have much higher inflation rates than richer countries.

There are three main lessons from our work. First, monetary velocity is not constant over the development process but falls with income. Most of the existing explanations of this observation are rooted in the non-agricultural sector - and focus on factors such as financial innovation or the expansion of the banking sector. The surprising finding of this paper is that the evolution of monetary velocity is driven almost exclusively by developments in the *agricultural* sector. It is the variation in agricultural productivity that influences the size of the non-agricultural sector and in turn influences monetary demand and hence velocity. Second, the cost of inefficient monetary policy varies with income and is higher in richer countries than in poorer countries. An inflation tax offers a relatively cheap source of income to poor-country governments, and its distortive effects are relatively small in economies that are dominated by large, non-monetary, agricultural sectors. This may help explain why we observe persistently higher inflation in poorer countries than in richer countries - despite recommendations of strong anti-inflationary policies by international financial institutions such as the International Monetary Fund (IMF).¹⁰ Finally, the third lesson of this paper is that, since velocity depends systematically on the composition of output - a country's price levels and inflation rates may not 'always and everywhere be a monetary phenomenon', as suggested

⁹ Capital is largely a credit good and so a variation in capital stock levels can influence the demand for money and hence velocity.

¹⁰ Weisbrot et al. (2009), for example, examine existing IMF loan agreements with 41 developing countries in 2009. They find that the IMF took a stance on monetary policy in 25 of these countries and in 22 of those 25 countries it recommended a contractionary or anti-inflationary monetary policy.

for example by the work of Friedman and Schwartz (1963). The price level in an economy, P , is defined as $P \equiv (M/Y)V$ where M is the money stock, Y is output and V is velocity. Two countries, with identical money stocks and identical output *levels* - but with different output *compositions* - will have entirely different velocities, V , and hence different price levels. The message to researchers from these findings is that a one-sector model cannot be successfully used to understand the long run dynamics of monetary velocity and hence the evolution of price levels or inflation rates. These findings should also be of interest to policymakers in developing countries who may have overlooked the importance of the agricultural sector in their monetary policy decisions.

In the next section we document the main facts regarding structural transformation, monetary velocity, and the extent of non-monetary production in agriculture. In section 3, we construct and solve our simple baseline model. In section 4 we calibrate the model to the experience of the United States and in section 5 we show the results for the US. Section 6 carries out the cross-country analysis by examining how the model performs in international, cross-country data and by running a number of counterfactuals to quantify the importance of the different mechanisms of the model. Section 7 performs a number of robustness checks and extensions. Section 8 examines the different costs of inflation in rich and poor countries. Finally, section 9 offers some concluding remarks on the importance of our findings to researchers and policy makers.

2 Facts

In this section we present three stylized facts. First, there is a positive relationship between the money to GDP ratio (or the inverse of monetary velocity) and income per capita. Second, the proportion of workers employed in agriculture tends to fall as income rises. Finally, non-monetary activities are most prevalent in countries with large agricultural sectors. These facts taken together suggest that as countries grow richer, they tend to use more money relative to the size of their economy, and that this process is potentially linked to the decline of a predominantly non-monetary sector - agriculture.¹¹

Money Shares We are interested in the pattern of the ratio of the stock of money to nominal GDP, over time and across countries. It is therefore crucial to define exactly what we mean by the stock of money. Throughout the paper all classifications of monetary data follow the definitions set out by the IMF in the International Financial Statistics (IFS). The IFS classifies money into ever broadening bands from M0 to M3.¹² We are not directly interested in the narrowest definition of the monetary stock such as M0 or M1: as argued by Ireland (1994), economies undergo a change in

¹¹ See Appendix A for details of sources and construction of all data.

¹² The narrowest definition of money stock, M0, is the value of currency and deposits in the central bank. The next classification, M1, includes all of M0 as well as transferable deposits and electronic currency. Next, M2, includes M1 but also measures time and savings deposits, foreign currency transferable deposits, certificates of deposit and securities repurchase agreements. Finally, M3 includes M2 as well as travelers checks, foreign currency time deposits, commercial paper, and shares of mutual/market funds.

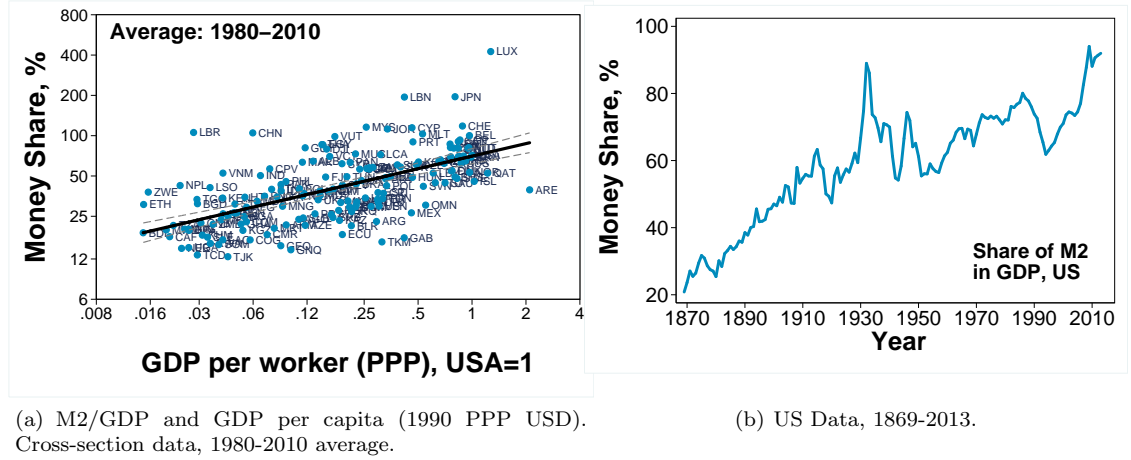


Figure 1: M2/GDP and income per capita. Share of money (M2) in GDP with income and over time. (Source: WDI, PWT, Anderson (2006))

monetary technologies as they develop, and they therefore use less currency and more sophisticated forms of exchange such as electronic currency or time deposits. These technological changes and the subsequent changes in the *composition* of money are not the focus of this paper. Instead we are interested in how the *quantity* of money evolves with income. This suggests that we should focus on the wider definitions of money. The data for M3 however is relatively scarce - both across countries and for longer periods of time. Consequently, we choose our measure of money stock to be M2.¹³

For each country we calculate the average money share (M2 divided by nominal GDP) and the average GDP per worker over the 1980-2010 period in order to focus on the long run relationship between these two variables. Figure 1(a) then plots the average money share versus GDP per worker (relative to the US average GDP) for the resulting cross-section of 166 countries. A strikingly positive relationship emerges: richer countries tend to have a higher money share (or a lower velocity) than poorer countries. This fact is statistically significant, as apparent from the 5% error bands on the regression line. Quantitatively, a 1% increase in GDP per worker is associated with a 0.16 percentage point increase in the money share. A similar fact can be observed in long run time series data. Figure 1(b) shows the share of M2 money stock relative to nominal GDP in the United States between 1869 and 2014. There is a clear rising trend in the money stock to GDP ratio. Since GDP per worker in the US roughly grew at a constant rate over the period, there is also a positive relationship between money share and income per worker in the US time-series data.¹⁴ Finally, notice that this is not an isolated case. Bordo et al. (1993) have documented similar historical patterns in long run data for Canada, the United Kingdom, Norway and Sweden.

¹³ See Dorich (2009) and McCallum and Nelson (2010) for a discussion of which monetary aggregate better captures the liquidity services of money.

¹⁴ The semi-elasticity of the money share with respect to GDP per worker in the US over the period is 0.19.

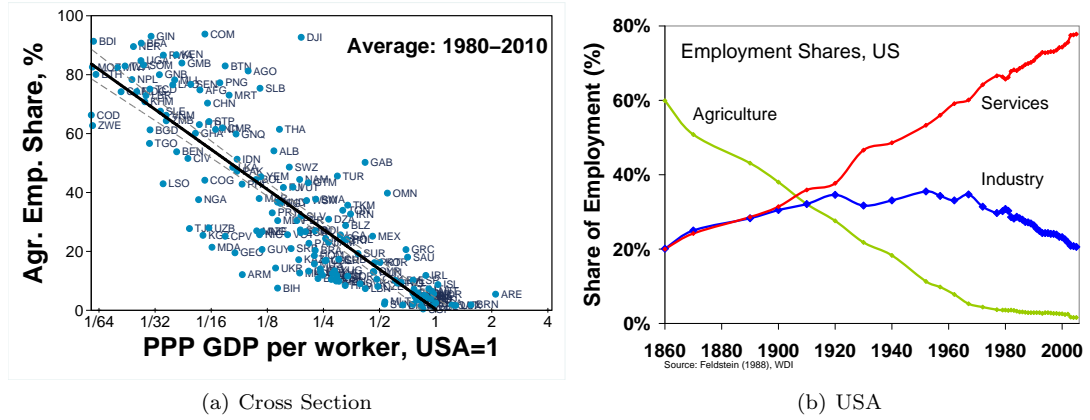


Figure 2: Share of employment in agriculture with GDP per worker and over time. (Source: FAOSTAT, Urquhart (1984) and WDI (2016).)

Structural Transformation It is a widely documented fact that agricultural employment shares fall with rising income in a process referred to as structural transformation.¹⁵ Figure 2(a) documents this pattern for a cross-section of 173 countries by plotting the average agricultural employment shares over the 1980-2010 period versus the average (1990, PPP) GDP per worker of each country. A similar pattern is visible within countries over time as their income per capita increases. Figure 2(b), for example, depicts falling agricultural employment shares in the US between 1860 and 2004 from Urquhart (1984) and WDI (2016). This process will prove to be the key element driving the rising monetary shares depicted in Figure 1.

The non-monetary economy Finally, we present evidence on the extent of non-monetary or subsistence activities, and their relationship with the agricultural sector. Blades (1975a,b) carries out an analysis for the OECD using survey data for 48 developing countries over the 1970-1975 period, documenting the proportion of GDP that can be classified as a non-monetary or subsistence activity¹⁶. His conclusions were quite strong: “Agriculture is obviously the main item in non-monetary production, accounting often for over 80 percent of the total.” Figure 3 plots the share of non-monetary value added in GDP from Blades (1975a,b) versus agricultural employment share in 1980.¹⁷ Notice the strong positive relationship: agricultural economies tend to have a far greater proportion of their value added in non-monetary sectors than more industrialized countries. As we argue in the remainder of the paper, this relationship will be crucial in explaining cross-country and within country differences in long run monetary velocity.

¹⁵ See for example, Maddison (1982), Echevarria (1997) or Duarte and Restuccia (2010).

¹⁶ See Appendix A for more details on this data.

¹⁷ We use the 1980 agricultural employment share, since earlier data is not available for all countries.

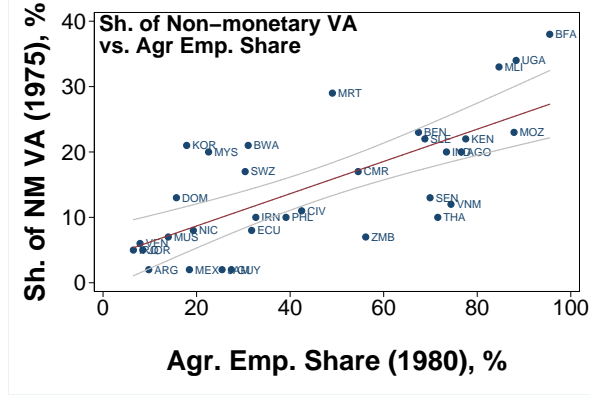


Figure 3: Share of non-monetary value added, relative to the agricultural employment share. (Source: WDI, Blades (1975a,b))

3 The Baseline Model

In this section we build a very simple cash-in-advance model of structural transformation guided by the three facts presented in the previous section: money shares that rise with income, agricultural employment shares that decline with income, and the dominance of non-monetary transactions in predominantly agricultural economies. Our model is a version of Cole and Kocherlakota (1998) but with two sectors: a completely non-monetary sector (agriculture) and a partially monetary sector (non-agriculture). In this baseline version, we assume that production technologies are linear in labor. In Appendix B.3, we extend the model to include capital accumulation in the non-agricultural sector, obtaining very similar results. Appendix B.4 relaxes the assumption that agricultural goods are only exchanged without money, introducing a money demand in agriculture; in this case results are also similar, therefore we focus on the simplest version of the model. In order to generate structural transformation, we impose non-homothetic preferences over agricultural goods. This assumption, in conjunction with rising agricultural productivity, generates a shift of labor from agriculture to non-agriculture. A calibrated version of the model will be used to quantify the role that structural transformation plays in driving across- and within- country differences in monetary shares.

Households The representative household maximizes welfare subject to its budget constraint, a cash in advance constraint, a non-negativity constraint on money holdings, and a non-Ponzi condition:

$$\begin{aligned}
 & \max_{a_t, c_{m,t}, c_{n,t}, b_{t+1}, m_{t+1}} \sum_{t=0}^{\infty} \beta^t (\alpha \log(a_t - \bar{a}) + (1 - \alpha - \gamma) \log(c_{m,t}) + \gamma \log(c_{n,t})) \\
 & \text{s.t. } p_{a,t}a_t + p_{c,t}(c_{m,t} + c_{n,t}) + b_{t+1} + m_{t+1} \leq w_t + (1 + r_t)b_t + m_t + T_t \\
 & \quad p_{c,t}c_{m,t} \leq m_t \\
 & \quad m_t \geq 0 \text{ and } b_t \geq -\bar{B}
 \end{aligned} \tag{1}$$

The household owns a unit of labor which it sells on the market for a wage w_t . In addition, the household comes into the period with money holdings m_t , bond holdings b_t (which it sells at a price $1 + r_t$) and it receives a helicopter transfer of money from the government of T_t . Given this income, the household purchases agricultural goods a_t (at the price $p_{a,t}$) as well as non-agricultural goods $c_{m,t}$ and $c_{n,t}$ (at the price $p_{c,t}$). It also purchases bonds b_{t+1} that promise to pay out $1 + r_{t+1}$ dollars next period, and chooses its (non-negative) stock of money, m_{t+1} , for the next period. We assume that there are two kinds of non-agricultural goods: those that can be bought without money (c_t^n) and those that must be bought with money ($c_{m,t}$). This establishes a role for money: the household needs to put aside a part of its income, m_{t+1} , each period in order to be able to buy monetary goods in the following period. To capture this idea, we impose the cash-in-advance (CIA) constraint (23) on $c_{m,t}$ goods. Only cash held from the previous period can be used to purchase monetary goods in the current period and cash transfers from government can only be used in the subsequent period. Finally, we impose a lower bound, $-\bar{B}$, on bond holdings to avoid Ponzi schemes.

Households have non-homothetic preferences for agricultural goods a_t . In other words, there exists a subsistence level of agriculture, \bar{a} , that must be consumed every period. Intuitively, households need to obtain a minimum quantity of food or calories in order to survive. A crucial assumption of our model is that agriculture is a non-cash good.¹⁸ The intuition here is that agriculture - especially traditional agriculture - is often characterized by compensation in kind or subsistence production. Farmers - especially in poorer countries - often work for landlords as sharecroppers and get paid in kind for their efforts, or produce much of their food at home for their own consumption (Blades, 1975a,b). Furthermore, barter is far more likely in agriculture than in non-agriculture, as most people wish to eat a varied basket of foods. Thus, when two agricultural producers (or households) meet to trade, double coincidence of wants for agricultural products is relatively likely. As such, little to no cash is needed to obtain agricultural goods in poorer countries. Of course, the above assumption is a simplification and in richer countries, cash will be far more readily used in the agricultural sector. However, in richer countries, the agricultural sector will also tend to be very small - and hence the influence of agricultural money demand on the overall money demand will also be very small. Therefore, as a first approximation, the assumption that all agricultural goods are traded without money is a reasonable one.

Money use in non-agriculture, however, will be important, since a double coincidence of wants in that sector is far less likely. Due to the massive variety of goods produced in non-agriculture, the probability that two randomly matching producers or consumers (say an economist and a car mechanic) would want each other's services will be far smaller. As such, money becomes necessary to overcome this mismatch problem. Of course, some goods in non-agriculture are nonetheless still traded without cash using credit arrangements or payments in kind (e.g. employer-provided cars,

¹⁸ This is an extreme and simplifying assumption. In appendix B.4, we relax it by having two sub-sectors in the agricultural sector: traditional agriculture, which is traded without cash, and modern agriculture, which is traded with cash. In this way, we generate a more realistic money demand for agricultural products. It turns out however, that agricultural money demand plays a small role and that results are very similar to the baseline version of the model.

housing, health insurance etc.). Hence, following Chari et al. (1996), in our model a fixed proportion γ of non-agricultural goods do not require cash.¹⁹

Firms There are two representative firms: agricultural (a) and non-agricultural (c). Each firm $s = a, c$, hires labor, $L_{s,t}$, and produces output, $Y_{s,t}$, using a simple linear technology that combines labor with exogenous, sector-specific total factor productivity, $B_{s,t}$. The profit maximization problems are:

$$\max_{L_{s,t}} p_{s,t} Y_{s,t} - w_t L_{s,t} \quad \text{s.t.} \quad Y_{s,t} = B_{s,t} L_{s,t}. \quad (2)$$

Output of non-agricultural firms, $Y_{c,t}$, can be sold as both a monetary and a non-monetary good, whereas all agricultural output is assumed to be non-monetary.

Money Supply The government is assumed to have a so-called helicopter monetary policy:

$$M_{t+1} = T_{t+1} + M_t. \quad (3)$$

Market Clearing Finally, in each period, markets clear in a standard fashion.

$$\begin{aligned} a_t &= Y_{a,t}, \quad c_{m,t} + c_{n,t} = Y_{c,t} \\ m_t &= M_t, \quad b_t = 0 \\ L_{a,t} + L_{c,t} &= 1. \end{aligned} \quad (4)$$

Competitive Equilibrium For a given monetary policy, $\{T_t\}_{t=0}^\infty$, a competitive equilibrium in this economy is a sequence of prices, $\{p_{a,t}, p_{c,t}, w_t, r_t\}$, and quantities, $\{a_t, c_{m,t}, c_{n,t}, b_t, m_t, L_{a,t}, L_{c,t}\}_{t=0}^\infty$, such that (1) given prices and monetary policy, households and firms solve their optimization problem, (2) the government budget constraint is satisfied and (3) markets clear.

Solution The first order conditions for the household are given by:

$$a_t: \frac{\alpha \beta^t}{a_t - \bar{a}} = \lambda_t p_{a,t}; \quad c_{m,t}: \frac{(1 - \alpha - \gamma) \beta^t}{c_{m,t}} = p_{c,t}(\lambda_t + \mu_t); \quad c_{n,t}: \frac{\gamma \beta^t}{c_{n,t}} = p_{c,t} \lambda_t \quad (5)$$

$$b_{t+1}: \lambda_t = (1 + r_{t+1}) \lambda_{t+1}; \quad m_{t+1}: \lambda_t = \lambda_{t+1} + \mu_{t+1} \quad (6)$$

$$\text{CIA: } \mu_t(p_{c,t} c_{m,t} - m_t) = 0 \quad \text{and} \quad \mu_t \geq 0 \quad (7)$$

¹⁹ In Appendix B.3, we follow Cole and Kocherlakota (1998) by adding capital to the model, and show that if we treat investment as a non-monetary, credit good we can go a long way in endogenizing the cash-credit split of the non-agricultural sector.

In the above, λ_t and μ_t are multipliers on the budget and CIA constraints respectively. The firms' first-order conditions are:

$$L_{a,t}: p_{a,t}B_{a,t} = w_t \quad \text{and} \quad L_{c,t}: p_{c,t}B_{c,t} = w_t. \quad (8)$$

The market clearing conditions are given by the equations in (4). Finally, the transversality conditions for the above problem are:

$$\liminf_{t \rightarrow \infty} \lambda_t(b_{t+1} + \bar{B}) = 0 \quad \text{and} \quad \liminf_{t \rightarrow \infty} \lambda_t m_{t+1} = 0. \quad (9)$$

Following Cole and Kocherlakota (1998), we solve the model by imposing the following assumption on interest rates:

Assumption 3.1. *Assume that interest rates are always positive, i.e. $r_t > 0$.*

The consequence of the above assumption is that the CIA constraint always binds since there is a positive opportunity cost of holding money.²⁰ We impose this assumption for two reasons. First and most importantly, despite recent negative nominal interest rates, this paper's focus is the very long run, and long-run interest rates have tended to be positive. Second, imposing this assumption allows us to find a simple, analytic and unique solution.

Given a sequence of government policies and Assumption 3.1 (so that the CIA constraint holds with equality), equations (5)-(8), in addition to the market clearing conditions, characterize the competitive equilibrium. In particular, sectoral employment is given by:

$$L_{a,t} = \frac{\alpha\tau_t + (1 - \alpha - \gamma(1 - \tau_t)) \frac{\bar{a}}{B_{a,t}}}{1 - (\alpha + \gamma)(1 - \tau_t)} \quad \text{and} \quad L_{c,t} = 1 - L_{a,t}, \quad (10)$$

where $\tau_t \equiv \frac{1}{\beta} \frac{M_{t+1}}{M_t}$. Non-agricultural output is divided between cash and non-cash goods:

$$c_{n,t} = (1 - \phi_t)Y_{c,t} \quad \text{and} \quad c_{m,t} = \phi_t Y_{c,t}, \quad (11)$$

where $\phi_t = \frac{1 - \alpha - \gamma}{(1 - \alpha - \gamma) + \gamma\tau_t}$. Finally, sectoral prices are

$$p_{a,t} = \frac{M_t}{\phi_t B_{a,t} L_{c,t}} \quad \text{and} \quad p_{c,t} = \frac{M_t}{\phi_t B_{c,t} L_{c,t}} \quad (12)$$

and the nominal interest rate and wage rate are

$$r_t = \tau_t - 1 \quad \text{and} \quad w_t = \frac{M_t}{\phi_t L_{c,t}}. \quad (13)$$

The impact of non-homothetic preferences on agricultural employment can be seen in equation (10). When productivity in agriculture, $B_{a,t}$, is low, a large fraction of the labor force must work

²⁰ To see this divide the equations in (6) by each other to obtain the expression for interest rates, $r_{t+1} = \frac{\mu_{t+1}}{\lambda_{t+1}}$. Thus, the nominal interest rate is positive if and only if money yields liquidity services ($\mu_{t+1} > 0$). In particular, if the nominal interest rate is positive, the CIA constraint is binding.

in agriculture in order to produce enough food for subsistence. As productivity in agriculture increases, the subsistence level can be met with a smaller fraction of the population working in the agricultural sector resulting in a shift of workers from agriculture to non-agriculture. In other words, from equation (10), $\partial L_{a,t}/\partial B_{a,t} < 0$.

