Quantitative Easing and the Loan to Collateral Value Ratio

Tatiana Damjanovic and Šarūnas Girdėnas

Centre for Dynamic Macroeconomic Analysis

University of St Andrews

Quantitative Easing and the Loan to Collateral Value Ratio[*]

Tatiana Damjanovic   Šarūnas Girdėnas[†]

University of Exeter   University of Exeter

17 May 2014

Abstract

We study monetary optimal policy in a New Keynesian model at the zero bound interest rate where households use cash alongside house equity borrowing to conduct transactions. The amount of borrowing is limited by a collateral constraint. When either the loan to value ratio declines or house prices fall, we observe a decrease in the money multiplier. We argue that the central bank should respond to the fall in the money multiplier and therefore to the reduction in house prices or the loan to collateral value ratio. We also find that optimal monetary policy generates a large and persistent fall in the money multiplier in response to the drop in the loan to collateral value ratio.

Keywords: optimal monetary policy, zero lower bound, quantitative easing, money multiplier, loan to value ratio, collateral constraint, house prices

JEL classification: E44, E51, E52, E58

1   Introduction

The recent economic crisis caused a significant decrease in credit availability (Dell Ariccia at al., 2008) which resulted in a sharp fall in house prices and output. Responding to worsening credit conditions, many developed countries significantly expanded their monetary bases. Several central banks engaged in quantitative easing (QE) taking "unconventional" assets onto their balance sheets (Gambacorta at al., 2012). In this paper, we provide a justification for QE and argue that monetary expansion is necessary for stabilizing price and output fluctuations when there is a marked tightening of credit conditions.

The general idea of connecting financial markets and business cycles can be traced back to Fisher (1933), Bernanke (1983) and Bernanke and Gertler (1989) who show that a contraction in the financial sector can lead to an economic slowdown. In this paper, we investigate whether, to what extent and how the monetary authorities should respond to worsening financial conditions in order to avoid an economic recession.

This question is not new to the academic literature.[1] On the one hand, in his review of recent monetary policy developments, Clarida (2012) argues that financial variables are not target variables and should not be included in monetary policy rules. The same opinion is also shared by Bernanke and Gertler (2001) and Iacoviello (2005) who claim that the government should not react to changes in asset prices as this does not improve the economy in terms of inflation and output stabilization.

On the other hand, Mishkin (2011) argues that after the 2007-2009 economic crisis, monetary policy makers understood that the financial sector has a considerably greater impact on economic activity than has previously been realized. Further to this, Svensson (2009) recognizes that credit capacity and asset prices may have a potentially negative impact on inflation and resource utilization and, therefore, including them in the monetary policy rule is entirely consistent with the stabilization of inflation and output gaps. We will also observe this particular feature in our model.

We study optimal monetary policy in a New Keynesian economy with sticky prices where households use cash alongside equity borrowing to conduct transactions. The amount of borrowing is limited by a collateral constraint as in Kiyotaki and Moore (1997) or Iacoviello (2005).

---

[1]For a comprehensive survey on macroeconomics with financial frictions, see Brunnermeier et al. (2012).

We simply assume that competitive financial intermediaries can costlessly create as much credit as they want. However, due to the lack of contract enforcement, each loan has to be collateralized.

We follow Iacoviello (2005) and Midrigan and Philippon (2011) and assume that households use houses as collateral. The households' credit capacity can deteriorate for two reasons: a reduction in collateral value or an exogenous shock which causes a decline in the average recovery rate of collateral. In the remainder of the paper, this is referred to as a "credit shock". When the loan to collateral value (LTV) declines, the credit capacity falls. Less inside money reduces nominal expenditure and, thus, nominal demand. In a flexible price economy, producers adjust their prices accordingly and recession is avoided. However, when prices are sticky, only incomplete adjustment is possible, and credit tightening results in both deflation and recession, unless an expansionary monetary policy is implemented.

The principal difference of our model to Iacoviello (2005), Monacelli (2009) and Carlstrom and Fuerst (1995) is our assumption that the economy may reach a liquidity trap. When the interest rate is at its zero lower bound, the monetary authorities cannot reduce it any further and are hence forced to stimulate the economy by providing direct monetary transfers to households. Unconventional monetary expansion at a zero bound interest rate has been advocated by Friedman (2000, 2006) and Bernanke at al. (2004). When the interest rate is at zero bound, direct monetary targeting cannot be criticized in the sense of McCallum (1985), because it does not cause any volatility in the short-term interest rate. To our knowledge, direct monetary expansion when the interest rate is at its zero lower bound (ZLB) has not been formally modeled and it is not considered in recent academic publications (see Adam and Billi, 2007; Coibion, Gorodnichenko and Wieland, 2012). The only exception, perhaps, is the recent paper by Benigno and Nistico (2013), which studies a similar shock to us and gives similar policy recommendations.