Next, notice that the composition of the economy affects sectoral prices. Since sectoral employment shares depend on agricultural productivity, from equations (10) and (12), we can write $\partial p_{s,t}/\partial B_{a,t} < 0$, $s = a, c$. In other words, *ceteris paribus*, prices of agricultural and non-agricultural goods decline with rising agricultural productivity and hence with structural transformation. Thus it is not only *monetary* factors that drive nominal prices, as has been suggested by the literature (Friedman and Schwartz, 1963). *Real* factors also play a crucial role. Rising agricultural productivity changes the composition of the economy, and hence influences the nominal price of goods. This feature of the model is particularly important at the early stages of industrialization. In section 5, we show that this process was key in generating the so-called “Great Deflation” in the late 19th century United States.

Finally, notice that monetary policy has an impact both on sectoral employment and the consumption of monetary goods. By choosing the transfers, the government directly controls the evolution of the money stock in the economy, M_t , and hence the variable τ_t , which determines how costly it is to hold cash from one period to another. From equation (13), the higher the τ_t , the higher the nominal interest rate, and hence the greater the opportunity cost of holding cash. Thus, higher τ_t results in workers shifting away from cash dominated sectors, $\partial L_{c,t}/\partial \tau_t < 0$ and consumers consuming less cash goods, $\partial c_{m,t}/\partial \tau_t < 0$. This first channel highlights a new cost of monetary policy in this two-sector CIA model - higher inflation taxes can reverse or delay structural transformation. Interestingly, the impact of monetary policy on the size of the agricultural sector is itself dependent on the level of agricultural productivity, given that $\frac{\partial(\partial L_{a,t}/\partial \tau_t)}{\partial B_{a,t}} > 0$. In other words, an inflation tax in a country with higher agricultural productivity will have a greater distortionary effect than in a country with lower productivity. Countries with high agricultural productivity have larger non-agricultural and hence monetary sectors. The inflation tax will therefore impact a larger fraction of those economies. This suggests that the same policies will have different effects in rich and poor countries. We quantify these effects in section 8, where we look at the welfare cost of inflation.

Velocity The share of monetary stock relative to the nominal GDP (or the *inverse* of monetary velocity) can be written as:

$$\begin{aligned} V_t^{-1} &= \frac{M_t}{p_{a,t}Y_{a,t} + p_{c,t}Y_{c,t}} = \phi_t \frac{p_{c,t}Y_{c,t}}{p_{a,t}Y_{a,t} + p_{c,t}Y_{c,t}} \\ &= \phi_t L_{c,t}. \end{aligned} \tag{14}$$

The first equality follows by definition. The second equality follows from the assumption that the CIA constraint binds, and from equation (11), which splits non-agricultural consumption into its monetary and non-monetary components. The final equality follows from equations (10)-(13).

There are two channels driving the money share. First, the term ϕ_t determines what proportion of non-agriculture is bought with cash. This variable itself is influenced by the preference parameter, γ , which captures (in a reduced form) the non-monetary activity in the non-agricultural sector, as well as by τ_t . Since $\partial\phi_t/\partial\tau_t < 0$, a higher inflation tax results in households wanting to purchase fewer cash goods, which in turn lowers money demand and the money share. Second, the money share crucially and positively depends on $L_{c,t}$ - the share of employment in the non-agricultural sector. As a greater proportion of workers shifts to a largely cash sector, the share of money in the economy rises - and the velocity falls.

From (14), two facts of interest emerge. First, since higher agricultural productivity results in greater non-agricultural employment, it also implies a higher money share. In other words, $\frac{\partial V_t^{-1}}{\partial B_t^a} > 0$. Second, a higher inflation tax means it is more costly to hold cash, which leads to a lower employment share in non-agriculture (as workers move away from a cash to a non-cash sector) and a lower ϕ_t (as consumers want to consume less cash goods). Together this means that the money share decreases in response to a higher τ_t so that $\frac{\partial V_t^{-1}}{\partial \tau_t} < 0$.

These two facts highlight that both monetary policy and agricultural productivity can influence a country's monetary demand and hence monetary shares. We quantify the role that each of these channels play in explaining within and across country variation in monetary shares, by calibrating the model in the next section.

4 US Calibration

We calibrate a benchmark economy to US data for the period from 1869 to 2007.²¹ Our calibration strategy involves choosing parameter values so that the equilibrium of the model matches the most important features of the structural transformation in the United States over this period.²² To stay consistent with the literature we follow as closely as possible the calibration of Duarte and Restuccia (2010). We assume that a period in the model is one year. We need to select parameter values for $\alpha, \bar{a}, \beta, \gamma$, the time series of the money stock M_t for t from 1869 to 2007 and the time series of productivity for each sector $B_{s,t}$ for t from 1869 to 2007 and $s \in \{a, c\}$.

We proceed as follows. First, we normalize the level of money to one in 1869; that is, $M_{1869} = 1$. Then we use data on the growth rate of M2 per worker in the United States to obtain the time paths of money per worker in the model. In particular, denoting as $g_{m,t}$ the growth rate of M2 per worker at time t , we obtain the time path of money in the model as $M_{t+1} = (1 + g_{m,t})M_t$.²³

Second, we use data on average money per worker growth rates and nominal interest rates in the United States to choose the discount factor. In particular, from equation (13), it follows that $\beta = \frac{g_{m,t}}{1+r_t}$. Denoting by \bar{g}_m the average annual growth rate in the US of M2 per worker between

²¹ We stop at 2007 to avoid the financial crisis period, when emergency monetary policy was implemented.

²² See Appendix A for details of sources and construction of all data.

²³ In Appendix B.1, we show that calibrating the model to observed nominal interest rates instead of money growth rates leaves the results unchanged.

Parameter	Values	Target	Target Value
$B_{s,1869}, M_{1869}$	1	Normalization, $s \in \{a, c\}$.	-
$g_a - 1$	0.028	Productivity growth in agriculture, 1869-2007.	2.79%
$g_c - 1$	0.012	Productivity growth in non-agriculture, 1869-2007.	1.23%
$\{B_{s,t}\}_{t=1869}^{2014}$	$B_{s,t} = g_s^{t-1869}$	Constant productivity growth in sector $s \in \{a, c\}$.	-
$\{M_t\}_{t=1869}^{2014}$	$\{\cdot\}$	Growth in money stock per worker.	$\{\cdot\}$
α	0.005	Long-run employment share in agriculture.	0.5%
\bar{a}	0.548	Employment share in agriculture in 1869.	55%
β	0.963	Average nominal interest rate & money growth, 1869-2007.	$\bar{r} = 8.85\%$ $\bar{g}_m = 4.79\%$
$1 - \gamma$	0.990	Long-run share of money in nominal value-added.	100lrmsshare%

Table 1: Baseline Model, calibrated parameters. See Appendix for detailed sources.

1869 and 2007 of 4.79% and by \bar{r} the average annual nominal returns of the Standard and Poor Composite Stock Price Index between 1871 and 2007 of 8.85%, we set $\beta = \frac{\bar{g}_m}{1+\bar{r}}$.

Third, we normalize productivity levels across sectors to one in 1869; that is, $B_{s,1869} = 1$ for $s \in \{a, c\}$. Then we use data on the average growth rate of sectoral value added per worker in the United States to obtain the time paths of sectoral labor productivity. In particular, denoting by g_s the average (annualized) growth rate of labor productivity between 1869 and 2007 in sector s , we obtain the time path of labor productivity in each sector as $B_{s,t+1} = (1 + g_s)B_{s,t}$.²⁴

Fourth, with positive productivity growth in all sectors and a monetary policy that converges to the optimum, the share of employment in agriculture converges to α whilst the ratio of money to nominal GDP converges to $1 - \gamma - \alpha$.²⁵ Because the share of employment in agriculture has been falling systematically and was about 1.7% in 2007, we assume a long-run share of 0.5%. The share of M2 relative to value added, on the other hand, has been rising and was about 75% in the 2000's. Using a non-linear regression, in Appendix A we fit an exponential function to the money-share data and find an implied asymptote of approximately 100 lrmsshare%. We take this as our long run money share. Both targets are somewhat arbitrary, however our main results are not sensitive to these choices.

Finally, \bar{a} is chosen to match the share of employment in agriculture in the United States in 1869 using equation (10). Table 1 summarizes the calibrated parameters and targets.

5 US Results

Next we show that despite its simplicity the model does well in capturing a number of real and monetary features of the US economy. We begin with the non-monetary or ‘real’ implications of the model. First, Figure 4(a) shows that the model predicts a decline of agricultural employment share from 55% in 1869 to 1.5% in 2013. Even though we only target the employment share in 1869, the

²⁴ We use average growth rates since earlier data exists only at decadal frequencies. In Appendix B.2, we allow for time varying growth rates by using interpolated data. This makes very little difference to our results.

²⁵ With positive productivity growth, agricultural employment converges to $\frac{\alpha}{(1+\gamma)(g_{m,t}/\beta-1)}$ and the money share converges to $\frac{1-\alpha-\gamma}{1+(\alpha+\gamma)(g_{m,t}/\beta-1)}$. In section 8, we show that monetary policy converging to the Friedman Rule, i.e. $g_{m,t} \rightarrow \beta$, is optimal. Under this policy agricultural and money shares converge to the values given in the text.

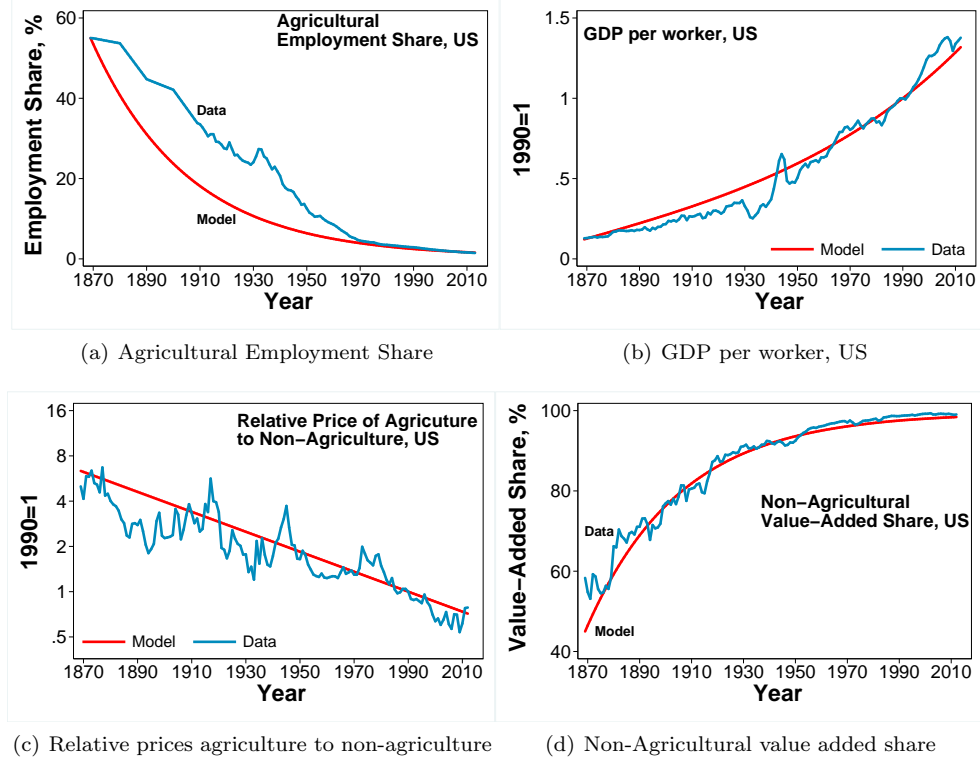


Figure 4: Simulations and data for the US, 1869-2012.

agricultural employment share in 2013 is very close to the value in the data of 1.6%. This decline is driven by rising agricultural productivity in equation (10). Second, since the model matches the evolution of sectoral employment well and since productivity growth comes directly from the data, the model also matches sectoral output (the product of labor productivity and sectoral employment). Agricultural output growth in the model is approximately 0.28% per year (in comparison to 0.29% per year in the data) whilst output growth in non-agriculture in the model is 1.79% per year (in comparison to 1.78% in the data). Figure 4(b) then shows output per worker for the entire economy. The model predicts an average annualized growth rate of 1.68% a year versus 1.69% in the data. Third, the model also has strong implications for the evolution of relative prices. From equation (12), the relative price of sector a to sector c goods is given by the ratio of sectoral labor productivities:

$$\frac{p_{a,t}}{p_{c,t}} = \frac{B_{c,t}}{B_{a,t}}. \quad (15)$$

Figure 4(c), shows that the relative price of agriculture in the data declined at approximately 1.51% a year between 1869 and 2013, whilst the model predicts a decline of 1.44% per year over the same period. The fit is thus remarkably close - even though the model is not calibrated to match the evolution of these prices. Finally, since the model matches relative prices and sectoral output well,

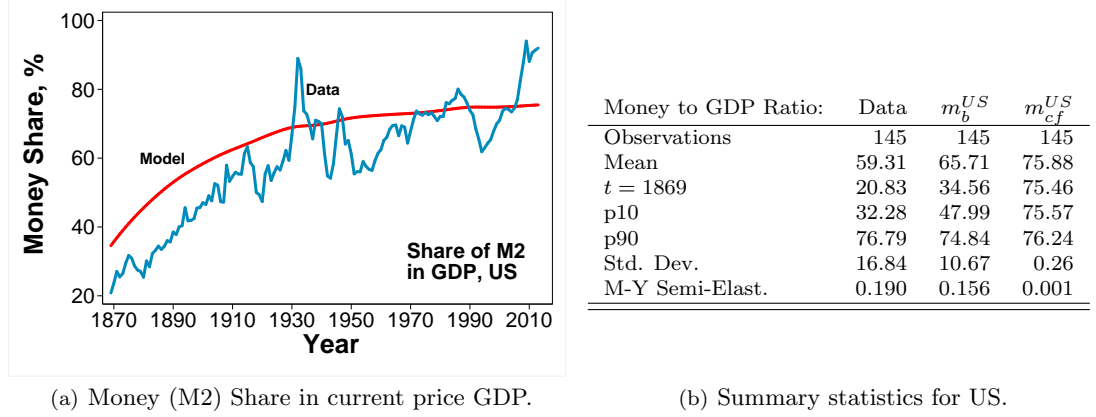


Figure 5: Money share of GDP and summary statistics. In the Table, columns show statistics for: money shares from the data (Data), the baseline model (m_b^{US}), and the US counterfactual (m_{cf}^{US}). Rows (1)-(6) show: the number of observations, the mean, the top-decile, the bottom-decile, the standard deviation as well as the semi-elasticity of money-share to income.

it follows that it must also match the evolution of sectoral value added shares.²⁶ Figure 4(d), shows that the share of non-agricultural value added in total value added in the data increased from approximately 54.2% in 1869 to 99.1% in 2012. The corresponding increase in the model was slightly larger from 45.0% in 1869 to 98.4% in 2012.

Next, we compare our predicted monetary or ‘nominal’ variables with the corresponding values in the data. Figure 5 shows the model’s striking ability to capture the evolution of US monetary share - our main variable of interest. This simple model generates 76% ($\approx \frac{77.6-34.56}{77.6-20.83}$) of the observed increase in the money share of the United States economy between 1869 and the implied long-run money share of approximately 77.6% and 60% ($\approx \frac{74.84-47.99}{76.79-32.28}$) of the observed increase between the top and bottom money-share deciles. The correlation between money shares in the model and the data is 0.91, the R^2 in a regression of the data on the model is 0.82 and the model captures 63% ($\approx \frac{10.67}{16.84}$) of the standard deviation in the data. Thus, whilst we do not capture short term fluctuations, the fit of the long-run trend is exceptional.

Importantly, notice that other than choosing γ in the calibration to target the implied long-run money share of 100 lrmsare%, we do not target the evolution of money shares at all. Performing a variance decomposition, it is easy to show that the increase in money share over time emerges entirely from the mechanism of the model, i.e. from a productivity driven reallocation of labor out of agriculture and into non-agriculture.²⁷ Finally, the fit of the model does not in any way come

²⁶ This is because $\frac{p_{c,t}Y_{c,t}}{p_{a,t}Y_{a,t}+p_{c,t}Y_{c,t}} = \frac{Y_{c,t}}{p_{a,t}Y_{a,t}+Y_{c,t}}$.

²⁷ To see this formally, recall that in the model, $V_t^{-1} = \phi_t L_{c,t}$. Taking logs of both sides of this equation we obtain, $\bar{V}_t^{-1} = \bar{\phi}_t + \bar{L}_{c,t}$, where the bar represents the log of a variable. Next, denoting the sample variance by $var(\cdot)$ and the sample standard deviation by $std(\cdot)$, we can calculate two variance decompositions of the money shares implied by the model: $\frac{std(\bar{L}_{c,t})}{std(\bar{V}_t^{-1})} = 1.01$ and $\frac{var(\bar{L}_{c,t})}{var(\bar{\phi}_t)+var(\bar{L}_{c,t})} = 0.99$. From this, we see that virtually all the variance in money share in the model is driven by variation in employment share. Importantly, this matches with what we observe in the US data where the corresponding values of the two decompositions between 1869 and 2013 are 0.77 and 0.76.

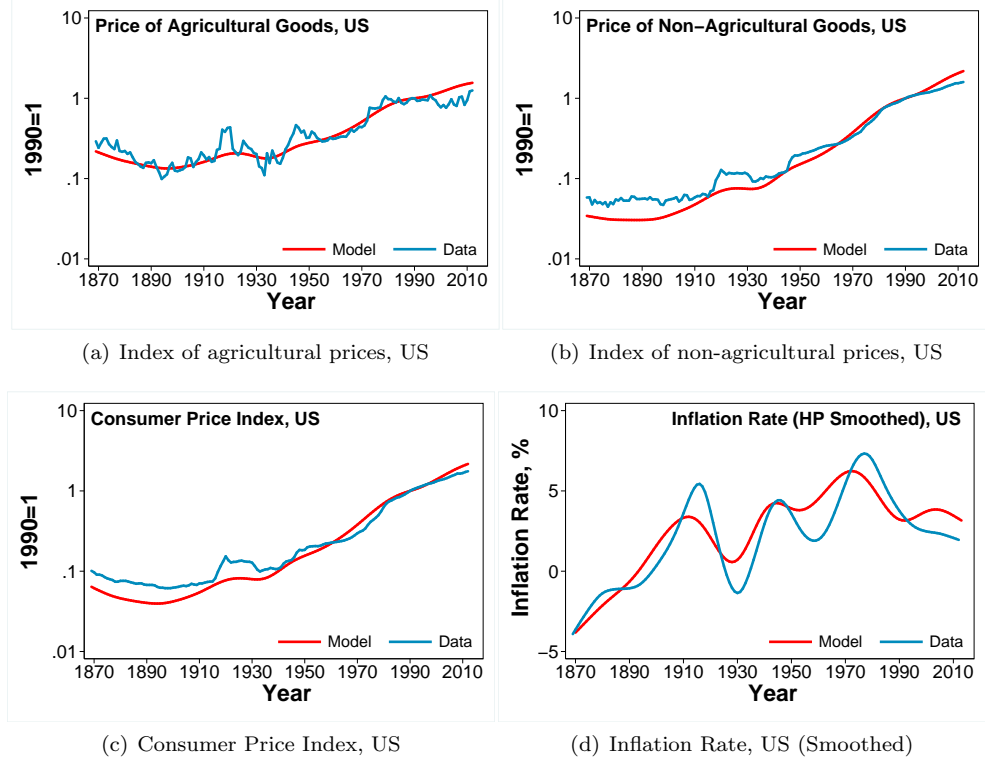


Figure 6: Simulations and data for US prices, 1869-2012.

from the fact that the money stock is taken directly from the data. In appendix B.1, we show that calibrating our baseline model to observed nominal interest rates rather than to the money stock directly, leaves our results quantitatively and qualitatively unchanged. Furthermore, to cement this intuition, in the following section we show that a one-sector model that also takes money stock directly from the data fails entirely in reproducing the observed dynamic of money shares. This is indicative that it is the model's two-sector structure that enables it to capture closely the evolution of money share over time.

We also examine the predictions of the model for *nominal* producer prices: the quantity of dollars needed to purchase one unit of sector s goods. From equations (12) and (14) we can derive an expression that pins down nominal prices for sectors $s \in \{a, c\}$:

$$p_{s,t} = \frac{M_t V_t}{B_{s,t}} = \frac{M_t}{B_{s,t} \phi_t L_{c,t}}.$$

This equation shows that the nominal price of sector s goods depends on the quantity of money in the economy, sector s productivity and the endogenously determined velocity of money. For a given velocity, nominal prices are driven by changes in the relative quantity of money and sectoral productivity: when money becomes relatively abundant the price of sector s goods increases and vice-versa. In our model however, monetary velocity is not constant but falls endogenously as the

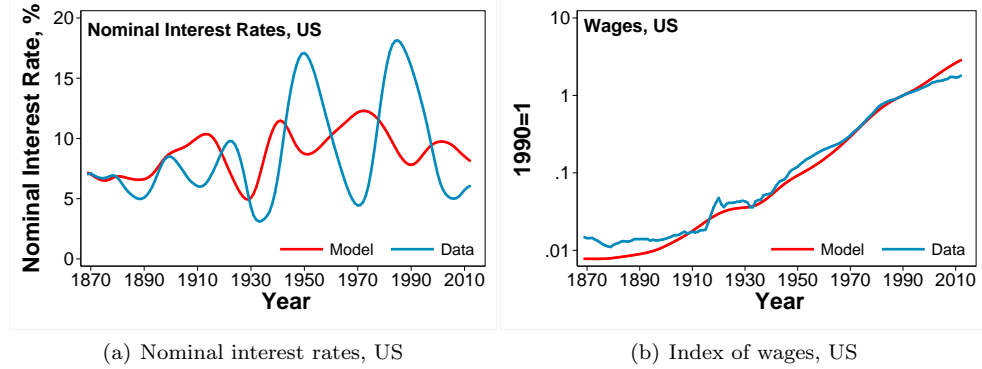


Figure 7: Simulations and data for US factor prices, 1869-2012.

economy shifts from non-monetary agriculture to monetary non-agriculture. This changing demand for money will play a key role in driving nominal prices - especially in the earlier parts of the sample when changes in velocity are largest. Comparing nominal prices in the model and the data provides us with a test of the model's mechanism. Figures 6(a) and 6(b) show sectoral price indices (in dollar terms) in the data and the model. Even though nominal prices are not targeted directly, the model does exceptionally well in capturing the non-linear trends of sectoral prices - both the flat (or falling) nominal sectoral prices in the initial part of the series and the subsequently growing prices in the second part of the series. The tight fit between model and data comes partially from the standard quantity theory of money: nominal prices are influenced by the quantity of money relative to sectoral productivity. However - as we shall see in the subsequent section - an important reason for the success (especially in the first part of the series) comes from the model's ability to capture monetary velocity. Figure 6(c) and 6(d) then shows the evolution of the aggregate consumer prices index and the corresponding inflation rates. Again our model predictions fit very well with the data. Interestingly, we also capture the deflationary period in the late 19th century. This turns out to be a key feature of our model discussed in more detail below.

Finally, the model also has implications for nominal factor prices, via equation (13). Figures (7(a)) and (7(b)) compare factor prices generated by the model with data. Recall that β was chosen to match the average nominal interest rates between 1869 and 2007 - and so it is unsurprising that the model does well in matching nominal interest rates. The model however also does well in matching the nominal wage rate, predicting an average annual increase of 4.2% between 1869 and 2013 (3.4% in the data). From equation (13), we see that nominal wages are given by $w_t = M_t V_t$. Thus, the good fit of nominal wages in the model and the data does not stem from the calibration, but rather emerges from the mechanism of the model.

US Counterfactual To highlight the mechanism driving the results, let us consider how an (effectively) one-sector version of the model would perform in replicating monetary shares. We can do this by assuming that productivity in the agricultural sector in 1869 is set equal to the agricultural

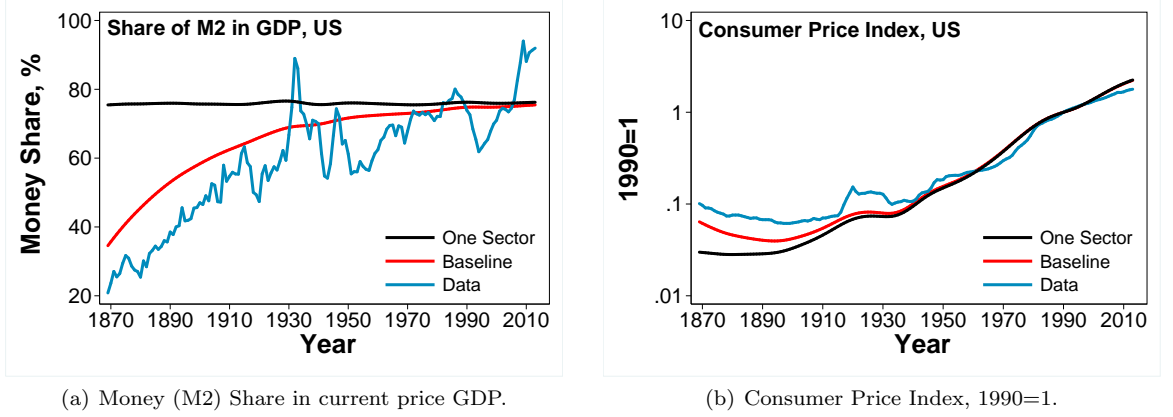
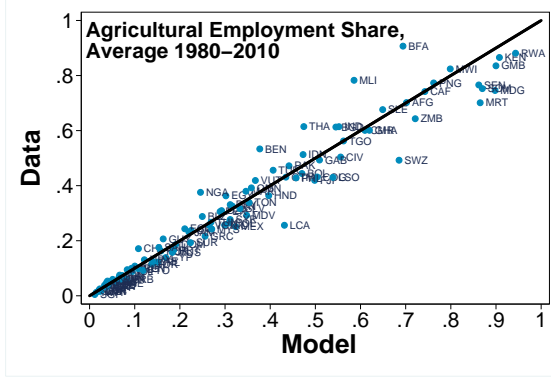


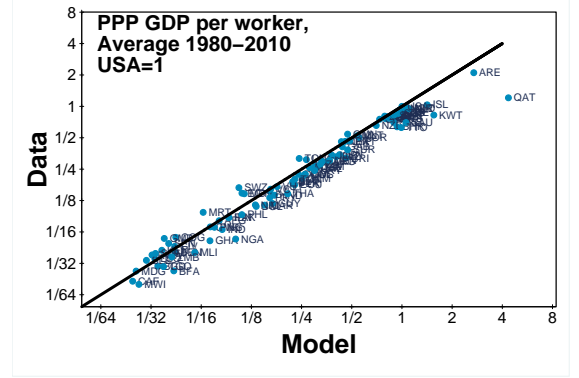
Figure 8: Results for US when we consider an (effectively) one-sector version of the model, 1869-2012.

productivity of the US in 2010. We can then re-calibrate the non-agricultural productivity to match the same (period-by-period) GDP-per-worker as in the baseline model - whilst keeping all other parameters identical to the calibration in Table 1. Given the new, high agricultural productivity, equation (10) suggests that the employment share in agriculture will be close to α - and hence very small. In other words, the agricultural sector will be of a negligible size. Thus, this counterfactual effectively replicates a one-sector version of the baseline model. Figure 8(a) shows that monetary shares in the one-sector model are almost constant over the entire period. Variations in monetary growth rates generate some limited fluctuations, but these are not large or systematic enough to generate the observed changes in money shares (see Table 5(b)). Why does the one-sector model fail? By definition, money share is $\frac{M_t}{P_t \times Y_t}$ - where M_t is the money stock, P_t is the aggregate price level and Y_t is GDP. In both models we take M_t directly from the data, whilst Y_t is chosen to be identical in both models. Thus the baseline and the one-sector models differ only in their ability to reproduce the evolution of aggregate prices P_t over time. Figure 8(b) shows the evolution of the price index in the baseline, the data and the one-sector counterfactual. The one-sector model fails to capture the evolution of aggregate prices in the early periods. In particular, from the equations in (12), an increase in the employment share of the non-agricultural sector in the baseline model decreases sectoral prices $p_{a,t}$ and $p_{c,t}$. This downward pressure on sectoral prices is stronger when $L_{c,t}$ is small, i.e. at early stages of the structural transformation process. This mechanism therefore helps the baseline model to closely match the observed deflation in the late 19th century, a task that the one-sector model cannot accomplish.

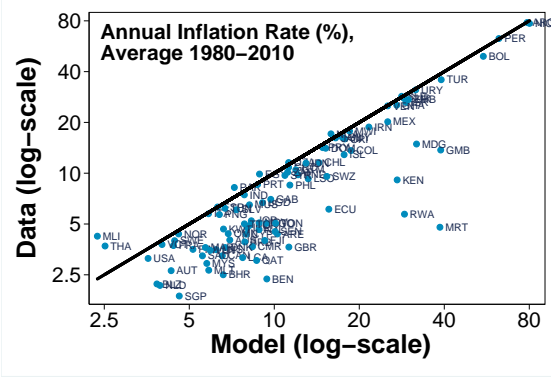
Historically, there has been an energetic debate among economists about the sources of this so-called ‘Great-Deflation’. One perspective, held by Tooke and Newmarch (1857), Wells (1891), Landes (1969), Rostow (1948, 1978) and Lewis (1978), highlights the importance of ‘real’ factors behind falling prices - such as the process of industrialization or globalization. The competing view - taken by Friedman and Schwartz (1963), Reti (1998), and Bordo and Schwartz (1980, 1981) - argues



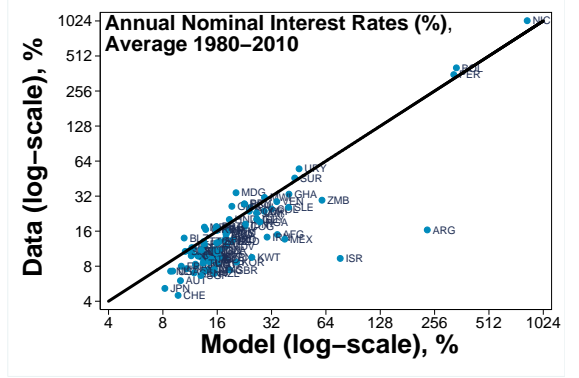
(a) Agricultural Employment Share, Average 1980-2010



(b) GDP per worker, Average 1980-2010



(c) Inflation Rate, Average 1980-2010



(d) Nominal Interest Rate, Average 1980-2010

Figure 9: Simulations and data for cross-section of the average for 1980-2010. Line depicts 45 degrees.

that it was inadequate growth of money supply relative to real output that drove observed prices at the end of the 19th century. Our model gives support to the proponents of the first explanation - by emphasizing the important role that structural transformation played in influencing monetary velocity and prices. Thus the lesson to be drawn from our results is that the multi-sector framework is essential in capturing the historical long-run evolution of monetary shares and prices in the US.

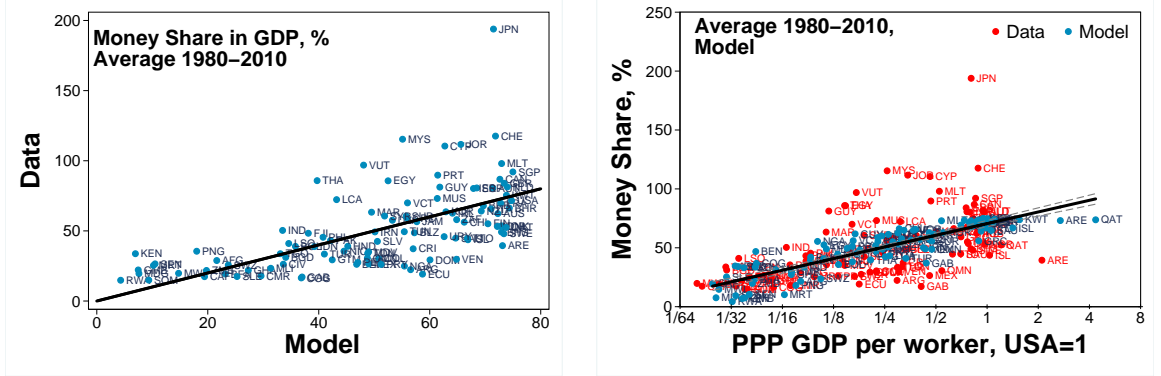
6 Cross-Country Analysis

In this section we test the model's ability to explain cross-country monetary shares, and we quantify the importance of structural transformation as a driver of the observed results. This is done in order to provide further external validity to the model and to examine its performance using modern data.

Cross-Country Calibration We focus on an international panel of 102 countries for 1980-2010.²⁸

We assume that each country is a closed economy with the same structural parameters of the US,

²⁸ We consider all countries for which all necessary data is available. Unless otherwise stated all data sources and



(a) Money (M2) relative to current price GDP, cross-country data, average 1980-2010. Line depicts 45 degrees.

(b) Money Share versus GDP (log-scale) in Model, Average 1980-2010

Figure 10: Money Share in model and data as well as the money share versus GDP (log-scale) in Model and Data, Average 1980-2010.

with the exception of sectoral productivity and monetary policy. Money stock per worker in each country and each year is taken directly from the data. We assume that labor productivity in sector s and in country i grows at a constant rate, $g_s^i - 1$, and is given by:

$$B_{s,t}^i = B_s^i \times (g_s^i)^{t-1980}. \quad (16)$$

We then choose g_s^i to match observed average sectoral labor productivity growth rates in each country between 1980-2010, and pin down sectoral productivity levels, B_s^i , by following the approach of Duarte and Restuccia (2010). B_a^i is chosen to match the agricultural employment share in country i in 1980, whilst B_c^i is chosen to match the ratio of GDP per worker in country i to that of the USA in 1980. Productivity levels are inferred using the model rather than directly from the data because, as explained by Duarte and Restuccia (2010), internationally comparable sectoral price levels are not as readily available and perhaps not as accurate as aggregate price levels - especially in the non-agricultural sector, which tends to be dominated by services.

Cross-Country Results We will focus on average values over the 1980-2010 period for each country, in order to emphasize the long-run cross-sectional fit of the model. Figure 9 compares the output, employment shares, inflation rates and nominal interest rates in the model to those in the data. Despite some variance, countries lie close to the 45-degree line and the model does well on all fronts. Next, we examine the model's performance in explaining money shares in the model and the data (presented in Figure 10(a)). Here too the model does well - although there is now more variation than in the US and the model is unable to generate money shares greater than 100% that occur in several European countries and Japan. Columns 1 and 2 of Table 2 illustrate the results by computing a few statistics. The model does well in predicting an average money share of 50.35% construction methodology is presented in Appendix A.

Money to GDP Ratio:	Data	m_b	m_{cf1}	m_{cf2}
Observations	102	102	102	102
Mean	51.19	50.35	74.54	72.59
p10	19.77	19.77	73.96	69.67
p90	85.81	72.93	75.01	74.53
Std. Dev.	29.14	19.66	0.37	2.59
M-Y Semi-Elast.	0.139	0.143	0.001	0.005

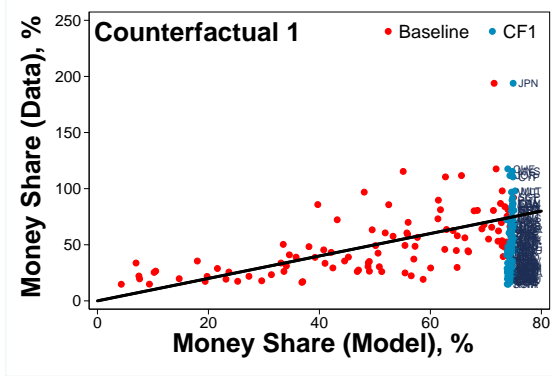
Table 2: Summary statistics for cross-section. Columns show statistics for: money shares from the data (Data), the baseline model (m_b), and counterfactuals 1 (m_{cf1}) and 2 (m_{cf2}). Rows (1)-(5) respectively show: the number of observations, the mean, the top-decile, the bottom-decile and the standard deviation of money shares in the data as well as those predicted by the baseline model and both counterfactuals. The final row shows the semi-elasticity of money-share to income.

in the model (51.19% in the data) and a money share of 19.77% in the bottom decile (19.77% in the data). The model slightly under-predict money-share in the top decile at 72.93% (versus 85.81% in the data), however we nonetheless capture 80% ($\approx \frac{72.93-19.77}{85.81-19.77}$) of the increase in money-shares observed in the data. There is more variation in money shares in the data, but the model still captures 67% ($\approx \frac{19.66}{29.14}$) of the standard deviation found in the data. Next, Figure 10(b) shows that the model performs exceptionally well in predicting the (semi-)elasticities of money-shares with respect to GDP per worker, which are approximately 0.14 in both the data and the model.²⁹ The simple model thus successfully captures the cross-sectional variation in monetary shares and the long-run, cross-sectional trend between monetary shares and income.

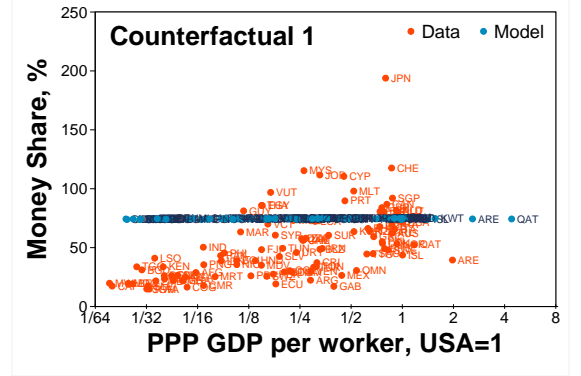
Cross-Country variation in velocity We now turn to understanding which specific mechanism or mechanisms drive the success of the model. Recall that countries are assumed to differ in their monetary policies M_t^i (and hence τ_t^i) and in their sectoral productivities, $B_{s,t}^i = B_s^i \times (g_s^i)^{t-1980}$. The following two counterfactuals quantify the extent to which variations in each of these factors contributes to explaining cross-country differences in velocities as well as in the velocity-income relationship.

Counterfactual 1 In the first counterfactual, we assume that each country follows US monetary policy, i.e. $\tau_t^i = \tau_t^{US}$. We also assume that all countries have the same agricultural productivity as the US in 1980, so that $B_a^i = B_a^{US}$. We re-calibrate non-agricultural productivity in 1980, B_c^i , to match each country's observed GDP per worker in 1980. Countries thus differ only in sectoral productivity growth rates and non-agricultural productivity levels. Since every country now has a high agricultural productivity (as the US), the agricultural sector will be vanishingly small in every country. As such, this experiment is akin to examining how well a one-sector model under US monetary policy in each country would match the data. Figure 11(a) compares money-shares in the counterfactual and the data, Figure 11(b) plots money-shares versus the (log) GDP per worker in the counterfactual and the data, whilst the column labelled m_{cf1} of Table 2 computes some summary

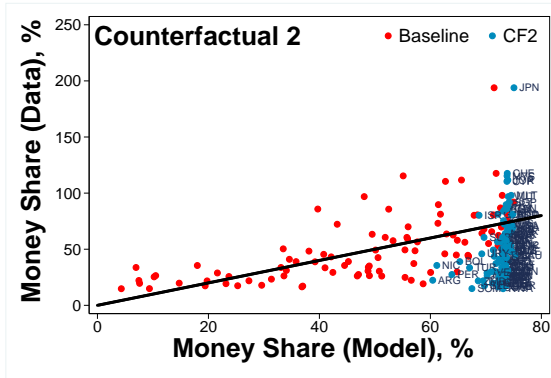
²⁹ In other words a doubling of GDP per worker in a country is associated with a 14 percentage point increase in the money share. The reason that this is different from the 0.16 elasticity found in Figure 1 is that we now have a smaller sample of countries.



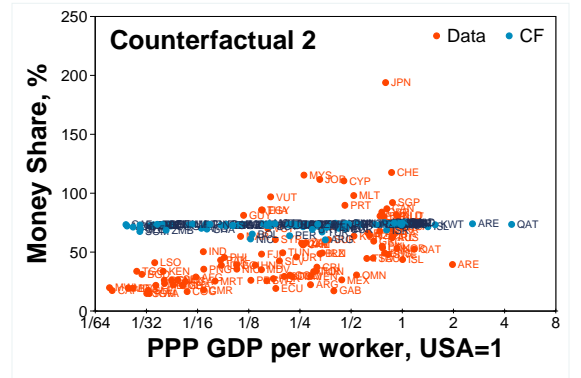
(a) Counterfactual 1 (US agr. productivity and monetary policy): Money shares in data and model.



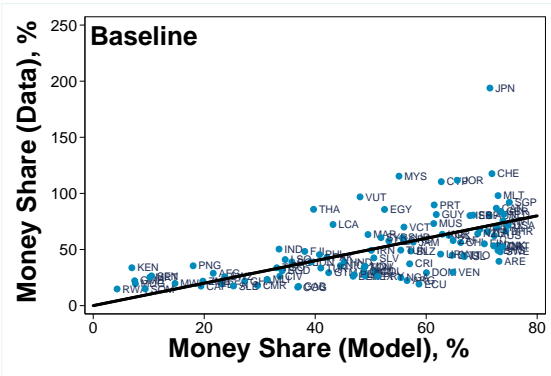
(b) Counterfactual 1 (US agr. productivity and monetary policy): Money shares versus GDP per capita.



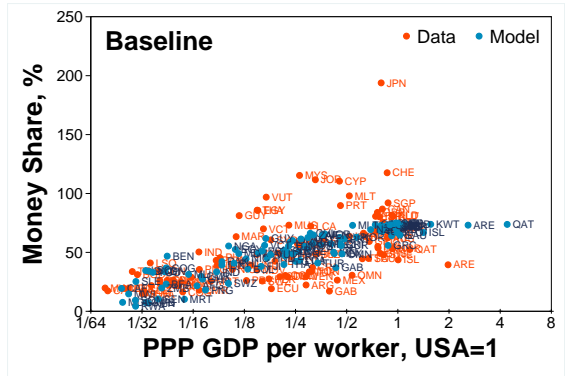
(c) Counterfactual 2 (US agr. productivity): Money shares in data and model.