We also find that monetary policy can ensure perfect stabilization of output and prices when a credit shock hits the economy. When the shock is small, the monetary policy maker can reduce the interest rate. However, a large negative credit shock can drive the interest rate to the ZLB. At that point monetary policy has to inject liquidity into the system.

The expansion of the monetary base in response to the deterioration in credit availability is necessary because of the fall in the money multiplier. The importance of the money multiplier

has been discussed in Bernanke and Blinder (1988), Freeman and Kydland (2000) and recently in Goodhart (2009) and Abrams (2011). Since the money multiplier reflects monetary transmission, optimal monetary policy should respond to changes in it. Our model shows how the multiplier depends on the LTV ratio and the relative price of collateral. Hence, if houses are used as collateral, monetary policy should respond to changes in house prices.

To evaluate monetary policy rules at the ZLB, we construct a second-order approximation as in Benigno and Woodford (2012)[2] and obtain a social loss function as in the conventional New Keynesian model (Benigno and Woodford, 2005). In our model, optimal monetary policy generates the same impulse responses to the cost-push shock for output and inflation as in the standard New Keynesian model. In order to achieve optimal dynamics, the social planner should conduct monetary expansion when there is a fall in the LTV ratio or if the relative price of collateral declines.

This paper is structured as follows. In Section 2, we present the model and define dynamic equations for the economy. In Section 3, we discuss optimal monetary policy and some other policy issues. We include a short discussion of what may happen if the social planner ignores changes in the credit constraint or fluctuations in house prices. We also underline the importance of the money multiplier and its connection to credit constraints and the relative price of collateral. In Section 4 we provide a short discussion of the factors which can affect the LTV ratio. In Section 5 we investigate an economy when money and loans are not perfect substitutes. Section 6 concludes the paper.

2 Model

In this section, we present a stylized New Keynesian economy with collateral constraint. Our model consists of several ingredients. First, we have households who consume final goods, provide labor to final good producers, earn wages, share the profit of the firms and take loans against collateral. Second, there are intermediate and final goods producers. The latter operate in a sticky price environment in the spirit of Calvo (1980). There are also monetary authorities which make decisions about interest rate and money supply.

Finally, we have a financial sector which creates inside money through lending. We simply

---

[2]See also Levine at al. (2008).

assume that a financial intermediary can costlessly create as much loans as is safe. That is a principal difference of our model compared to Benigno and Nistico (2013) or Iacoviello (2005), where financial intermediaries do not participate in the creation of inside money. To hedge against the risk of default, the lender issues debt against valuable collateral, represented by houses in our model. If borrowers repudiate their debt, the lender can repossess the borrowers' assets and recover a fixed proportion of their value. The only interesting outcome of financial intermediation for our model is the loan to value ratio. Micro modelling of the financial sector could explain/endogenise LTV; however, for simplicity, we treat it as exogenous but stochastic.

2.1   Households

A representative household has a utility function that includes the consumption of goods, Y_t, valuable collateral (house), h_t, and labor, L_t,

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t u(Y_t, h_t, L_t) = E_t \sum_{t=0}^{\infty} \beta^t \left(\log Y_t + \eta \log h_t - \lambda \frac{L_t^{v+1}}{v+1} \right), \tag{1}$$

where v is the labor supply elasticity parameter, η captures individual household preferences towards units of housing and λ defines the value of leisure.

For their transactions, households can use cash, i.e. outside money, M_t, and the money created by the banking system, i.e. inside money, B_t. The broad money can be used to buy consumption goods and invest in collateral

$$P_t Y_t + Q_t(h_t - h_{t-1}) \leq M_t + B_t, \tag{2}$$

where P_t is the price of final goods, Q_t is the price of collateral, and $h_t - h_{t-1}$ is investment in collateral.