(d) Counterfactual 2 (US agr. productivity): Money shares versus GDP per capita.



(e) Baseline Model: Money shares in data and model.



(f) Baseline Model: Money shares versus GDP per capita.

Figure 11: Money shares in the baseline model and the counterfactuals (average for 1980-2010). Drawn versus corresponding data and (log of) GDP per worker.

statistics. Predicted monetary shares are almost constant. Thus, an (effectively) one-sector version of the model with no variation in monetary policy cannot replicate the data. The reasons for this are clear from equation (14): $(V_t^i)^{-1} = \phi_t^i L_{c,t}^i$. First, non-agricultural productivity does not influence velocity. Therefore, even if in this counterfactual there is cross-country heterogeneity in B_c^i , the effect on velocity is zero. Second, since monetary policies are the same across countries, $\phi_t = \frac{1-\alpha-\gamma}{(1-\alpha-\gamma)+\gamma\tau_t^{US}}$ will be identical across countries. Finally, since each country's agricultural productivity is assumed to be high and the preference weight on agriculture, α , is small under our calibration, we have that $L_{a,t} \approx \frac{\alpha\tau_t^{US}}{1-(\alpha+\gamma)(1-\tau_t^{US})} \approx 0$. Consequently, $L_{c,t}^i \approx 1$, and hence approximately constant. Thus, differences in productivity growth rates and non-agricultural productivity levels generate very little cross-country variation in monetary shares.

Counterfactual 2 In the second counterfactual we keep agricultural and non-agricultural productivity exactly as in Counterfactual 1, but we allow each country to have its own monetary policy taken directly from the data. Thus, this experiment examines whether differences in monetary policies in an (effectively) one sector model are capable of explaining cross-country differences in money shares and the velocity-income relationship. The results are shown in Figures 11(c) and 11(d) and column m_{cf2} of Table 2. Whilst there is some additional variation coming from differences in monetary policies across countries - especially in some high inflation countries like Argentina or Peru - monetary shares do *not* change significantly with respect to the previous counterfactual. Thus, whilst differences in monetary growth rates do technically influence money shares across countries, the message from this experiment is that, *quantitatively*, velocity and the velocity-income relationship is overwhelmingly *not* driven by differences in monetary policies across countries.

Baseline Finally, maintaining all the assumptions from Counterfactual 2, we also allow countries to differ in their agricultural productivity levels - and hence in the size of their agricultural sectors. In other words, we revert to the baseline economy where countries differ in their monetary policies as well as in agricultural and non-agricultural productivity growth rates and levels. The results are striking and shown in Figures 11(e) and 11(f) and column m_b of Table 2. As was argued before, the model does well in capturing the cross-country variation in money-shares and very well in capturing the relationship between money-shares and income. We can therefore conclude that most of the variation in money shares in the model comes entirely from the multi-sector framework, and in particular from the cross-country differences in agricultural productivity levels, which generate agricultural sectors of different sizes.³⁰ Since the model captures practically the entire trend in cross-country velocity-income variation, this suggests - perhaps somewhat surprisingly - that structural transformation, and the decline of the agricultural sector in particular, is a key driver of the long run monetary velocity.³¹

³⁰ In particular, the (effectively) one-sector model with country-specific monetary growth can only explain 13% of the standard deviation of the baseline model. This means that 87% of the standard deviation of the baseline comes from the two-sector framework alone.

³¹ Of course, there are some striking outliers. Countries like Cyprus, Switzerland or Japan have monetary shares

7 Robustness

This section shows that our baseline results are robust to different setups and different calibration strategies. Details are largely left to Appendix B.

Matching interest rates The per-capita money-stock in our calibration is chosen to match money growth rates in data directly. One may worry that since we are trying to explain the money-to-GDP ratio, this calibration strategy does not put enough distance between outcomes and the calibration. Section 5 demonstrated that this is not the case, since a one-sector model - even one that takes money directly from the data as an input - does not generate the observed monetary shares. Nonetheless, in Appendix B.1 we re-calibrate the model to match observed nominal interest rates instead of money growth rates. We show that the money-to-GDP ratio under this calibration is nearly indistinguishable to that of the baseline since money shares are driven by changes in money demand that stem from structural transformation rather than changing money supply.³²

Observed sectoral productivity Our baseline calibration matches average sectoral labor productivity growth rates in the US between 1869 and 2007 rather than period-by-period sectoral labor productivity growth rates. We take this approach since historical data on sectoral value added is available only at irregular intervals (especially pre-1929). Another way to deal with this is to interpolate missing data and re-calibrate the model to reproduce period-by-period growth rates. We perform this exercise in appendix B.2. The fit of the model along the ‘money-share’ dimension is unchanged, whilst certain ‘real’ features (such as agricultural employment or output per worker) exhibit an even better fit than in the baseline.

Capital accumulation The baseline model captures non-agricultural, non-monetary demand by assuming a preference for non-cash goods. We endogenize this reduced-form, non-monetary demand by extending our framework to include capital accumulation in the non-agricultural sector. In this framework, capital is largely a non-monetary, credit-good. As capital accumulates over the growth process, non-agricultural non-monetary demand increases. Modelling capital accumulation may also be important in and of itself, since periods of industrialization are typically characterized by heavy investment in plants, machines and equipment. The details of this extension are reported in Appendix B.3. The results remain largely unchanged. Structural transformation remains by far the main driver of money shares in the model whilst variation in the capital stock accounts for a very small part of the evolution of money shares over time and across countries.

far above their predicted values. A limitation of our model is that it cannot generate money shares larger than 1. However, the focus here is on broad cross-country trends, rather than the specifics of individual countries - and in this respect the model does very well. A model with intermediate goods would presumably do even better. We leave this for future research.

³² Changes in money supply are instead almost entirely reflected into changes in prices. As a consequence, money shares exhibit almost no change at all between the baseline and this re-calibration.

Agricultural money demand To preserve simplicity, the baseline model assumes an entirely non-monetary agricultural sector.³³ We extend our model and allow agriculture to transition from a non-monetary sector in poorer countries to a monetary sector in richer countries. The idea is to introduce two agricultural sub-sectors: a ‘traditional’, non-monetary agricultural sector with low productivity growth and a ‘modern’, monetary agricultural sector with higher productivity growth. Since these sectors are close substitutes in consumption, the difference in productivity growth rates generates a transition from the non-monetary to the monetary agricultural sector as the economy grows richer. Details of the model and calibration are presented in Appendix B.4. Again, our main results are unchanged. Intuitively, in poor countries where agriculture is an important part of the economy, traditional, non-cash agriculture dominates the agricultural sector and hence the agricultural sector does not play a large role in driving money demand. In richer countries, the agricultural sector as a whole tends to be small, and hence it also does not contribute much to money demand. This exercise confirms that agricultural monetary demand is not crucial to our results.

Alternative tests of the model In the main body of the paper we test our theory by comparing data with money-to-GDP ratios that emerge from a fully calibrated model. A more direct test of the theory, that does not require a complete calibration, can be constructed using the equations in (14). In particular, we only need to calibrate the interest elasticity of money demand (as captured by the ϕ function) and then we can feed in non-agriculture labor and value added shares directly from the data to construct model predicted money-shares. These ‘predicted’ money-shares can then be compared directly to those in the data.³⁴ We perform this secondary test in Appendix B.5. Since our baseline calibration does well in replicating sectoral employment and value-added shares, we find that this alternative test corroborates the close fit between model and data found in the main body in the paper. Despite the simplicity of the above exercise, we believe that the baseline calibration is preferable as it helps us better understand the mechanism of the model, provides additional external validity by allowing us to compare additional implications of the model to data (such as sectoral prices) and gives us an environment in which counterfactuals and welfare analyses - such as those in the next section - can be performed.

8 Welfare Cost of Inflation

In this section we examine the welfare costs of inflation in rich and poor countries. We first define optimality in our framework, and show that the Friedman rule holds. We then calculate a compensating variation measure of welfare that determines how much higher a household’s income would have to be in order to compensate for permanently higher inflation.

³³ We thank an anonymous referee for making this point and suggesting a source for US non-monetary agricultural value-added data.

³⁴ We thank an anonymous referee for suggesting this test.

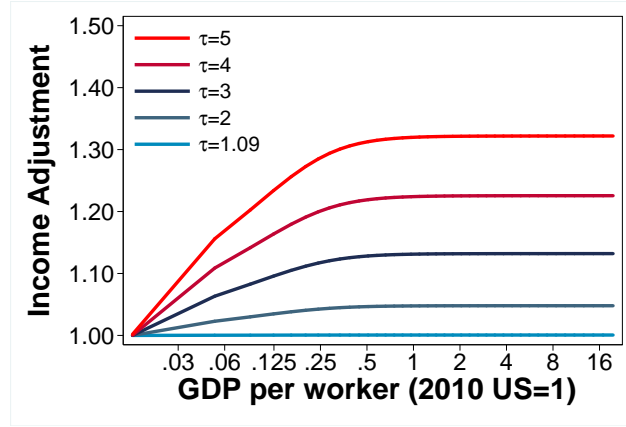


Figure 12: Income adjustment necessary to make a household indifferent between living in a world with sub-optimal monetary policy and a world with optimal monetary policy at different income levels.

Optimality The distortion in this environment arises - as in the standard CIA model - from the lag between households being paid their wage income and their ability to buy non-agricultural cash goods with that income (Cole and Kocherlakota, 1998). In particular, households can only use last period's money holdings to purchase current period non-agricultural cash goods. This forces households to hold a low-yield asset (money) instead of a higher yield asset (bonds) in order to have money holdings to purchase cash goods in the future. Thus, as long as nominal interest rates are positive (i.e., the CIA constraint binds), the economy will not reach the first best. If, however, nominal interest rates were set to zero, then households would be indifferent between being paid today or being paid in the future (and indeed between holding money and a bond), and the distortion associated with the trading arrangement would be eliminated.³⁵ Hence, since the CIA binds if and only if $r_t > 0$, and, as we showed above, $r_t = \tau_t - 1$, we need $\tau_t \rightarrow 1$ to eliminate the distortion. In other words, we need to implement the Friedman rule, i.e. we must have $\frac{M_{t+1}}{M_t} \rightarrow \beta$.³⁶

Welfare cost of inflation The lifetime indirect utility of a household is given by:

$$V(\{w_t, p_{a,t}, p_{c,t}, \tau_t\}_{t=T}^{\infty}) = \sum_{t=T}^{\infty} \beta^t u(a(w_t, p_{a,t}, \tau_t), c_n(w_t, p_{a,t}, p_{c,t}, \tau_t), c_m(w_t, p_{a,t}, p_{c,t}, \tau_t)). \quad (17)$$

In equation (17), $a(\cdot)$, $c_n(\cdot)$ and $c_m(\cdot)$ are standard demand functions for agricultural goods as well as monetary and non-monetary non-agricultural goods.³⁷ Suppose that $V(\cdot; \lambda)$ denotes a household's

³⁵ More specifically, money does not expand the production possibility frontier. As such, the Pareto optimal allocations can be found by solving the corresponding social planner's problem without money. It is then easy to show that the decentralized problem and the social planner's problem are identical when nominal interest rates are zero.

³⁶ A word of caution is needed here. Whilst it is true that as $\tau_t \rightarrow 1$, the equilibrium allocations approach the Pareto optimal allocations, directly setting $\tau_t = 1$ in equations (10)-(13) violates Assumption 3.1. It is relatively easy to show that it is nonetheless true that the allocations and prices implied by the above equations when $\tau_t = 1$ are also a competitive equilibrium. However, as is argued by Cole and Kocherlakota (1998), an equilibrium such as this (i.e. one where $r_t = 0$) can be achieved with a large set of monetary policies - including but not restricted to the policy $M_{t+1}/M_t = \beta$. Thus, whilst the limit is indeed an equilibrium, Pareto optimal, and can be implemented with a policy $\tau_t = 1$, it is not necessarily unique as other monetary policies could also sustain zero nominal interest rates.

³⁷ In particular, from the household's first order conditions and its budget constraint we can show that: $a(w_t, p_{a,t}, \tau_t) =$

lifetime indirect welfare when income, w_t , is multiplied by a factor λ in each time period:

$$V(\{w_t, p_{a,t}, p_{c,t}, \tau_t\}_{t=T}^{\infty}; \lambda) \equiv \sum_{t=T}^{\infty} \beta^t u(a_t(\lambda w_t, p_{a,t}, \tau_t), c_{n,t}(\lambda w_t, p_{a,t}, p_{c,t}, \tau_t), c_{m,t}(\lambda w_t, p_{a,t}, p_{c,t}, \tau_t)). \quad (18)$$

We wish to know by what factor, λ , income must be multiplied to make a household indifferent between living in a world with some sub-optimal monetary policy, $\{\hat{\tau}_t\}_{t=0}^{\infty}$, and a world with optimal monetary policy. The answer to this question satisfies the following equation:

$$V(\{w_t, p_{a,t}, p_{c,t}, 1\}_{t=T}^{\infty}) = V(\{w_t, p_{a,t}, p_{c,t}, \hat{\tau}_t\}_{t=T}^{\infty}; \lambda), \quad (19)$$

where prices are equilibrium prices for the optimal economy (i.e. one with $\tau_t = 1$) from equations (12) and (13).³⁸

We calculate such λ for a hypothetical economy that is identical in all respects to the US economy at different stages of its development, with the exception of its inflation tax.³⁹ In particular, we set different values of this tax, ranging from $\tau_t = 1.08$, (the average rate in the US) up to $\tau = 5$. The results are shown in Figure 12. A higher inflation tax results in higher welfare costs - independent of a country's level of income. However the cost of a sub-optimal policy will be *lower* in poorer countries. The reason for this is that most of the output in poorer countries is concentrated in (non-monetary) agriculture. An inflation tax on the small monetary sector will thus have relatively little effect. In richer countries, where more of the output is produced in (monetary) non-agriculture, inflation taxes can have larger welfare effects. For example, in a country with GDP per worker equal to the US GDP in 2010, a monetary policy of $\tau_t = 5$ will require an increase in income of approximately 32% to make a household indifferent to a world with optimal monetary policy. However, the same monetary policy in a country that has approximately 3% of the US's GDP in 2010 will only require an increase in income of 5%. Thus, the same inflationary policies have very different costs in rich and poor countries. This suggests why we see higher inflation in poorer countries: these countries have a lower welfare cost of implementing inflationary policies.

An alternative approach for highlighting this result is to find the maximum inflation tax, $\bar{\tau}$, in each country that makes the country's welfare cost of inflationary policy, λ , identical to the welfare cost of the US's inflationary policy, λ^{US} . This inflation tax satisfies the following equation:

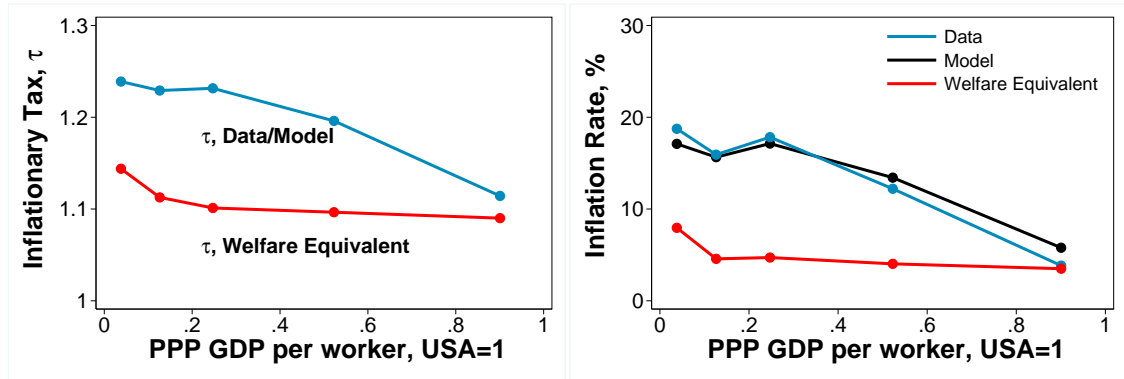
$$V(\{w_t, p_{a,t}, p_{c,t}, 1\}_{t=T}^{\infty}) = V(\{w_t, p_{a,t}, p_{c,t}, \bar{\tau}\}_{t=T}^{\infty}; \lambda^{US}). \quad (20)$$

The red line in Figure 13(a) shows this average welfare-equivalent $\bar{\tau}$ in each income quintile of the sample. In the lowest income quintile $\bar{\tau}^{q5} = 1.12$, whilst in the highest $\bar{\tau}^{q1} = 1.08$. The red line in Figure 13(b) then shows how each of these inflation taxes translate into observed average, annual

³⁸ $\bar{a} + \frac{\alpha(w_t - p_{a,t}\bar{a})\tau_t}{p_{a,t}(1-\alpha-\gamma+(\alpha+\gamma)\tau_t)}$, $c_n(w_t, p_{a,t}, \tau_t) = \frac{\gamma(w_t - p_{a,t}\bar{a})\tau_t}{p_{c,t}(1-\alpha-\gamma+(\alpha+\gamma)\tau_t)}$ and $c_m(w_t, p_{a,t}, \tau_t) = \frac{(1-\alpha-\gamma)(w_t - p_{a,t}\bar{a})}{p_{c,t}(1-\alpha-\gamma+(\alpha+\gamma)\tau_t)}$.

It does not matter whether we choose prices from the optimal or the sub-optimal economy, as the demand functions $a(\cdot)$, $c_n(\cdot)$ and $c_m(\cdot)$ depend only on relative prices and these are entirely independent of τ_t .

³⁹ Thus, for this exercise only, we assume each country follows the same growth path as the US. Poorer countries can thus be simply thought of as the US at some point in the past.



(a) Observed and welfare equivalent measures of τ in each income quintile.

(b) Observed and welfare equivalent measures of average, annual inflation rates in each income quintile.

Figure 13: Observed and welfare equivalent measures of inflation taxes and rates across income quintiles.

inflation rates. Welfare-equivalent inflation rates in the lowest and highest income quintiles are 7.9% and 3.9% a year, respectively. Thus the poorest countries can have inflation rates that are twice as high compared to rich countries, and yet have the same welfare costs as rich, low-inflation countries. The blue lines in Figure 13 show the corresponding inflationary taxes and the average, annual inflation rates in the data. Notice from Figure 13(b) that the observed inflation rate in the richest countries is 3.7 times higher than the inflation rate in the poorest countries. Thus, whilst inflation rates vary enormously between rich and poor countries, the welfare costs of these policies do not vary nearly as much.

9 Conclusions

We put forward a new theory of money demand in the long run, where velocity depends on a country's GDP level. The main drivers are structural transformation - i.e. the reallocation of labor from agriculture to non-agriculture associated with development - and an endogenous share of monetary transactions in different sectors. Despite its stark simplicity, our theory is very successful at matching both within- and across-country changes in monetary velocity and the velocity-income relationship. We can replicate - almost perfectly - the long run monetary velocity in the United States between 1869 and 2013 and in a large cross-sectional data set of 92 countries.

There are three lessons to be learned from our work. First, our findings suggest that the evolution of monetary velocity is driven almost exclusively by developments in the *agricultural* rather than the non-agricultural sector. Second, we show that the costs of bad monetary policy are disproportionately higher in richer than in poorer countries. Third, we demonstrate that, since velocity depends systematically on the composition of output, a country's price levels and inflation rates may not 'always and everywhere be a monetary phenomenon' (Friedman and Schwartz, 1963). These lessons

should be of utmost interest to central bankers, especially in developing countries, who may downplay the role of the agricultural sector in their policy decisions. Moreover, our findings may help explain why we observe persistently higher inflation rates in poorer countries than in richer countries - despite strong anti-inflationary policies recommended by international institutions. Finally, our results are also important to researchers, as they highlight that one-sector models cannot be successfully used to understand the long run dynamics of monetary velocity and nominal price levels. Our work on the evolution of long-run monetary velocity thus offers key insights into understanding the secular trends in sectoral and aggregate price levels, with strong implications for policy and future research.

A Data Appendix

A.1 US and Cross-Country Data

GDP (PPP) and Labor Force Data We use data from Penn World Tables version 7.1. (see Heston et al. (2012)) to construct annual time series of PPP-adjusted GDP in constant 1990 prices, PPP-adjusted GDP per worker in 1990 constant prices, and total employment between 1950-2010. For each country, we construct total employment L using the variables Population (POP), PPP Converted GDP Chain per worker at 2005 constant prices (RGDPWOK), and PPP Converted GDP Per Capita (Chain Series) at 2005 constant prices (RGDPCH) as $L = (RGDPCH \times POP)/RGDPWOK$. We then construct PPP-adjusted GDP in constant 2005 prices using the variables “PPP Converted GDP Per Capita (Laspeyres), derived from growth rates of c, g, i, at 2005 constant prices (RGDPL)” and “Population (POP)” as $GDPK = RGDPL \times POP$. We then re-base the 2005 data to 1990 prices.

We extend the labor force and GDP data calculated above back in time for the period 1869-1950 using growth rates from Maddison (2007). In particular, we calculate the pre-1950 growth rates for Maddison’s GDP measure (in 1990 Geary-Khamis dollars) and use it to extend the GDP measure calculated above. We also calculate the population growth rates from Maddison’s data and assume that the growth rate of the labor force pre-1950 is the same as the growth rate of the population (which is true if the labor-force-to-population ratio stays constant over time). We then use these population growth rates to extend the labor force data calculated above back in time. Finally, we extend the series for GDP and labor force forward in time between 2011 and 2014 using growth rates for labor force and constant price PPP GDP from the WDI (2016). The series for PPP-adjusted GDP per worker in constant prices is then computed as $y = GDP/L$.