The amount of private credit is subject to a collateral constraint

$$B_t \leq \theta_t Q_t h_t, \tag{3}$$

which implies that households cannot borrow more than a fraction θ_t of their collateral value $Q_t h_t$. Parameter θ_t denotes the tightness of the borrowing constraint. A smaller value of θ_t

implies a smaller loan size, whereas a high value means that a household may obtain a relatively large loan. The government implements monetary policy by printing new bills and distributing them across households as a lump-sum transfer

$$M_t^s = M_{t-1} + T_t. \tag{4}$$

The loan must be repaid immediately after households obtain their wage and dividend income. Let W_t be the nominal wage and Π_t be the profit of firms owned by households and paid in the form of dividends. Then, at the end of the period, the liquidity position of the household is

$$M_t^d = W_t L_t + \Pi_t + T_t - (1 + r_{t-1}) B_{t-1}. \tag{5}$$

2.1.1 Household's optimization

In the appendix, we show that maximization of household utility (1) subject to constraints (2, 3 and 5) results in the following Euler equation

$$U_{ht}' + \theta_t \frac{Q_t}{P_t} \left(U_{ct}' - \beta E_t U_{ct+1}' \frac{P_t}{P_{t+1}} (1 + r_t) \right) = U_{ct}' \frac{Q_t}{P_t} - \beta E_t U_{ct+1}' \frac{Q_{t+1}}{P_{t+1}}. \tag{6}$$

The left-hand side of the equation shows the marginal benefit from an extra unit of collateral: it consists of a direct boost to utility, U_{ht}', as well as an effect due to the possibility of using collateral to secure a loan. The value of the second source is proportional to credit tightness θ_t. In other words, a smaller θ_t reduces the loan size and, as a result, there is a fall in the benefits from using a house as a collateral .

Moreover, constraint (2) is always binding, while constraint (3) is binding when

$$U_{ct}' - \beta E_t U_{ct+1}' \frac{P_t}{P_{t+1}} (1 + r_t) > 0. \tag{7}$$

In a deterministic steady state the credit constraint will be binding if and only if $\beta \frac{(1+r_t)}{\pi_{t+1}} < 1$. As we see, either a reduction of the borrowing rate or an increase in inflation can move the economy from the state when the credit constraint is not important to the state when it is binding. In our main presentation we assume the interest rate to be at the zero bound, $r_t = 0$.

In this case, if the price level and the consumption level are stable, inequality (7) is satisfied. Following Iacoviello (2005), we will consider that the uncertainty is "small enough" to preserve inequality (7).

Using the particular functional form of utility (1), the assumption of the zero interest rate, and normalizing the units of housing, $h_t = 1$, we transform equation (6) into the following form

$$\eta + \beta E_t \left[q_{t+1} \right] = \left(1 - \theta_t \left(1 - \beta E_t \left(1 + r_t \right) \left[\frac{P_t Y_t}{P_{t+1} Y_{t+1}} \right] \right) \right) q_t, \tag{8}$$

where q_t is relative housing expenditure, which is defined as

$$q_t = \frac{Q_t h_t}{P_t c_t}.$$

Finally, the first-order condition with respect to L_t defines labor supply

$$-\frac{U_L'(C_t, L_t)}{U_C'(C_t, L_t)} = \lambda L_t^v Y_t = \frac{W_t}{P_t}. \tag{9}$$

2.2 Final good producers

We assume that final goods are imperfect substitutes and that consumption is defined over the Dixit-Stiglitz (1977) basket of goods, $Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$. The average price-level, P_t, is known to be $P_t = \left[\int_0^1 p_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$. The demand for each good is given by $Y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\varepsilon} Y_t$, where $p_t(i)$ is the nominal price of the final good produced in industry i and Y_t denotes aggregate demand. Each good is produced according to a linear technology using labor as the only input, $Y_t(i) = L_t(i)$.

There is an economy-wide labor market so that all firms pay the same wage for the same labor, $w_t(i) = w_t$, $\forall i$. All households provide the same share of labor to all firms, so that the total labor supply in (1) is defined as $L_t = \int_0^1 L_t(i) di$, which in combination with the production function and demand relates output to labor income. $L_t = \int L_t(i) di = Y_t \int \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} di = Y_t \Delta_t$, where Δ_t is the measure of price dispersion: $\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} di$. The real wage, $w_t = \frac{W_t}{P_t}$, is then $w_t = \beta \lambda Y_t^{v+1} \Delta_t^v$.

2.2.1 Price-setting of a representative firm

We will model price stickiness according to Calvo (1983). A fixed proportion of firms adjusts prices in each period. These firms choose the nominal price which maximizes their expected profit given that they have to charge the same price in k time periods with probability α^k. The real profit can be written as $\Pi(i) = \frac{p_t(i)}{P_t} Y_t(i) - \phi_t w_t L_t(i)$, where ϕ_t is a cost-push shock. We assume that firms are price takers and cannot affect any aggregate variables. Let p'_t denote the choice of the nominal price by a firm that is permitted to re-price in period t. Then the firm's objective is to choose p'_t to maximize the following sum

$$\max E_t \sum_{k=0}^{\infty} \alpha^k \beta^k \left[\frac{p'_t}{P_{t+k}} \left(\frac{p'_t}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} - \phi_t w_t \left(\frac{p'_t}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \right]. \tag{10}$$

The first-order condition implies

$$\left(\frac{p'_t}{P_t} \right) = \frac{\left(\frac{\varepsilon}{\varepsilon-1} \right) E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[\phi_{t+k} w_{t+k} Y_{t+k} (P_t/P_{t+k})^{-\varepsilon} \right]}{E_t \sum_{k=0}^{\infty} (\alpha\beta)^k (P_t/P_{t+k})^{1-\varepsilon} Y_{t+k}}. \tag{11}$$

It is useful to introduce new variables, X_t and Z_t, for the discounted expected real revenue and costs of the firm. We define them as $X_t = E_t \sum_{k=0}^{\infty} (\alpha\beta)^k (P_t/P_{t+k})^{1-\varepsilon} Y_{t+k}$, $Z_t = E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[\phi_{t+k} w_{t+k} Y_{t+k} (P_t/P_{t+k})^{-\varepsilon} \right]$. The price index will evolve according to the following law of motion, $P_t = \left[(1-\alpha) p_t^{1-\varepsilon} + \alpha P_{t-1}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$, which can be rewritten as $\frac{p'_t}{P_t} = \left[\frac{1-\alpha\pi_t^{\varepsilon-1}}{1-\alpha} \right]^{\frac{1}{1-\varepsilon}}$.

2.3 The government's optimization problem

The policy maker maximizes the household's utility function with the awareness that the supply of houses is constant and normalized to 1, $h_t = 1$,

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t \left(\log Y_t - \frac{Y_t^{v+1} \Delta_t^{v+1}}{v+1} \right), \tag{12}$$

subject to a set of constraints imposed by private agents' behavior (13-20), where a block of three equations (14-16) represents formula (11) in a VAR form and (17) is the law of motion for the measure of price dispersion.

$$\eta + \beta E_t \left[q_{t+1} \right] = \left(1 - \theta_t \left[1 - \beta E_t \left(1 + r_t \right) \left(\frac{Y_t}{\pi_{t+1} Y_{t+1}} \right) \right] \right) q_t; \qquad (13)$$

$$X_t = Y_t + a \beta E_t \pi_{t+1}^{\varepsilon - 1} X_{t+1}; \qquad (14)$$

$$Z_t = \phi_t \beta \lambda Y_t^{v+2} \Delta_t^v + a \beta E_t \pi_{t+1}^{\varepsilon} Z_{t+1}; \qquad (15)$$

$$X_t \left(\frac{1 - \alpha \pi_t^{\varepsilon - 1}}{1 - a} \right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon - 1} Z_t; \qquad (16)$$

$$\Delta_t = \alpha \Delta_{t-1} \pi_t^{\varepsilon} + (1 - \alpha) \left(\frac{1 - \alpha \pi_t^{\varepsilon - 1}}{1 - a} \right)^{\frac{\varepsilon}{\varepsilon - 1}}; \qquad (17)$$

$$\frac{M_t}{P_t Y_t} = 1 - \theta_t q_t; \qquad (18)$$

$$T_t = M_t - M_{t-1}; \qquad (19)$$

$$\pi_t = \frac{P_t}{P_{t-1}}. \qquad (20)$$

As in Midrigan and Philippon (2011), we obtain that, in the steady-state output, Y, does not depend on the credit constraint, θ. However, the value of θ will positively affect relative housing expenditure, $q = \frac{\eta}{(1-\theta)(1-\beta)}$ and, therefore, equilibrium real house price $\frac{Q}{P} = qY$. It will also define the broad money multiplier, $m = \frac{M+B}{M}$. Since broad money, $M_t + B_t$, equals total expenditure, we can compute the money multiplier from (18)

$$m_t = \frac{M_t + B_t}{M_t} = \frac{P_t Y_t}{M_t} = \frac{1}{1 - \theta_t q_t}. \qquad (21)$$

This positive relation between the money multiplier, m_t, the credit constraint, θ_t, and the relative collateral value, q_t, will drive our results.