Agricultural and Non-Agricultural Employment We construct contemporary (1980-2014) agricultural employment share using data from the FAOSTAT (2012) by taking the ratio of economically active population (labor force) in agriculture to total economically active population (labor force). For the US, we then combine this with the agricultural employment share from Alvarez-Cuadrado and Poschke (2011) for 1909-1980 and Lebergott (1966) for 1869-1908. Missing data is interpolated. We then obtain total employment in agriculture by multiplying the agricultural employment shares calculated above by the labor force data found in the previous paragraph. Non-agricultural employment is then the difference between total employment and agricultural employment. These are the $L_{a,t}$ and $L_{c,t}$ referred to in the main body of the text.

Sectoral Value Added, Constant price For the period 1970-2014 we construct constant price (1990) value added data for agriculture and total value added in Local Currency Units from UN (2016). For the US, we then extend this data backwards using sectoral growth rates. First, we obtain constant price agricultural and total value added data for the period 1947-1970 from the

10-sector database constructed by Timmer et al. (2014) and use this data to extend our value added measures back to 1947. Then, we extend these concatenated data back to 1869 using the Historical National Accounts database constructed by the Timmer et al. (2014). This database provides historical constant price indices of GNP for agriculture and the total economy. Missing values are interpolated. Notice that - like Duarte and Restuccia (2010) - we do *not* directly use the resulting series in our model. Instead, we use the above constant (1990) price agricultural and total value added data to calculate constant price shares of agricultural value added in total value added. Then, in order to remain consistent with our aggregate GDP data calculated above, we multiply these constant price shares by the aggregate PPP GDP data calculated above. This gives us a consistent estimate of 1990 value added of agriculture. Subtracting this estimate from the GDP PPP gives us an estimate of non-agricultural value added. These are the $Y_{a,t}$ and $Y_{c,t}$ referred to in the main body of the text.

Labor Productivity Growth Rates To calculate labor productivity growth rates we first calculate sectoral labor productivity for each sector $s = a, c$ as $y_{s,t} \equiv Y_{s,t}/L_{s,t}$ using values described in the above paragraphs. Next, we HP-filter these series and for each country and sector $s = a, c$ we calculate the average annualized sectoral labor productivity growth rates, $g_s - 1$, between 1980 and 2010. For the US we follow the same procedure but focus on the 1869-2007 period.

Price Indices For the cross country data and the period 1970-2014 we can construct agriculture and non-agriculture price indices by dividing sectoral value added in current prices by constant price (1990) value added data from UN (2016). Sectoral prices between 1929 and 2013 for the United States are obtained by dividing sectoral value added in current prices (obtained from the BEA and (Timmer et al., 2014)) by constant price sectoral value added found in the previous paragraphs. For the 1970-2014 period, these are identical to the indices that can be obtained using UN data. Obtaining pre-1929 prices is more complicated. In particular, as far as we know, no dependable series of data on sectoral prices exists. As such, we use nominal wheat prices obtained from Table Cc205-266 in Carter et al., eds (2006) to extend the agricultural price index back in time between 1869 and 1929. Then, using data on constant- and current-price aggregate GDP, constant price sectoral value added and the agricultural price index calculated above, we can infer a non-agricultural price index as well. In particular, multiplying the constant-price agricultural value added by the agricultural price index, gives us an estimate of current price agricultural value added, $p_{a,t}Y_{a,t}$. We can then subtract this from current price GDP, to obtain an estimate of current price non-agricultural value added, $p_{c,t}Y_{c,t}$. Then, taking the ratio of current price non-agricultural value added to constant price non-agricultural value added we obtain an estimate of the price index of the non-agricultural sector. We obtain price indices for the economy as a whole in the same manner -

Money Data on M2 for the period 1980-2014 comes from the IMF (2015) and is conveniently collected in the World Bank’s WDI database. For the US, we follow Anderson (2006) in the construction of the long run money stock series. The source of the data for the years 1959-2014 are lines 34 and 35 in the International Financial Statistics (IMF, 2015).⁴⁰ For the year 1948-1958 we use data constructed by Rasche (1990) and available online on the Historical Statistics of the United States website (Carter et al., eds, 2006). We extend this for 1869-1947 using data from Friedman and Schwartz (1963) also reported in the Historical Statistics of the United States.⁴¹

Nominal Interest Rates Nominal interest rates for the period 1980-2014 are constructed as the sum of the real interest rates from the IMF (2015) and the GDP deflator. Both data series are conveniently collected in the World Bank’s WDI. For the US, we take nominal interest rates for the period 1871 and 2014 as the nominal returns (including dividends) on the Standard and Poor Composite Stock Price Index from an updated version of Chapter 26 of Shiller (1989), available online at <http://www.econ.yale.edu/~shiller/data/chapt26.xlsx>.

Aggregate Capital We follow Kuralbayeva and Stefanski (2013) and Caselli (2005) in constructing capital and make use of the perpetual inventory method. Capital is accumulated according to:

$$K_{t+1} = (1 - \delta)K_t + I_t, \quad (21)$$

where I_t is investment and δ is the depreciation rate. We measure I_t from the PWT 7.1 as real aggregate investment in PPP terms.⁴² As is standard, we compute the initial capital stock K_0 as $I_0/(g + \delta)$, where I_0 is the value of the above investment series in the first period that it is available, and g is the average geometric growth rate for the investment series in the first twenty years the data is available. Finally, we set the depreciation rate, δ , to 0.05 to match the depreciation rates in the US.⁴³ The results are not very sensitive to choices in either δ , g or initial capital stock. The above process gives us sequences of capital stocks, K_t , derived from PWT data.

We extend our US capital data back in time using Piketty (2014) who provides data on the capital-output ratio in the US between 1770 and 2010 (measured in current-period prices). We consider only the reproducible part of his capital measure by subtracting the value of land from his measure of ‘national capital’. The resulting capital series corresponds much more closely to our

⁴⁰ Notice, that for 1980-2014 this coincides with the IFS/WDI data used above.

⁴¹ Notice that Friedman and Schwartz (1963) have slightly different definitions of monetary aggregates to those currently used. According to Anderson (2006), Friedman and Schwartz’s ‘M4 resembles, in many respects, the Federal Reserve’s current M2’ and ‘(h)ence, from an economic viewpoint, Friedman and Schwartz’s M4 and the currently published Federal Reserve M2 aggregates are more similar than first appearances might suggest’. As such, for the 1898-1947 period - when it is available - we use Friedman and Schwartz’s measure of M4. For the period 1869-1897 only the M3 measure is available. As such, we use the growth rate of M3, to extend the data back in time.

⁴² So that $I_t \equiv RGDPL \cdot POP \cdot KI$, where $RGDPL$ is real income per capita obtained with the Laspeyres method, POP is the population and KI is the investment share in real income. In the above we have re-based $RGDPL$ into 1990 dollars.

⁴³ The value of δ is chosen so that the average ratio of depreciation to GDP using the constructed capital stock series matches the average ratio of depreciation to GDP in the data over the corresponding period. The OECDs Annual National Accounts report depreciation in the data as “Consumption of Fixed Capital.”

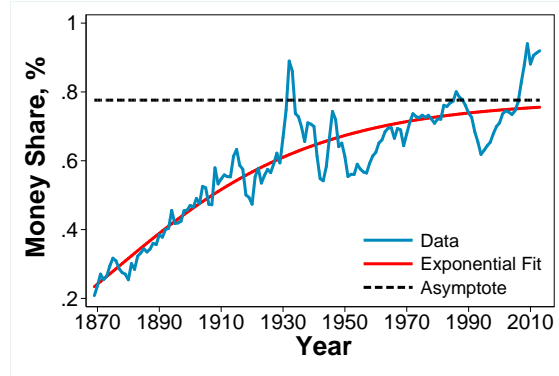


Figure 14: Money Share in US and exponential best-fit line.

modern measure of capital (Jones, 2015). We use the implied growth rates in the capital-output ratio from Piketty (2014) to extend our capital-output ratio data backwards in time and maintain comparability. We find that the implied (reproducible) capital-output ratio in the US in 1869 (in current-period prices) was approximately 1.97. This is of course a very crude estimate and should be approached cautiously - nonetheless this is the best we can do over this long time horizon.

Capital Shares Caselli and Feyrer (2007) estimate reproducible capital shares for 57 countries. They start by taking aggregate capital shares using data from Gollin (2002) and Bernanke and Gurkaynak (2001). They then make use of the World bank’s “Where is the Wealth of Nations?: Measuring Capital for the 21st Century” database (WB, 2005) to adjust these shares by excluding non-reproducible capital. We take these capital shares as our ν ’s. For countries not included in their data set, we assign the average capital shares value of the 57 countries, which is 0.19.

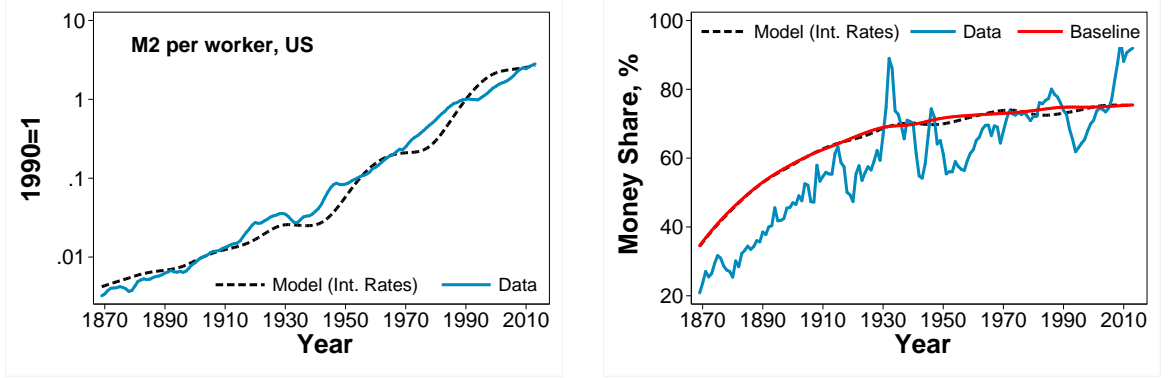
TFP growth rates In the version of the model with capital, we use total factor productivity growth rates of the non-agricultural sector directly from the data. To calculate these, we assign all capital in the economy to the non-agricultural sector. Then, taking $Y_{c,t}$, $L_{c,t}$ and K_t from the above, and using the country-specific capital share, ν , the TFP in non-agriculture in a country is simply calculated as a Solow residual: $Y_{c,t}/(K_t^\nu L_{c,t}^{1-\nu})$. Calculating the annualized average growth rate of this term between 1980 and 2010, gives us the TFP growth rate used in the model.

Long Run US Money Share To obtain an estimate of long-run US money share, we fit an exponential function with an asymptote to the US money-share data. In particular we choose a non-linear first order difference equation with an asymptote defined by: $m_{t+1} = (ae^{-b(t-1869)} + 1)m_t$ and $m_{1869} = c$ and find parameters a, b and c that minimize the mean squared error between the fitted money share, m_t , and the money share in the US between 1869 and 2013. We find that $a = 0.031$, $b = 0.026$ and $c = 0.234$. The predicted function and the data are shown in Figure 14. The implied asymptote of this difference equation is approximately 0.776.

Cross-country, non-monetary value added Blades (1975a,b) collect their data through a survey questionnaire sent to national statistical offices in 1973. They describe the process as follows: “The information on country practices in this report was mainly obtained from a mail enquiry made by the OECD Development Centre during the first part of 1973. Questionnaires requesting information on the coverage, valuation, and relative importance of non-monetary activities were sent to national statistical offices in just under 100 of the more important countries on the DAC list and 70 of them returned, completed questionnaires. (...) During the course of this study data were obtained from 48 countries on the percentage contribution of non-monetary activities to total GDP for a recent year. This information is, of course, readily available for countries which make separate estimates for non-monetary activities, but the majority of countries do not attempt to distinguish the non-monetary component of GDP. These countries were asked to make a ‘best guess’, or to indicate the probable range for the share of non-monetary output.” The estimated shares for the 48 countries are the ones used in Figure 3.

Importantly, in their analysis Blades and his team focus on a particular type of non-monetary activity: goods produced for own-use or subsistence only. They characterize goods as follows: “The most important of these activities is the production for own consumption of crops and livestock, but we also consider a number of other activities which are related to primary production and which are undertaken mostly in rural areas. These include such things as fishing, hunting and forestry activities, building and construction, manufacture of simple household articles, ownership of dwellings, water collection and crop storage.”

Thus, the Blades data does *not* consider payments in kind received by employees, or barter trade. As such - if anything - the data presented in the paper represent an under-estimate of the importance of non-monetary goods used across countries. Since we expect barter and payments in kind to be more prevalent in poorer countries with larger agricultural sectors, including these transactions would likely make Figure 3 even steeper, supporting further our results.



(a) Money (M2) per worker in the data and implied by matching interest rates.

(b) Implied money share.

Figure 15: Results for US when matching nominal interest rate rather than money, 1869-2012.

B Theoretical Appendix

B.1 Matching interest rates

The per-capita money-stock in our baseline calibration is chosen to match money growth rates in data. Here we recalibrate the baseline model by matching observed nominal interest rates instead of money growth rates. Normalizing the initial level of money per worker to one, $M_{1869} = 1$, taking nominal interest rates from the data and using equation (13), the model implies a series for money per worker over time pictured in Figure 15(a).⁴⁴ This derived money series is very similar to the observed monetary series. Figure 15(b) then compares the evolution of the money share under this calibration, under the baseline calibration and in the data. Results are nearly indistinguishable between the two calibrations. The key reason is that the evolution of money shares is driven by changes in money demand stemming from structural transformation rather than money supply.

B.2 Observed Productivity Results

In the baseline model we chose to match *average* sectoral productivity growth. We did this since much of the early (pre-1929) data was interpolated. Here we show the results for a version of the baseline model calibrated to reproduce the period-by-period evolution of (interpolated) sectoral productivity. We show that qualitatively and quantitatively all previous results go through.

For the calibration, all parameters (besides sectoral productivity) remain identical to those of the baseline. To calibrate sectoral productivity, we normalize productivity levels across sectors to one in 1869; that is, $B_{s,1869} = 1$ for $s \in \{a, c\}$. Then we use data on the period-by-period growth rate of sectoral value added per worker in the United States to obtain the time paths of sectoral

⁴⁴ Since the calibration of the remaining parameters is independent from the monetary growth rate, the other parameters remain identical to the baseline.

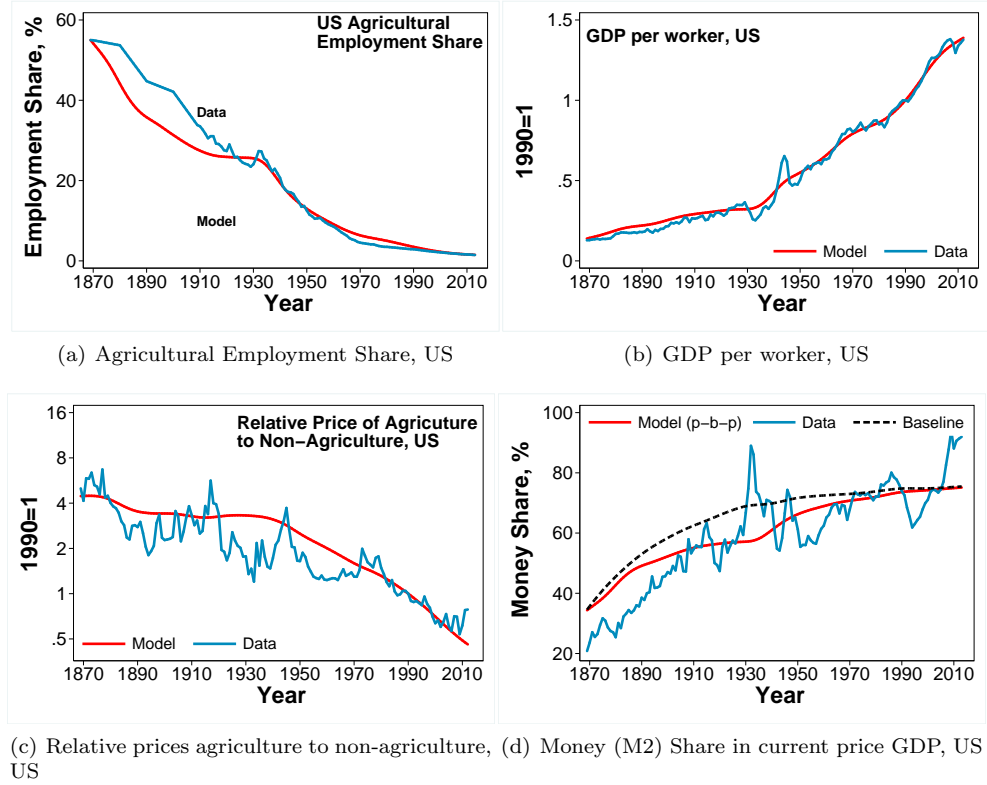


Figure 16: Simulations (period-by-period model) and data for US, 1869-2012.

labor productivity. In particular, denoting by $g_{s,t}$ the growth rate of labor productivity between period t and period $t + 1$ in sector s , we obtain the time path of labor productivity in each sector as $B_{s,t+1} = (1 + g_{s,t})B_{s,t}$. Figure 16 shows the key results of this version of the model. First, in Figure 16(a) we reproduce the employment share in agriculture. With this calibration strategy we do well - especially in the second part of the sample - where data has not been interpolated. This is as expected, since labor productivity is the major determinant of employment in agriculture in our model. Matching productivity more closely allows us to reproduce employment shares more closely. Analogously, this calibration strategy matches the GDP per worker slightly better than the baseline model (Figure 16(b)). Figure 16(c) shows the relative price of agriculture to non-agriculture goods. Once more, since relative prices are driven by relative productivity (as in equation (15)), we now do a better job at capturing the evolution of relative prices. Nominal prices and factor payments evolve in a very similar fashion to the baseline, and are omitted to save space. Finally, we compare the money share from this calibration with the baseline version and the data in Figure 16(d). Notice that the model with data calibrated period-by-period does better in the second half of our sample, when data is not interpolated. Importantly however, as expected, the money shares predicted by both models in 1869 and 2007 are the same. Thus, both qualitatively and quantitatively this extension makes very little difference to our results.

B.3 Introducing Capital

In this section we show that our results remain unchanged when we allow for capital accumulation in the non-agricultural sector. Adding capital allows us to capture a potentially important part of the development process and to endogenize the non-monetary demand of the non-agricultural sector which in the baseline is captured entirely by preference for credit goods. We find that adding capital does not significantly change our results.

Household's problem The representative households's problem is now given by:

$$\begin{aligned} \max_{\substack{a_t, c_{m,t}, c_{n,t}, \\ b_{t+1}, m_{t+1}, K_{t+1}}} \quad & \sum_{t=0}^{\infty} \beta^t (\alpha \log(a_t - \bar{a}) + (1 - \alpha - \gamma) \log(c_{m,t}) + \gamma \log(c_{n,t})) \\ \text{s.t.} \quad & p_{a,t}a_t + p_{c,t}(c_{m,t} + c_{n,t} + x_t) + b_{t+1} + m_{t+1} \leq w_t + r_{k,t}K_t + (1 + r_t)b_t + m_t + T_t \\ & K_{t+1} = x_t + (1 - \delta)K_t \\ & p_{c,t}c_{m,t} \leq m_t \\ & K_t, m_t \geq 0 \text{ and } b_t \geq -\bar{B} \text{ and } K_0 \text{ given} \end{aligned} \tag{22}$$

The problem is very similar to the baseline model. The household owns a unit of labor and a stock of capital K_t which it rents out on the market for a wage w_t and a rental rate $r_{k,t}$ respectively. In addition, the household enters period t with money holdings m_t , bond holdings b_t , and a helicopter transfer of money from the government, T_t . With these resources, it can buy agricultural goods a_t (at the price $p_{a,t}$), cash- and non-cash non-agricultural goods (respectively, $c_{m,t}$ and $c_{n,t}$, at the price $p_{c,t}$), investment goods x_t (at the price $p_{c,t}$), bonds b_{t+1} for a gross return of $1 + r_{t+1}$ dollars next period, and it can decide to hold a (non-negative) stock of money m_{t+1} for the next period. Notice that we follow Cole and Kocherlakota (1998) by treating investment as a non-monetary, credit good which helps endogenize the cash-credit split of the non-agricultural sector.

Firms There are representative agricultural and non-agricultural firms. As in the baseline model, the agricultural firm hires labor, $L_{a,t}$, and produces output, $Y_{a,t}$, using a simple linear technology that combines labor with exogenous, sector-specific total factor productivity, $B_{a,t}$. Its profit maximization problem is given by equation (2). Non-agricultural firms however, are now assumed to hire labor, $L_{c,t}$, and to rent capital, K_t^f , from households. With these inputs they produce output, $Y_{c,t}$, using a Cobb-Douglas technology that combines labor and capital with exogenous, sector-specific total factor productivity, $B_{c,t}$. Their profit maximization problem is therefore given by:

$$\max_{K_t^f, L_{c,t}} p_{c,t}Y_{c,t} - w_tL_{c,t} - r_{k,t}K_t^f \quad \text{s.t. } Y_{c,t} = B_{c,t}(K_t^f)^\nu (L_{c,t})^{1-\nu} \tag{24}$$

Output of non-agricultural firms, $Y_{c,t}$, can be sold as both a monetary and non-monetary consumption good as well as a non-monetary investment good. As in the baseline model, agricultural output is assumed to be a non-monetary, consumption good.

Money Supply The government is assumed to have a helicopter monetary policy as before:

$$M_{t+1} = T_{t+1} + M_t. \quad (25)$$

Market Clearing Finally, market clearing is standard.

$$\begin{aligned} a_t &= Y_{a,t}, \quad c_{m,t} + c_{n,t} + x_t = Y_{c,t} \\ m_t &= M_t, \quad b_t = 0 \\ L_{a,t} + L_{c,t} &= 1, \quad K_t = K_t^f. \end{aligned} \quad (26)$$

Solution We follow the same solution strategy as in the baseline model. In particular, we assume that nominal interest rates are always positive and we use the first order conditions of the household and firms problems, in combination with the market clearing conditions and government's money supply to obtain three equations that (together with transversality conditions for bonds, capital and money) pin down the equilibrium solutions of the problem.

The first equation defines a split of non-agricultural consumption between monetary and non-monetary goods. Defining $C_t \equiv c_{m,t} + c_{n,t} = Y_{c,t} - (K_{t+1} - (1 - \delta)K_t)$ we can write:

$$c_{n,t} = (1 - \phi_t)C_t \quad \text{and} \quad c_{m,t} = \phi_t C_t, \quad (27)$$

where $\phi_t = \frac{1-\alpha-\gamma}{(1-\alpha-\gamma)+\gamma\tau_t}$. Second, given initial capital endowment, K_0 , the path of capital is pinned down by a transversality condition and the following Euler equation:

$$\frac{\tau_{t+1}}{\tau_t} \frac{\phi_{t+1}}{\phi_t} \frac{C_{t+1}}{C_t} = \beta \left(\nu B_{t+1}^c (K_{t+1})^{\nu-1} (L_{t+1}^c)^{1-\nu} + 1 - \delta \right). \quad (28)$$

Finally, employment in the non-agricultural sector, $L_{c,t}$, is determined by the following:

$$\frac{\alpha\tau_t}{(1 - L_{c,t}) - \frac{\bar{a}}{B_{a,t}}} = \frac{(1 - \nu)(1 - \alpha - \gamma)B_{c,t}K_t^\nu(L_{c,t})^{-\nu}}{\phi_t C_t}. \quad (29)$$

We solve the model following the strategy of Echevarria (1997). We assume that after some point in time, T , the variables M_t , $B_{a,t}$ and $B_{c,t}$ grow at constant rates ($g_m - 1$, $g_a - 1 > 0$ and $g_c - 1$ respectively) and hence $\tau_t \rightarrow \bar{\tau}$ and $\phi_t \rightarrow \bar{\phi}$ converge to constants. Given these assumptions, the role of the non-homotheticity disappears in the limit as $\lim_{t \rightarrow \infty} \frac{\bar{a}}{B_{a,t}} = 0$ in equation (29). The model thus converges asymptotically to a balanced growth path where capital, investment and non-agricultural consumption grow at the rate $g_c^{\frac{1}{1-\nu}}$, agricultural consumption grows at a rate g_a and employment in agriculture and non-agriculture are constant. Consequently, we can re-write the model in terms of variables that are stationary in the long run. In particular, defining $\tilde{k}_t \equiv K_t/B_{c,t}^{\frac{1}{1-\nu}}$ and $\tilde{c}_t \equiv C_t/B_{c,t}^{\frac{1}{1-\nu}}$ equations (28) and (29) become:

$$\left(\frac{B_{c,t+1}}{B_{c,t}} \right)^{\frac{1}{1-\nu}} \frac{c_{t+1}}{c_t} \frac{\tau_{t+1}}{\tau_t} \frac{\phi_{t+1}}{\phi_t} = \beta \left(\nu \tilde{k}_{t+1}^{\nu-1} (L_{t+1}^c)^{1-\nu} + 1 - \delta \right) \quad \text{and} \quad (30)$$

Parameter	Values	Target	Target Value
$B_{s,1869}, M_{1869}$	1	Normalization, $s \in \{a, c\}$.	-
$g_a - 1$	0.028	Labor productivity growth in agriculture, 1869-2007.	2.79%
$g_c - 1$	0.010	Labor productivity growth in non-agriculture, 1869-2007.	1.23%
$\{B_{s,t}\}_{t=1869}^{2014}$	$B_{s,t} = g_s^{t-1869}$	Constant productivity growth in sector $s \in \{a, c\}$.	-
$\{M_t\}_{t=1869}^{2014}$	$\{\cdot\}$	Growth in money stock per worker.	-
K_0	1.298	Reproducible-capital to output ratio in 1869, Piketty (2014).	1.97
α	0.004	Long-run employment share in agriculture.	0.500%
\bar{a}	0.547	Employment share in agriculture in 1869.	55.5%
β	0.963	Average nominal interest rate & money growth, 1869-2007.	$\bar{r} = 8.85\%$; $\bar{g}_m = 4.79\%$
δ	0.050	Consumption of fixed capital relative to GDP, 1964-2007.	0.050
ν	0.229	Reproducible-capital income share, Caselli and Feyrer (2007).	22.9%
$1 - \gamma$	0.907	Long-run share of money in nominal value-added.	77.6%

Table 3: Model with capital, calibrated parameters. See Appendix for detailed sources.

$$\frac{\alpha \tau_t \phi_t}{(1 - L_{c,t}) - \frac{\bar{a}}{B_{a,t}}} = \frac{(1 - \nu)(1 - \alpha - \gamma) \tilde{k}_t^\nu (L_{c,t})^{-\nu}}{(1 - \delta) \tilde{k}_t - \left(\frac{B_{c,t+1}}{B_{c,t}} \right)^{\frac{1}{1-\nu}} \tilde{k}_{t+1} + \tilde{k}_t^\nu (L_{c,t})^{1-\nu}} \quad (31)$$

Since $\lim_{t \rightarrow \infty} \frac{\bar{a}}{B_{a,t}} = 0$, using the above it is easy to show that $\tilde{k}_t \rightarrow \tilde{k}^*$ and $L_t^c \rightarrow L_c^*$, where:

$$L_c^* = \frac{(1 - \nu) \left(g_c^{\frac{1}{1-\nu}} - \beta(1 - \delta) \right)}{\left(g_c^{\frac{1}{1-\nu}} - \beta(1 - \delta) \right) + \frac{\alpha}{1-\alpha-\gamma} \bar{r} \bar{\phi} \left(g_c^{\frac{1}{1-\nu}} (1 - \beta\nu) - \beta(1 - \delta)(1 - \nu) \right)}, \text{ and} \quad (32)$$

$$\tilde{k}^* = \beta^{-\frac{1}{1-\nu}} (g_c^{\frac{1}{1-\nu}} - \beta(1 - \delta))^{\frac{1}{1-\nu}} \nu^{\frac{1}{1-\nu}} L_c^*. \quad (33)$$

We then use standard numerical methods to solve for the sequences \tilde{k}_t and $L_{c,t}$ that converge to L_c^* and \tilde{k}^* using equations (30) and (31). Given these, we can obtain solutions for K_t and the other non-detrended variables.

Calibration and Results We follow the same calibration strategy used to calibrate the baseline model. Table 3 summarizes the parameters' values. The only additional parameters are δ , ν and initial capital stock. Total factor productivity growth in non-agriculture, g_c , is also calibrated slightly differently. We choose δ to match the average ratio of consumption of fixed capital relative to GDP in the US between 1964 and 2007 from the BEA. We choose a country specific ν - or capital share - directly from Caselli and Feyrer (2007) who provide estimates of reproducible capital shares for 57 countries (see Appendix A for details). A crucial aspect of this calibration is the choice of initial capital in 1869. We use Piketty (2014)'s data to determine this value, by choosing the initial capital stock in the model to replicate his (reproducible-)capital-output ratio in the US in 1869 (details on our calculation are in Appendix A). Although this is most likely an imprecise measure of capital, it is the best we can do given the lack of historical data. Finally, since we do not have the entire capital stock series for the period, rather than calibrating total factor productivity growth in non-agriculture directly from the data, we instead choose the growth rate g_c so that our model replicates

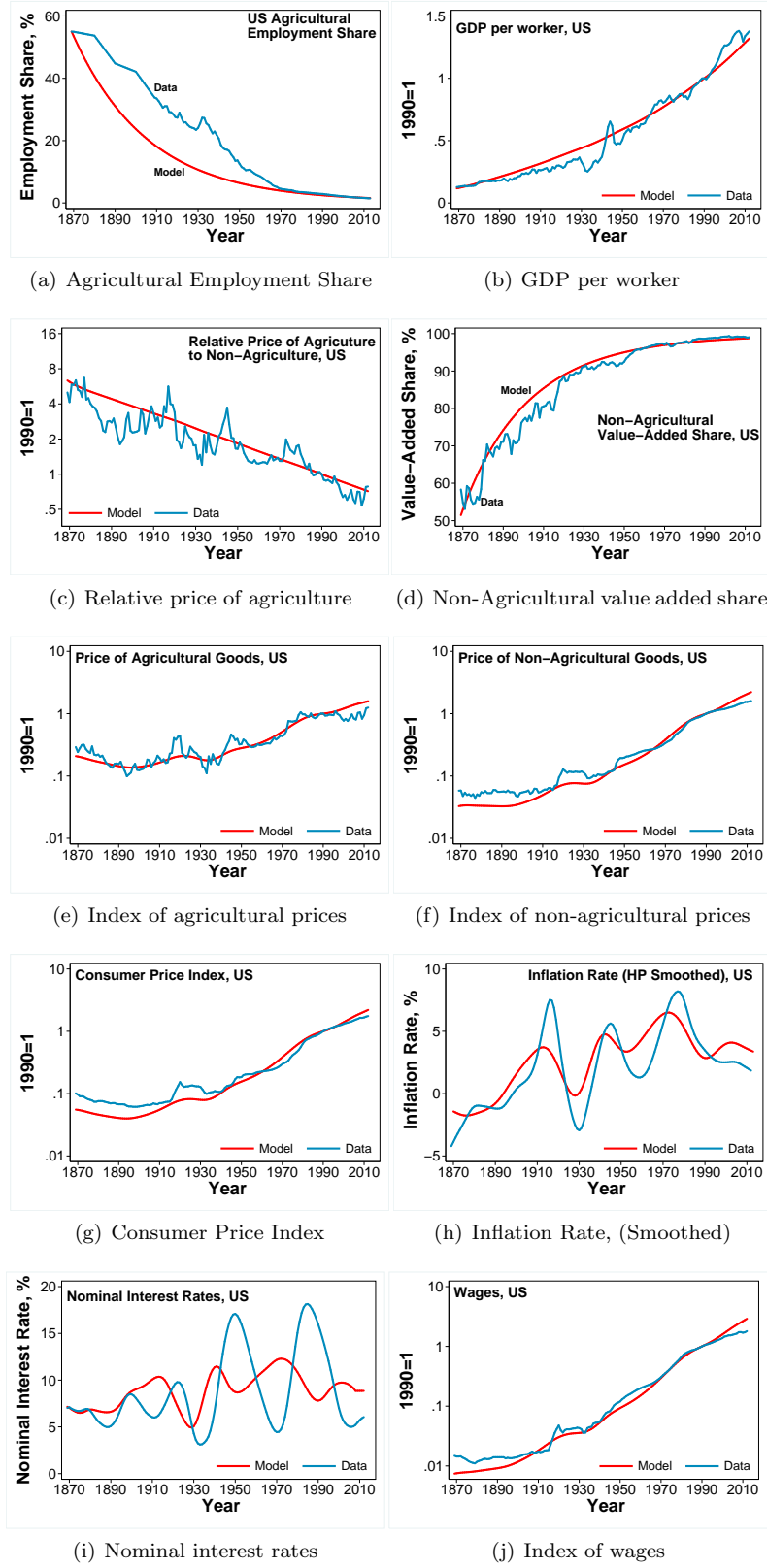


Figure 17: Simulations and data for the US in the Capital Model, 1869-2012.

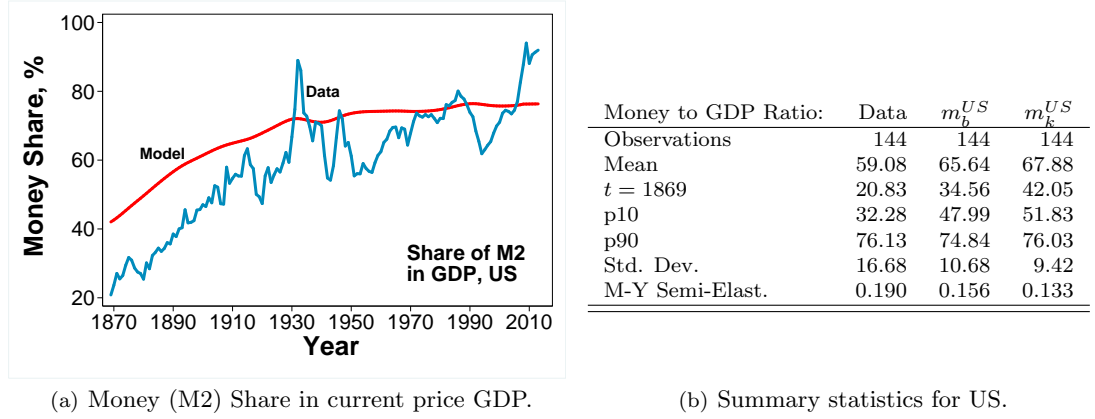


Figure 18: Money share of GDP and summary statistics for US. In the Table, columns show statistics for: money shares from the data (Data), the baseline model (m_b^{US}), and the capital model (m_k^{US}). Rows (1)-(6) show: the number of observations, the mean, the top decile, the bottom decile, the standard deviation as well as the semi-elasticity of money-share to income.

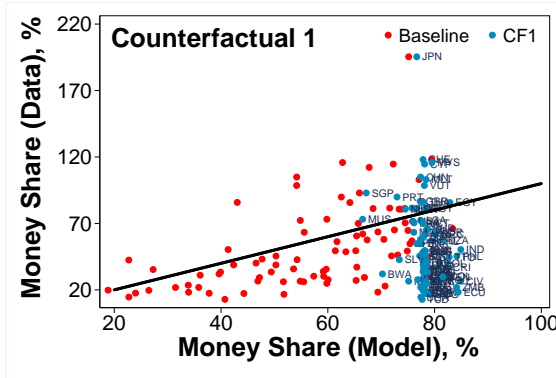
observed *labor* productivity growth between 1869 and 2007 in the data. Keeping in mind our caveats for capital stock data, Figure 17 illustrates that our model with capital does a good job in fitting the long-run data for US, and that outcomes are not very different to the baseline model. In Figure 18(a) we can see that our model predicts money shares trends quite accurately. Our initial money share in 1869 is slightly higher than before, however this is somewhat sensitive to the initial capital stock level - which is only roughly estimated. Importantly, we capture a large part of the increase in money shares - in particular we explain 63% of the increase in money share between 1869 and 2007. This is quite remarkable especially in light of the caveats regarding the measure of capital used.

Cross-Country Calibration Next, we consider how well the model performs in the cross-country setting. As in the baseline we focus on an international panel of 88 countries for 1980-2010.⁴⁵ We assume that each country is a closed economy with the same structural parameters of the US, with the exception of sectoral productivity, monetary policy, capital share and initial capital stock. Money stock per worker in each country and each year is taken directly from the data. The country i specific capital share, ν_i , comes directly from Caselli and Feyrer (2007). We assume that total factor productivity in sector s and in country i grows at a constant rate, $g_s^i - 1$, and is given by:

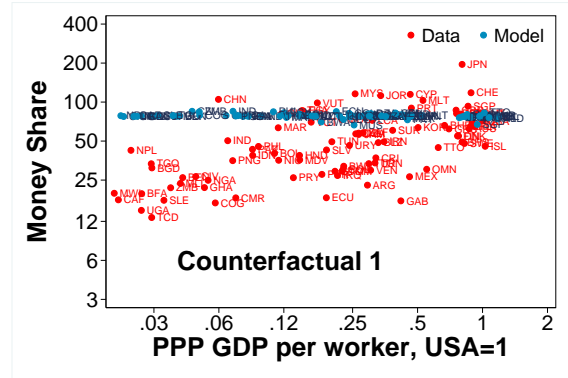
$$B_{s,t}^i = B_s^i \times (g_s^i)^{t-1980}. \quad (34)$$

We then choose g_s^i to match observed average sectoral total factor productivity growth rates in each country between 1980-2010 directly from the data, and pin down sectoral productivity levels, B_s^i , as in the baseline: B_a^i is chosen to match the agricultural employment share in country i in 1980,

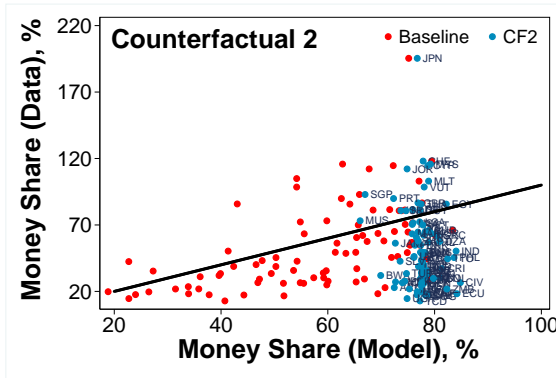
⁴⁵ We consider all countries for which all necessary data is available. Unless otherwise stated all data sources and construction methodology is presented in Appendix A.



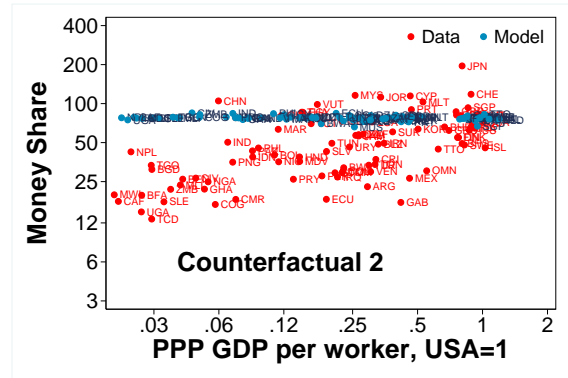
(a) Counterfactual 1 (US agr. productivity and monetary policy, steady state capital level): Money shares in data and model.



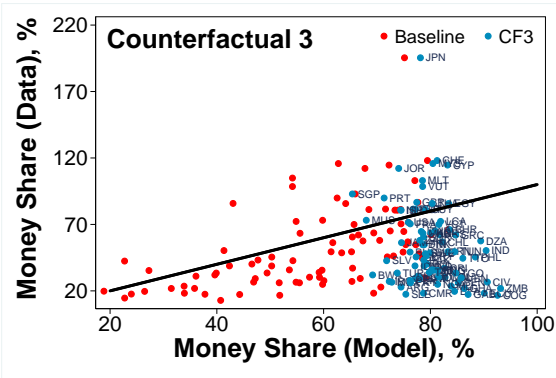
(b) Counterfactual 1 (US agr. productivity and monetary policy, steady state capital level): Money shares versus GDP per capita.



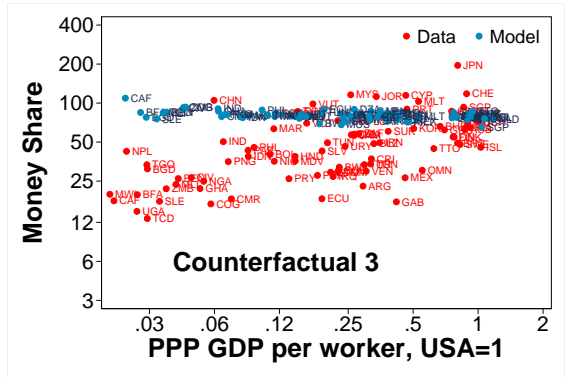
(c) Counterfactual 2 (US agr. productivity, steady state capital level): Money shares in data and model.



(d) Counterfactual 2 (US agr. productivity, steady state capital level): Money shares versus GDP per capita.

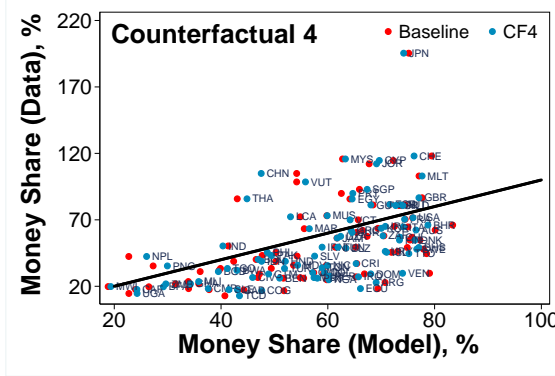


(e) Counterfactual 3 (US agr. productivity, country-specific capital level): Money shares in data and model.

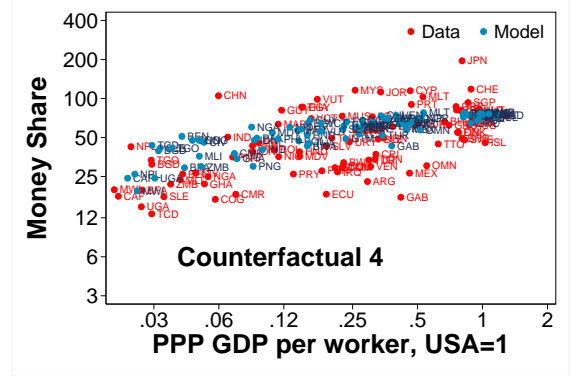


(f) Counterfactual 3 (US agr. productivity, country-specific capital level): Money shares versus GDP per capita.

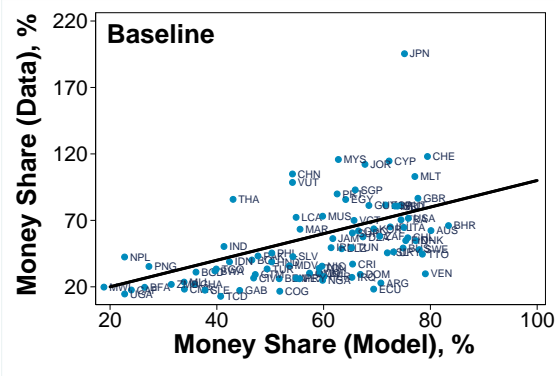
Figure 19: Money shares in the the baseline model and the counterfactuals (average for 1980-2010). Drawn versus corresponding data and (log of) GDP per worker.