2.4 Negative Credit Shock and ZLB

When a small negative credit shock hits an economy, the monetary authority can easily mitigate it by reducing the interest rate with or without applying direct monetary transfers. Consider a simple case when an economy is initially in an optimal steady state. Suddenly, the Loan to Value ratio, θ, declines. If the interest rate rate is positive and the credit shock is relatively small, the government can use conventional interest rate policy only, without providing monetary transfers. Precisely, we formulate it in the following proposition

Proposition 1 *When interest rate is positive, $r_t > 1$, and the negative credit shock is small, there exists such stabilization policy that $T_t = T_{t+1} = 0$, $\pi_t = \pi_{t+1} = 1$; $Y_{t+1} = Y_t = Y_{t-1}$. That policy implies the following dynamics for interest rate*

$$r_t = r_{t-1} - \frac{1}{\beta}\left(\frac{1}{\theta_t} - \frac{1}{\theta_{t-1}}\right) - \left(E_{t-1}\frac{1}{\theta_t} - E_t\frac{1}{\theta_{t+1}}\right) \tag{22}$$

Proof. *See appendix* ∎

The exact amount of the interest rate reduction depends on three factors: i) the direct negative effect from the difference in inverse LTV ratio, $\frac{1}{\beta}\left(\frac{1}{\theta_t} - \frac{1}{\theta_{t-1}}\right)$; ii) the surprise effect or the difference between actual and expected value $s_t = \left(\frac{1}{\theta_t} - E_{t-1}\frac{1}{\theta_t}\right)$, and iii) the expected change, $e_t = \left(E_t\frac{1}{\theta_{t+1}} - \frac{1}{\theta_t}\right)$

$$r_t = r_{t-1} - \frac{1}{\beta}\left(\frac{1}{\theta_t} - \frac{1}{\theta_{t-1}}\right) + s_t + e_t \tag{23}$$

For example, in an extremely myopic case when the fall in LTV is completely unexpected, $E_{t-1}\frac{1}{\theta_t} = \frac{1}{\theta_{t-1}}$, the surprise effect is big, $s_t = \frac{1}{\theta_t} - \frac{1}{\theta_{t-1}}$, however it is smaller than a negative direct effect. In a less extreme case the surprise effect should be smaller. The expectation effect depends on how permanent the fall in LTV is expected to be. Consider the extreme case when the public expects the LTV to return back to its original value, $E_t\frac{1}{\theta_{t+1}} = \frac{1}{\theta_{t-1}}$. In this case the expectation and surprise effects cancel each other and the optimal change in interest is

$$r_t = r_{t-1} - \frac{1}{\beta}\left(\frac{1}{\theta_t} - \frac{1}{\theta_{t-1}}\right). \tag{24}$$

While policy (22) stabilises the economy perfectly, it is subject to the zero lower bound constraint on interest. Consider case (24) with the data of September 2008. The LTV ratio before the subprime mortgage crisis was above 95%, and then it sudden dropped below 85%. According to (24) the corresponding reduction of the interest rate would exceed 12%. However, the pre-crisis interest rate was below 5% and the optimal reduction was impossible without hitting the zero lower bound.

When the fall of the LTV is significant the interest rate adjustment can be insufficient to stabilise the economy due to the ZLB constraint. In that case, additional expansionary monetary policy is required through non-conventioanl methods.

10

In the next section we will show that the monetary authorities can mitigate a negative credit shock with direct monetary expansion, $T_t > 0$. The conventional interest rate policy is easier to implement, however it can be restricted by ZLB condition. The non-conventional policy of QE raises lots of questions about its technical implementation: Which assets the central bank should purchase? How much risk it should put on the central bank balance sheet? how easy will it be to reverse QE in the future? In the opposite situation, when the LTV ratio increases, the direct monetary contraction could raise even more implementation issues. Whether the government should increase the interest rate or sell unconventional assets in this case is an interesting topic for future research.