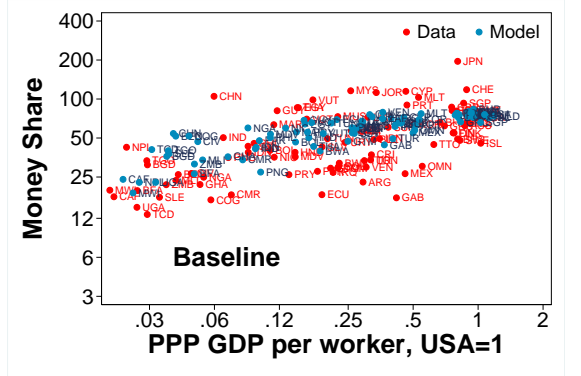
(a) Counterfactual 4 (Country-specific agr. productivity, steady state capital level): Money shares in data and model.



(b) Counterfactual 4 (Country-specific agr. productivity, steady state capital level): Money shares versus GDP per capita.



(c) Baseline Model: Money shares in data and model.



(d) Baseline Model: Money shares versus GDP per capita.

Figure 20: Money shares in the baseline model and the counterfactuals (average for 1980-2010). Drawn versus corresponding data and (log of) GDP per worker.

whilst B_c^i is chosen to match the ratio of GDP per worker in country i to that of the USA in 1980.⁴⁶ For more details on how we construct the international series for sectoral total factor productivities, capital, capital shares and monetary aggregates, see Appendix A. Next, given this calibration and in order to disentangle the mechanisms at work, we perform a set of four counterfactuals.

Counterfactual 1 The aim of the first counterfactual is to isolate the effect of heterogeneous sectoral productivity levels and growth rates across countries. We assume that each country has the same monetary policy as the US. We then choose $B_a^i = B_a^{US}$ for each country and choose B_c^i to match the ratio of GDP per worker in country i to that of the US in 1980. Furthermore, we choose a country's initial capital in 1980 to be equal to its balanced growth path level, i.e. $K_{1980}^i =$

⁴⁶ Since this is a dynamic model, we must also specify future values of productivity and money stock. We assume that all exogenously growing variables continue to grow at their country specific averages for a hundred years after 2007. After this point it is assumed that the growth rates of these variables converge to the long run US average growth rates. This assumption is made for convenience and quantitatively plays no role in our results.

$\tilde{k}_i^* (B_{c,1980}^i)^{\frac{1}{1-\nu_i}}$. Countries thus vary only in their country-specific, sectoral productivity growth rates and their initial level of non-agricultural productivity. Given that agricultural productivity is very high, the agricultural sector is small, and therefore this experiment is akin to having a one-sector model where all countries start with balanced-growth-path capital levels and the same monetary policy, while varying only in their productivity growth rates and initial levels. Figure 19(a) compares the results of our model in the counterfactual 1 with the data, while Figure 19(b) shows money shares predicted by the model as a function of the GDP per worker. As expected, and analogously to the model without capital, predicted money shares are about constant.

Counterfactual 2 In this counterfactual, we make the same assumptions as in the counterfactual 1, but we let countries have their own monetary policy. We therefore aim to see if differences in monetary policy can help explaining the variation in money shares across countries. In other words, in this counterfactual, our model is akin to a one-sector model in which all countries start from balanced-growth-path capital level, and they differ in productivity levels and growth rates as well as money growth rates. From Figures 19(c) and 19(d) it is evident that predicted money shares remain approximately constant. Hence, differences in monetary policies have only a small effect on money shares.

Counterfactual 3 In this exercise, we test if variations in initial capital endowments drive cross-country differences in money shares. The calibration is therefore the same as for counterfactual 2, but we assume country-specific initial levels of capital, taken directly from the data. Therefore, our exercise is akin to a one-sector model in which countries differ in growth rates and productivity, money growth rates and initial capital endowments. In this case, Figures 19(e) and 19(f) show that the model helps in explaining differences for rich countries, but it does a poor job for less developed countries.

Counterfactual 4 In our last counterfactual, we aim to isolate the effect of the two-sector structure of the economy. We therefore modify counterfactual 2, by re-calibrating B_c^i to match the ratio of GDP per worker in each country relative to the US and by calibrating B_a^i for each country to match agricultural labor shares in 1980. Therefore, this counterfactual is akin to the baseline two-sector model, but with all countries starting with their balanced-growth-path level of capital. From Figures 20(a) and 20(b) we observe that the counterfactual does a very good job in matching money-shares. In particular, this counterfactual suggests that differences in capital across countries do not drive our results but rather it is the two sector structure that is key.

Summary Our fully fledged model with capital is very good at predicting money shares across countries and the money share-income relationship. The crucial lesson is that variation in agricultural productivity levels is still the dominant driver, although of course capital accumulation now also plays a role. Table 4 provides summary statistics for the baseline and the counterfactuals, and

Money to GDP Ratio:	Data	m_b^k	m_{cf1}^k	m_{cf2}^k	m_{cf3}^k	m_{cf4}^k
Observations	88	88	88	88	88	88
Mean	52.71	58.43	78.15	77.32	80.26	58.31
p10	19.63	33.92	75.96	73.68	74.06	35.81
p90	92.92	77.09	81.82	81.05	87.06	75.96
Std. Dev.	31.16	16.06	3.00	3.11	6.19	15.03
M-Y Semi-Elast.	0.143	0.123	-0.005	-0.005	-0.025	0.116

Table 4: Summary statistics for cross-section (with capital). Rows (1)-(5) respectively show: the number of observations, the mean, the top-decile, the bottom-decile and the standard deviation of money shares in the data as well as those predicted by the baseline model and both counterfactuals. The final row shows the semi-elasticity of money-share to income.

allows us to decompose the money share’s standard deviation into the contribution of each counterfactual. In the data, the standard deviation of money share is 31.16, while it is 16.06 in our baseline model. The baseline thus captures 52% of the standard deviation of the data. We can also calculate the contribution of each counterfactual to the baseline model’s results. By comparing counterfactuals 3 and 4 with counterfactual 2, we can observe the role played by capital variation and the two-sector framework respectively. Adding capital heterogeneity helped explain an additional 19.2% ($\approx \frac{6.19-3.11}{16.06}$) of the deviation, whilst adding the two-sector framework helped explain an additional 74% ($\approx \frac{15.03-3.11}{16.06}$) of the deviation. Thus, the two sector framework is crucial in obtaining the baseline result. Finally, notice that, in the data, the money share-income (semi-)elasticity is approximately 0.143, whilst the baseline model predicts 0.123 - or 86% of the observed value. Again, it is the two-sector framework - rather than variation in capital stock - that overwhelmingly contributes to the success of the model along this dimension.

B.4 Traditional and modern agriculture

The baseline model in section 3 assumes that all transactions in the agricultural sector are non-monetary. We make the assumption both for simplicity and since data on non-monetary agricultural consumption is generally unavailable - especially in the cross-sectional setting. Nonetheless, we do not expect this assumption to make too much of a difference to our results. The agricultural sector tends to be relatively small in rich countries, and hence will not contribute much to money demand in those countries anyway. In poor countries where agriculture is an important part of the economy, we would expect traditional, barter/home production-style and importantly non-cash agriculture to dominate and hence, again, agriculture would not play a large role in driving money demand.

Nonetheless, in this section we use US data to examine agriculture’s contribution to the evolution of money demand. The USDA collects data on non-monetary value added for crops and animals consumed in farms for 1910-2016, and for non-cash payment to labour for the period 1919-2016. By summing up these two measures, we can obtain a rough estimate of the share of non-monetary transactions in the agricultural sector in the US.⁴⁷ According to these data, around 25% of agricultural value added was exchanged without cash in 1910. This figure declined at a roughly constant

⁴⁷ We thank an anonymous referee for this suggestion.

rate of 4.37% a year and was approximately 0.5% in 2016. Based on this evidence, we extend the model to incorporate cash and non-cash transactions in the agricultural sector.

We continue to assume there are two final-goods sectors (agriculture and non-agriculture). The setup of the non-agricultural sector remains identical to the baseline model.⁴⁸ However, we now assume that there are *two* intermediate sub-sectors in agriculture: a traditional and a modern sector. These two sub-sectors differ along two dimensions. First, the traditional sector is characterized by lower productivity growth. In fact, due to lack of better data, we will assume throughout that productivity in this sub-sector remains constant over time. This is in line with the argument put forward by Lucas (2004) that “traditional agricultural societies are very like one another”.⁴⁹ Second, we assume that households use cash to pay for the “modern” portion of their agricultural purchases whilst traditional agricultural goods are bought without cash.

The outputs of these two sub-sectors are close, but imperfect substitutes. The higher productivity growth in modern agriculture combined with the substitutability generates a transition from traditional to modern agriculture, on top of the standard shift from agriculture to non-agriculture. The changing composition of agriculture introduces an agriculture-specific money demand which lets us match the evolution of non-monetary transactions in the US. Crucially, we find that money shares and velocities are very similar to the benchmark model.⁵⁰ In what follows, we first present the setup and the analytical results of the model outlined above. We then explain how we calibrate the model to US data, and show that our results are close to the baseline model.

Household’s problem The representative households’s problem is now given by:

$$\begin{aligned} \max_{\substack{a_t, c_{m,t}, c_{n,t}, \\ b_{t+1}, m_{t+1}}} & \sum_{t=0}^{\infty} \beta^t (\alpha \log(a_t - \bar{a}) + (1 - \alpha - \gamma) \log(c_{m,t}) + \gamma \log(c_{n,t})) \\ \text{s.t. } & p_{a,t}a_t + p_{c,t}(c_{m,t} + c_{n,t}) + b_{t+1} + m_{t+1} \leq w_t + (1 + r_t)b_t + m_t + T_t \\ & p_{c,t}c_{m,t} + \theta_t p_{a,t}a_t \leq m_t \\ & b_t \geq -\bar{B} \end{aligned} \tag{35}$$

The only difference relative to the baseline model is that now a proportion of agricultural goods purchased by the household, $0 < \theta_t < 1$, will also be paid for with cash. We assume that this θ_t is exogenous to the household (but not to the firm) to capture the idea that sellers - rather than buyers - decide whether to take cash for their goods or not.⁵¹

⁴⁸ For simplicity, we continue to abstract from capital accumulation in this sector.

⁴⁹ Notice that this is not a crucial assumption. All that is required is that productivity growth in the traditional sector is smaller than in the modern sector.

⁵⁰ Notice that the above setup is stylized and adopted primarily for simplicity. We will get qualitatively and quantitatively very similar results - at the cost of greater complexity (i.e. corner solutions) - by assuming a transition from traditional to modern agricultural sector like in Gollin et al. (2007). There, both agricultural sub-sectors are perfect substitutes, but traditional agriculture is assumed not to use capital whilst modern agriculture does use capital. This results in poorer countries that have smaller capital endowments using traditional methods in agriculture, and later transitioning to modern farming techniques as capital holdings and modern agriculture productivity increase.

⁵¹ Letting households rather than firms decide has almost no quantitative effect on the results but complicates

Firms The final agricultural good is produced using two types of intermediate agricultural goods: modern (monetary) agricultural goods, denoted by m and traditional (non-monetary) agricultural goods denoted by n . Production of the non-agricultural final good (denoted by c) remains exactly as in the baseline. Agricultural intermediate goods and non-agricultural final goods are produced using the following production functions: $Y_{s,t} = B_{s,t}L_{s,t}$, for $s \in \{m, n, c\}$. Here, $B_{s,t}$ is labor productivity and $L_{s,t}$ is the labor input. The profit maximization problem for each firm $s \in \{m, n, c\}$ is given by:

$$\max_{L_{s,t}} p_{s,t}Y_{s,t} - w_tL_{s,t} \quad \text{s.t. } Y_{s,t} = B_{s,t}L_{s,t}. \quad (36)$$

The final agricultural good is an aggregate of the intermediate m and n goods with a production function given by: $Y_{a,t} = \left(\eta(a_{n,t})^{\frac{\sigma-1}{\sigma}} + (1-\eta)(a_{m,t})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$. In this expression σ is the elasticity of substitution between traditional and modern agriculture, whilst $0 < \eta < 1$ determines the relative importance of traditional agriculture. Traditional and modern agriculture are assumed to be close substitutes so that $\sigma > 1$. The profit maximization of the final good firm is standard:

$$\max_{a_{n,t}, a_{m,t}} p_{a,t}Y_{a,t} - p_{n,t}a_{n,t} - p_{m,t}a_{m,t}. \quad (37)$$

An *endogenous* proportion of the value of final agricultural goods, $\theta_t \equiv \frac{p_{m,t}a_{m,t}}{p_{a,t}Y_{a,t}}$ will be ‘modern’ goods, whilst the remainder will be ‘traditional’.

Money Supply The government is assumed to have a helicopter monetary policy as before:

$$M_{t+1} = T_{t+1} + M_t. \quad (38)$$

Market Clearing Finally, markets clear in the usual way:

$$\begin{aligned} a_{n,t} &= Y_{n,t}, \quad a_{m,t} = Y_{m,t}, \quad a_t = Y_{a,t}, \quad c_{m,t} + c_{n,t} = Y_{c,t} \\ Y_{a,t} &= \left(\eta(a_{n,t})^{\frac{\sigma-1}{\sigma}} + (1-\eta)(a_{m,t})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \\ m_t &= M_t, \quad b_t = 0 \\ L_{m,t} + L_{n,t} + L_{c,t} &= 1. \end{aligned} \quad (39)$$

Solution Assuming positive nominal interest rates, we can solve for the optimal allocations. From the first order conditions of the agricultural intermediate- and final-good firms we can derive an expression for θ_t given by:

$$\theta_t = \frac{x_t}{1+x_t}, \text{ where } x_t \equiv \left(\frac{1-\eta}{\eta} \right)^{\sigma} \left(\frac{B_{m,t}}{B_{n,t}} \right)^{\sigma-1}. \quad (40)$$

Given our assumptions that $\sigma > 1$ and that productivity growth in modern agriculture is higher than in traditional agriculture, the share of modern agriculture, θ_t , will rise over time, whilst the

the solution as changes in monetary growth will then effect θ_t over time so that this variable becomes dependent on future and current outcomes rather than just current outcomes, much like the relative size of the monetary sector in non-agriculture, ϕ_t , described below.

share of traditional agriculture, $(1 - \theta_t) = \frac{1}{1+x_t}$, will fall. Denoting total agricultural employment by $L_{a,t} \equiv L_{m,t} + L_{n,t}$, from the same first order conditions we can also show that:

$$L_{m,t} = \theta_t L_{a,t} \text{ and } L_{n,t} = (1 - \theta_t) L_{a,t}.$$

Non-agricultural output is divided between cash and non-cash goods:

$$c_{n,t} = (1 - \phi_t) Y_{c,t} \quad \text{and} \quad c_{m,t} = \phi_t Y_{c,t}, \quad (41)$$

We can then use households' first order conditions and the previous equations to get the labor share in agriculture

$$L_{a,t} = \alpha \frac{\phi_t}{1 - \alpha - \gamma + \alpha \phi_t} + \eta^{\frac{\sigma}{1-\sigma}} \frac{\bar{a}}{B_{a,t}} \frac{1 - \alpha - \gamma}{1 - \alpha - \gamma + \alpha \phi_t} (1 - \theta_t)^{\frac{1}{\sigma-1}} \quad (42)$$

Using first order conditions for consumption in the non-agricultural sector, we get a dynamic first-order difference equation that determines ϕ_t :

$$\frac{\phi_{t+1} + L_{a,t+1}(\theta_{t+1} - \phi_{t+1})}{\phi_t + L_{a,t}(\theta_t - \phi_t)} = \frac{\gamma(1 - L_{a,t})\phi_{t+1}\tau_t}{(1 - \alpha - \gamma)(1 - L_{a,t+1})(1 - \phi_t)} \quad (43)$$

In steady state, $\phi_t = \phi^{SS}$, i.e.

$$\phi^{SS} = \frac{1 - \alpha - \gamma}{1 - \alpha - \gamma + \gamma \tau^{SS}} \quad (44)$$

where τ^{SS} is determined by the steady state money growth rate. Given sequences for $B_{i,t}$, $i = a, m, c$ and M_t , we can easily solve for the optimal sequence of ϕ_t with a shooting algorithm, assuming that, for a large enough T , the economy is in steady state and hence $\phi_T = \phi_{T+1} = \phi^{SS}$. Next, from firms first order conditions, prices for sector $s \in \{m, n, c\}$ goods are:

$$p_{s,t} = \frac{w_t}{B_{s,t}}. \quad (45)$$

Similarly, the price for final agricultural goods is given by:

$$p_{a,t} = \frac{w_t}{B_{n,t}} \eta^{\frac{\sigma}{1-\sigma}} (1 - \theta_t)^{\frac{1}{\sigma-1}}, \quad (46)$$

and the nominal interest rate and wage rate are:

$$r_t = \tau_t \left(\frac{1 + \frac{\theta_t L_{a,t}}{\phi_t L_{c,t}}}{1 + \frac{\theta_{t+1} L_{a,t+1}}{\phi_{t+1} L_{c,t+1}}} \right) - 1 \quad \text{and} \quad w_t = \frac{M_t}{\theta_t L_{a,t} + \phi_t L_{c,t}}. \quad (47)$$

Finally, the share of monetary stock relative to the nominal GDP (or the *inverse* of monetary velocity) can be written as:

$$\begin{aligned} V_t^{-1} &= \frac{M_t}{p_{a,t} Y_{a,t} + p_{c,t} Y_{c,t}} = \theta_t \frac{p_{a,t} Y_{a,t}}{p_{a,t} Y_{a,t} + p_{c,t} Y_{c,t}} + \phi_t \frac{p_{c,t} Y_{c,t}}{p_{a,t} Y_{a,t} + p_{c,t} Y_{c,t}} \\ &= \theta_t L_{a,t} + \phi_t L_{c,t}. \end{aligned} \quad (48)$$

Parameter	Values	Target	Target Value
$B_{s,1869}, M_{1869}$	1	Normalization, $s \in \{m, n, c\}$.	-
$g_{m,a} - 1$	0.038	Labor productivity growth in agriculture, 1869-2007.	2.79%
$g_{n,a} - 1$	0	Productivity growth in traditional agriculture.	0%
$g_c - 1$	0.012	Labor productivity growth in non-agriculture, 1869-2007.	1.23%
$\{B_{s,t}\}_{t=1869}^{2014}$	$\{g_s^{t-1869}\}$	Constant productivity growth in sector $s \in \{m, n, c\}$.	-
$\{M_t\}_{t=1869}^{2014}$	$\{\cdot\}$	Growth in money stock per worker.	-
α	0.005	Long-run employment share in agriculture.	0.5%
\bar{a}	0.306	Employment share in agriculture in 1869.	55.5%
β	0.963	Average nominal interest rate & money growth, 1869-2007.	$\bar{r} = 8.85\%$ $\bar{g}_m = 4.79\%$
$1 - \gamma$	0.776	Long-run share of money in nominal value-added.	77.6%
σ	2.330	Change in proportion of traditional agriculture, 1910-2007.	-4.37%
η	0.660	Proportion of traditional agriculture, 2007.	0.52%

Table 5: Model with modern agriculture, calibrated parameters. See Appendix for detailed sources.

The first equality follows by definition. The second equality follows from the assumption that the CIA constraint binds. The final equality follows from the above price equations and the perfectly competitive nature of the problem. In contrast to the baseline, the money share in this model now depends on money demand originating both in the agricultural sector ($\theta_t L_{a,t}$) and the non-agricultural sector ($\phi_t L_{c,t}$). The importance of money demand in each sector is determined by the monetary component and the size of the respective sector (in terms of sectoral value added shares or - equivalently - sectoral employment shares). The above clearly demonstrates why agricultural money demand is unlikely to be important. In poorer countries, where productivity tends to be low, we will observe employment in agriculture $L_{a,t}$ close to one (equation (42)). However, given low productivity in the modern agricultural sector, traditional agriculture will dominate this sector and hence θ_t will be close to zero (equation (40)). Thus, the product of the two terms will also be close to zero. In richer countries, productivity will tend to be higher, so we will observe θ_t closer to one, but $L_{a,t}$ closer to zero - thus again, the product of the two terms will be close to zero. As such, it is the second part of the expression, $\phi_t L_{c,t}$, that plays a key role in driving money shares over time both in both the baseline and this model.

Calibration We calibrate the model to US data for the period from 1869 to 2007, following the same calibration strategy as in the baseline model. Table 5 summarizes all the parameter values. To remain brief, below we focus only on the calibration of parameters that do not appear or differ from the baseline calibration: σ , η and the productivity parameters in modern and traditional agriculture: $B_{s,t}$ for $s \in \{m, n\}$.

First, we normalize productivity in modern and traditional agriculture to one - so that $B_{s,1869} = 1$ for $s \in \{m, n\}$. As argued above we also assume that productivity growth in traditional agriculture is zero, so that $B_{n,t} = 1$. Then we use data on the average growth rate of agricultural value added per worker in the United States to obtain the time paths of labor productivity in the modern agricultural sector. In particular, we choose g_m - the average labor productivity growth in the modern agricultural sector in the model between 1869 and 2007 - so that the model reproduces the

observed growth of labor productivity in *total* agriculture (both modern and traditional) in the data over the same period. The time path of labor productivity in the modern agricultural sector is then given by $B_{m,t+1} = (1 + g_m)B_{m,t}$.

Second, we choose σ and η to match the evolution of the share of traditional agricultural value added in total agricultural value added (i.e. $1 - \theta_t$) over time. We use USDA data to construct the series of traditional agriculture value added. From equation (40), for a given sequence of productivities in modern and traditional agriculture, η determines the level of $1 - \theta_t$, whilst σ determines the extent to which a change in relative productivity translates into a change in $1 - \theta_t$ over time. Consequently, we choose η so that the model reproduces the share of traditional agriculture in 2007 and we choose σ so that the model matches the average annual rate of decline in the share of traditional agriculture between 1910 (the first year available) and 2007. Notice that given greater productivity growth in modern agriculture than in traditional agriculture, the calibration implies an elasticity of substitution between modern and traditional agriculture that is greater than one. The calibration of all remaining parameters is identical to the baseline.

Results Results from this model are in line with the baseline setup. Furthermore, most explanations behind the results remain identical to those of the baseline. Figures 21(a) and 21(b) show a very good fit for the series for agricultural labor shares and GDP per worker. The fit in this case is even better than in the baseline setup - as we are now effectively allowing for an endogenously evolving productivity in agriculture, resulting from a changing composition of the agricultural sector from a (zero productivity growth) traditional sector to a (positive productivity growth) modern sector. Figure 21(c), shows this changing composition of employment within agriculture. Notice that over time, traditional agriculture shrinks. This occurs for two reasons: not only because workers are moving away from traditional agriculture into modern agriculture due to the higher productivity growth in the modern sector, but also because of the flow of workers out of agriculture all together. Notice also that modern agricultural employment share now forms a hump shape over time. This happens since traditional workers initially flow into modern agriculture, resulting in a rising employment share in that sector. However, as productivity in modern agriculture improves, fewer agricultural workers are needed to feed the population and so workers shift out of agriculture (including modern agriculture) altogether and into non-agriculture. Whilst we do not have data on the size of employment in traditional and modern agricultural sector, we do have data on the share of traditional agricultural value added in total agricultural value added. This is shown in Figure 21(d), together with the predicted values for the model. This fit emerges largely from the calibration - recall that we have chosen parameters (η and σ) to match the traditional agricultural share in 2007 as well as the average rate of decline of the traditional agricultural share. However, since our focus is on monetary velocity, this is not a big concern. The remaining figures show the evolution of selected price variables. The relative price of agriculture to non-agriculture shown in Figure 21(e) is now nonlinear, first going up slightly at the end of the 19th century and then steadily declining.

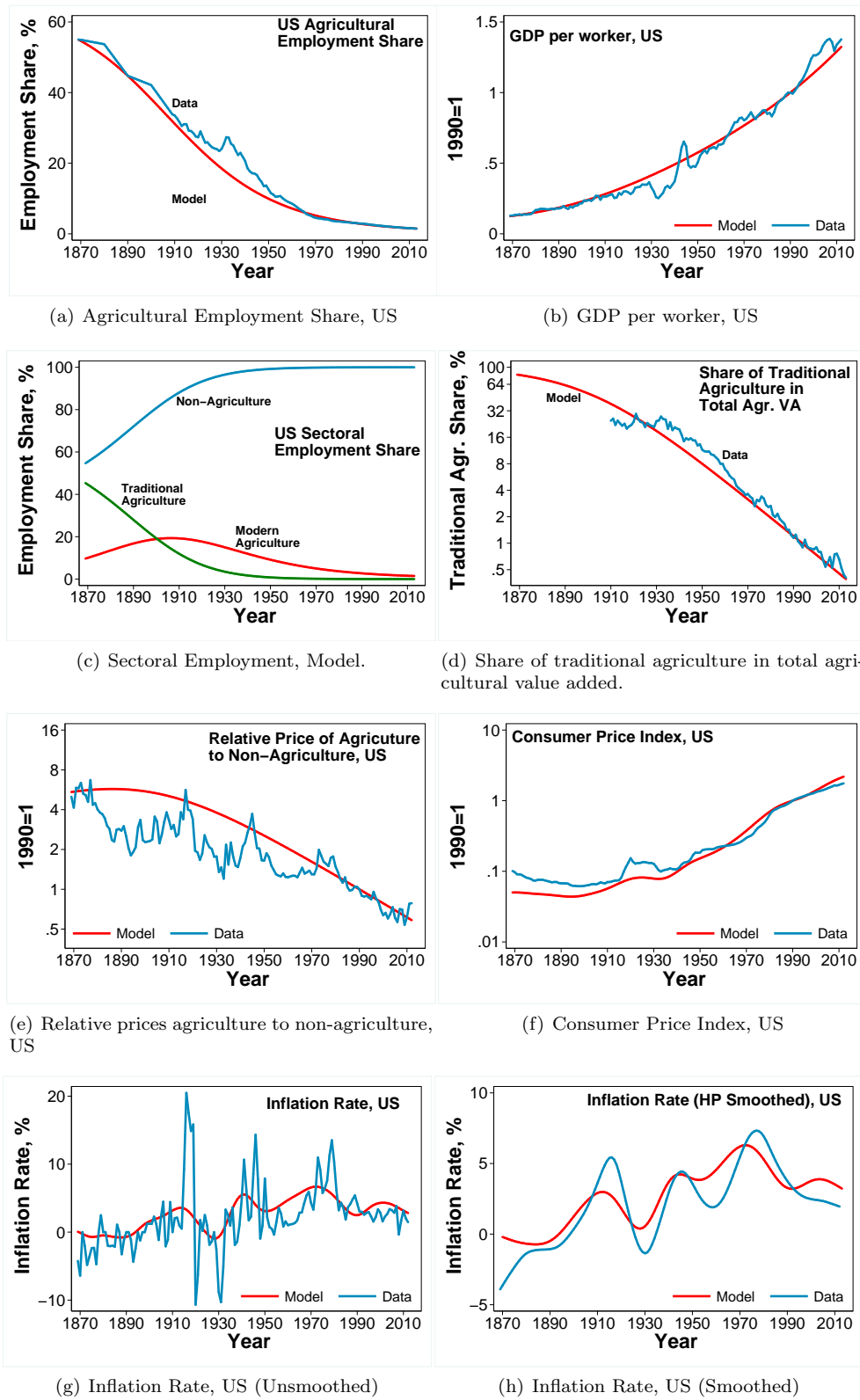


Figure 21: Simulations and data for US, 1869-2012.

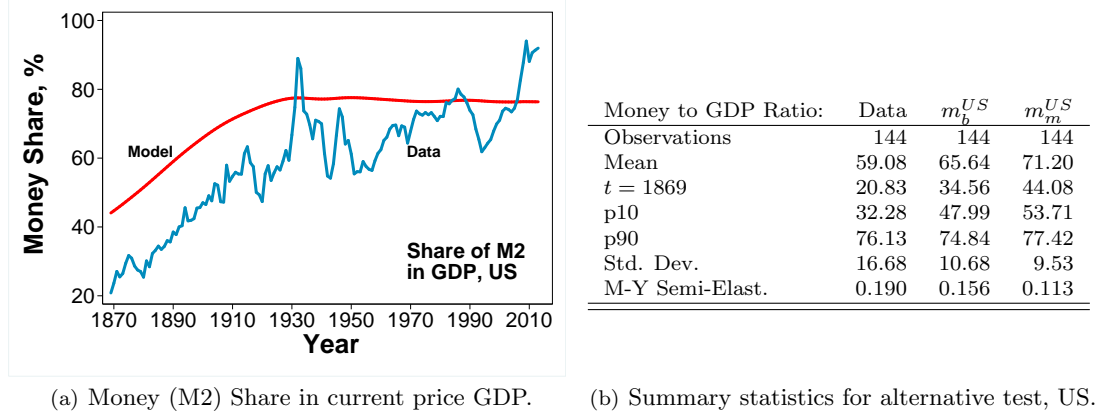


Figure 22: Money share of GDP and summary statistics for US Alternative Measures. In the Table, columns show statistics for: money shares from the data (Data), the baseline model (m_b^{US}), and the model with modern and traditional agriculture (m_m^{US}). Rows (1)-(6) show: the number of observations, the mean, the top-decile, the bottom-decile, the standard deviation as well as the semi-elasticity of money-share to income.

This feature arises from the fact that relative prices of agriculture to non-agriculture now not only depend on relative productivity in both sectors but also the composition of the agricultural sector - which is changing over time. This feature reduces the ability of the model to reproduce the Great Deflation at the end of the 19th century (see Figure 21(f)): although a deflation is still present, the magnitude predicted by the model is smaller.

Finally, Figure 22(a) shows the result for the money shares. Notice that the model in 1869 predicts a slightly higher money share than in the baseline. Importantly though, we still explain 59% of the increase of money share over the period relative to the long-run trend. Thus, even accounting for money demand in the agricultural sector, our results are in line with the those obtained from the baseline model.

B.5 Alternative tests of the model

Next, we present an alternative test of the baseline model. In the standard one-sector setup, money demand (M) is assumed to be proportional to nominal GDP (\bar{Y}) so that $M = \phi(r)\bar{Y}$. As an alternative, we propose that money demand originates (largely) in the non-agricultural sector. Our theory can thus be summarized with the following equations describing the money share:

$$\frac{M}{\bar{Y}} = \phi(r) \frac{\bar{Y}_c}{\bar{Y}_a + \bar{Y}_c} \quad (49)$$

$$= \phi(r) \frac{L_c}{L_a + L_c}. \quad (50)$$

Equations (49)-(50) are identical to equation (14), where for notational convenience we have dropped time subscripts, we have defined $\bar{Y}_s \equiv p_s Y_s$ for $s = a, c$, and we have emphasized that ϕ in equilibrium depends on the nominal interest rate. It is clear from both expressions that the money-to-GDP ratio

is increasing in the level of development, since both the value-added share and the employment share of the non-agricultural sector increase with structural transformation. A straightforward check of our theory, therefore, would be to use equations (49) and (50) to plot the observed money-GDP ratio from the data (left hand side of the equation) against the predicted money-GDP ratios (right hand sides of the above equations). The only parameter that would then need to be calibrated is the interest elasticity of money demand.⁵² In this section we perform this check for US time series and the cross-country sample, and show that doing so does not significantly change our baseline results.

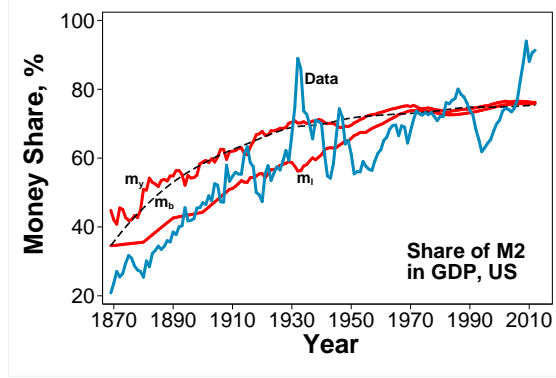
Comparison We start by calibrating the interest elasticity of money demand. In our baseline model, $\phi(r) = \frac{1}{1+a(1+r)}$, where $a \equiv \frac{\gamma}{1-\alpha-\gamma}$, which implies that interest elasticity of money demand is $|\frac{\partial \log(\phi(r))}{\partial \log(r)}| = \frac{ar}{1+a(1+r)}$. To pin down this elasticity we only need to know the parameter a . As we argued in the main body of the paper, under reasonable assumptions, the ratio of money to nominal GDP predicted by the model converges to $1 - \alpha - \gamma$ in the long run and the agricultural employment share converges to α . If we assume - as in the baseline - that the money-to-GDP ratio converges to a long run value of 0.776 predicted by the US data and employment share in agriculture converges to a reasonable 0.05%, then it follows that, $a = 0.282$, which allows us to pin down $\phi(r)$. Finally, taking nominal interest rates (r), current-price non-agriculture value added shares ($\frac{Y_c}{Y_a+Y_c}$) and non-agriculture employment shares ($\frac{L_c}{L_a+L_c}$) directly from the data we can construct ‘predicted’ money shares, $m_y \equiv \phi(r) \frac{Y_c}{Y_a+Y_c}$ and $m_l \equiv \phi(r) \frac{L_c}{L_a+L_c}$ and compare these with those predicted by our baseline model, m_b , and the data.⁵³

Figure 23(a) examines these predictions for the US between 1869 and 2012: m_y and m_l are plotted in red, our baseline model money share, m_b , is the black dashed line and the data is shown in blue. Table 23(b) provides summary statistics for each estimate. Given our calibration, in the long run all three estimates converge to the same value of money share or 77.6%. We can thus measure our success by comparing the change in predicted and observed money share between 1869 and the long run value of 77.6%. The m_b and m_l explain the largest part of the increase - each capturing 76.4% of the increase. Next, m_y generates a slightly smaller increase as it misses the money shares in the early part of the data - explaining approximately 58% of the increase - however it generates a better fit over the remainder of the period. Finally, m_b captures 65% of the standard deviation of the data, m_y captures 61% of the standard deviation whilst m_l does best and captures 84% of the variation. Overall, the three predicted measures give qualitatively identical and quantitatively very similar results for the US - although m_l and m_b do best in capturing the evolution of money shares found in the data.

Next, we turn to the cross country data. As before we will focus on average values over the 1980-2010 period for each country, in order to emphasize the long-run cross-sectional fit of the model.

⁵² We would like to thank an anonymous referee for this suggestion.

⁵³ In our model $r_t = \tau_t - 1$. Hence, instead of using nominal interest rates directly, we could use the fact that $\tau_t = \frac{M_{t+1}}{\beta M_t}$ to infer the model-predicted r_t . This however, would require us to calibrate another parameter - β - so we will not go down this route. Moreover, choosing this option would give practically indistinguishable results.



(a) Money (M2) Share in current price GDP. The black, dashed line is the baseline model. The two red lines are the alternative measures using value added and labor shares.

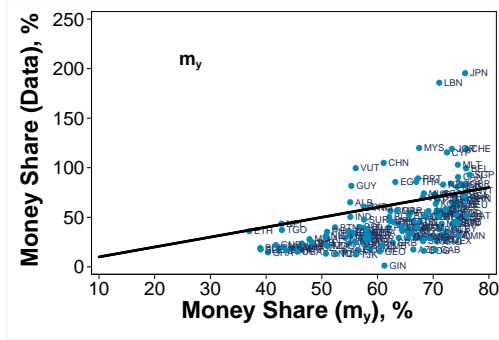
	Data	m_b^{US}	m_l^{US}	m_y^{US}
Observations	144	144	144	144
Mean	59.08	65.64	59.63	66.43
$t = 1869$	20.83	34.56	34.56	44.81
p10	32.28	47.99	37.72	52.14
p90	76.13	74.84	74.77	75.79
Std. Dev.	16.68	10.68	13.78	9.93
M-Y Semi-Elast.	0.190	0.156	0.183	0.123

(b) Summary statistics for alternative test, US.

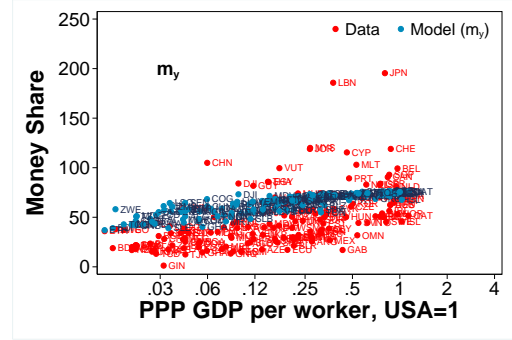
Figure 23: Money share of GDP and summary statistics for US Alternative Measures. In the Table, columns show statistics for: money shares from the data (Data), the baseline model (m_b^{US}), the model fitting employment shares directly (m_l^{US}) and the model fitting value-added shares directly (m_y^{US}). Rows (1)-(6) show: the number of observations, the mean, the top-decile, the bottom-decile, the standard deviation as well as the semi-elasticity of money-share to income.

Figure 24 shows observed money shares versus predicted money shares (left column) and model predicted money shares versus GDP per worker (right panel) in the cross-section. Notice that all three measures capture a substantial portion of the variation of money shares in the data - as well as a positive money-share to GDP relationship. However, once more, it is the baseline model and m_l that do the best - capturing most of the variation and best replicating the money-share income elasticity. This is highlighted in Table 6 which provides summary statistics and allows us to quantify the success of individual models. The m_y model captures about a third of the standard deviation in the data, 36-40% of the observed increase between the highest and the lowest money-shares deciles and it explains 50-54% of the money-share to income semi-elasticity. The m_l and m_b models fare about the same. They both capture about two-thirds of the standard deviation in the data, 80-88% of the observed increase between the highest and the lowest money-shares deciles and explain the entire money-share to income semi-elasticity. Thus, quantitatively m_y does slightly worse in explaining the cross-country data than either m_b or m_l - but qualitatively all three measures capture similar patterns. These findings support our choice to calibrate the baseline model to employment shares rather than value-added shares. Value added shares - especially in poorer countries and in historical US data - are of poor quality (they can be self-reported, based on surveys, or imputed) or they simply do not exist. Instead, by focusing on labor shares rather than value added shares, we avoid some of these problems as it is easier to count 'bodies' employed in a sector than the output and prices of hundreds of individual products.

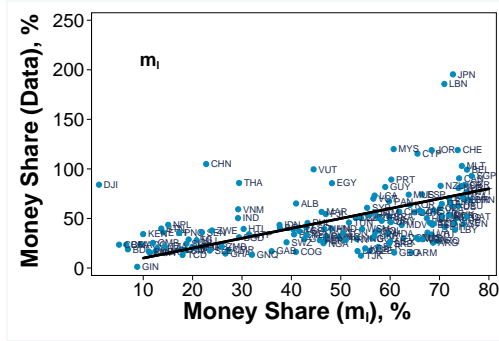
Despite the simplicity of the above exercise, we believe that the baseline calibration in the main body of the paper is preferable as it helps us better understand the mechanism of the model and



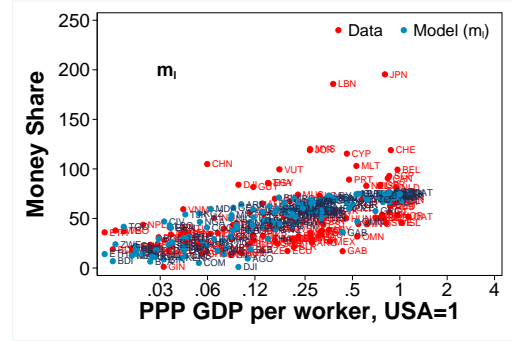
(a) Money Share, alternative measure versus Data. (VA)



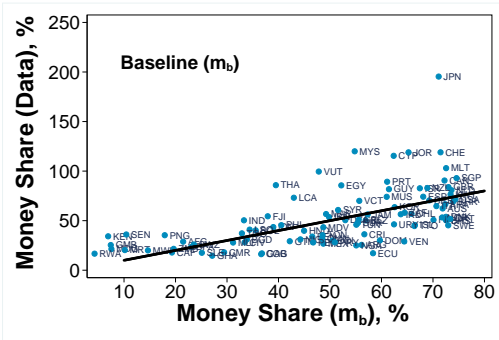
(b) Money Share, alternative measure and data versus GDP. (VA)



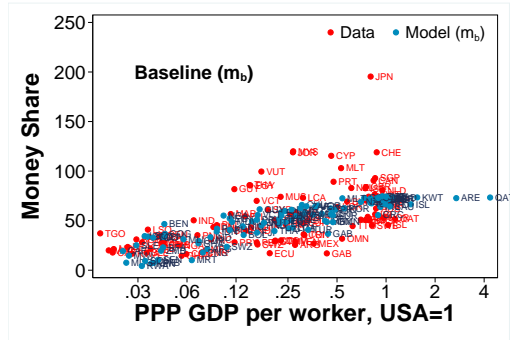
(c) Money Share, alternative measure versus Data. (EMP)



(d) Money Share, alternative measure and data versus GDP. (EMP)



(e) Money Share, alternative measure versus Data. (Baseline)



(f) Money Share, alternative measure and data versus GDP. (Baseline)

Figure 24: Money shares in different calibrations (average for 1980-2010). Drawn versus corresponding data and (log of) GDP per worker. ALL AVAILABLE DATA

	Data	m_b	m_l	m_y	Data	m_b	m_l	m_y
Observations	153	102	153	153	94	94	94	94
Mean	47.82	50.07	49.41	63.16	52.41	51.09	52.08	64.52
p10	17.84	19.64	15.06	47.78	21.00	19.65	21.88	50.60
p90	84.04	72.54	73.46	74.42	86.80	72.55	73.83	74.56
Std. Dev.	30.02	19.55	21.24	10.26	29.92	19.41	19.12	9.90
M-Y Semi-Elast.	0.135	0.142	0.152	0.074	0.140	0.138	0.142	0.070

Table 6: Summary Statistics. Left part of table shows all data, whilst the right part of the table shows only data that is available for all measures.

provides additional external validity. First, our baseline model specifies and quantifies the exact mechanism that drives employment, value added shares and nominal interest rates which in turn drive money shares. Instead, the exercise in this section takes shares and interest rates as exogenous. Knowing the mechanism is important as it allows us to determine - perhaps surprisingly - that agricultural productivity growth is largely responsible for the observed money-share patterns.

Second, having a fully calibrated and specified model allows us to perform counterfactuals and welfare analyses. This provide us with insights that the simple exercise in this section could not deliver. In section 8 for example, we find that inflationary policies tend to be far more damaging in rich countries than in poor countries, which helps to explain why we tend to observe higher inflation rates in poorer countries than in richer countries.

Finally, and most importantly, the calibrated baseline model has strong implications on the evolution of nominal prices, which can then be compared with the data. As we argue in section 5, this turns out to be especially interesting in light of the so-called ‘Great Deflation’ period in the United States at the end of the 19th century that our model, in contrast to more standard models, is able to replicate.

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