2.5 Linear Quadratic Model

In order to make our work easier, we linearize the constraints of private behavior presented in equations (13-14)[3]. In the appendix, we show that the optimal steady state is achieved under price stability, i.e. $\pi = 1$. Applying the Benigno and Woodford (2012) method to the non-linear problem (12, 13-20), we receive a pure quadratic approximation of the social objective. As we can see, it consists of the squares of output and inflation gaps similar to those of Benigno and Woodford (2005),

$$U_t = -\frac{1}{2} E_t \sum_{s=1}^{\infty} \beta^s (\alpha_C \left(\widehat{Y}_{t+s} + \alpha_\phi \widehat{\phi}_{t+s} \right)^2 + \alpha_\pi \widehat{\pi}_{t+s}^2) + O3 + tip, \tag{25}$$

where α_C and α_π are the policy maker's preferences towards the output gap and inflation, respectively, $-\alpha_\phi \widehat{\phi}_t$ is the target level of output, which is inversely related to the cost push shock and tip denotes the terms that are independent of the policy maker's choices. Coefficients α_C, α_π and α_ϕ are all positive and computed in the Appendix.

2.5.1 Private-sector behavior constraints

Linearized versions of equations (14-16) can be combined to form a New Keynesian Phillips Curve (26). A linearization of equation (17) shows that the relative price dispersion term is of second-order importance and can be ignored. Finally, (18-20) can be combined into (28), which

---

[3]See appendix 7.2-7.4.

relates the monetary policy instrument, T_t, to inflation and output. Therefore, as a result of simplification and linearization, we obtain a reduced system of three equations

$$\widehat{\pi}_t = \frac{1-a}{a}(1-\alpha\beta)\left[(v+2)\widehat{Y}_t + \widehat{\phi}_t\right] + \beta E_t\widehat{\pi}_{t+1};$$ (26)

$$\beta\widehat{q}_{t+1} = \widehat{q}_t(1-\theta(1-\beta(1+r))) - \widehat{\theta}_t\theta(1-\beta(1+r)) + r\beta\theta\widehat{r}_t + \beta\theta(1+r)\left(\widehat{Y}_t - E_t\widehat{\pi}_{t+1} - E_t\widehat{Y}_{t+1}\right);$$ (27)

$$\widehat{\pi}_t = \widehat{Y}_{t-1} - \widehat{Y}_t + \frac{\theta q}{1-\theta q}\left(\widehat{\theta}_t - \widehat{\theta}_{t-1} + \widehat{q}_t - \widehat{q}_{t-1}\right) + \widehat{T}_t,$$ (28)

where variables with hats denote percentage deviations from the steady state, and variables without time subscript denote the steady state values.

Expression (28) is the essence of the paper: whatever is the target for inflation and output dynamics, one cannot neglect the fluctuations in relative house value, $\widehat{q}_t$, or credit availability, $\widehat{\theta}_t$. In other words, for given dynamics of $\widehat{\pi}_t$ and $\widehat{Y}_t$, monetary policy, $\widehat{T}_t$, should be adjusted to the shock in the credit constraint and the change in relative house expenditure ($\widehat{q}_t$).

Our objective is to find the first-order approximation to the optimal policy reaction function. We will allow two shocks to perturb our economy: a cost-push shock, $\widehat{\phi}_t$, and a credit shock, $\widehat{\theta}_t$. We assume that $\widehat{\phi}_t$ and $\widehat{\theta}_t$ follow two independent $AR(1)$ processes

$$\widehat{\theta}_t = \rho_\theta\widehat{\theta}_{t-1} + \xi_{\theta t};$$ (29)

$$\widehat{\phi}_t = \rho_\phi\widehat{\phi}_{t-1} + \xi_{\phi t}.$$ (30)

The linear approximation to optimal policy can be found by maximizing the second-order approximation to social welfare (25), subject to linear constraints (26)-(28).

3 Optimal monetary policy

3.1 Reaction to credit shock

Consider the case when only credit shocks hit the economy, that is when the price markup is constant, $\widehat{\phi}_t = 0$. If the credit shock is the only source of instability, the government can achieve zero losses, perfectly stabilizing both output and inflation. We formalize this statement in Proposition 2.

Proposition 2 *In the absence of the cost-push shock, credit market contraction can be perfectly neutralized. Indeed, the policy maker can achieve perfect price and output stabilization, i.e. $\widehat{\pi}_t = 0$ and $\widehat{Y}_t = 0$.*

Proof. *If $\widehat{\phi}_t = 0$, output and price stability are not in contradiction with the system of constraints (26)-(28).* ∎

Note that complete price and output stabilization delivers the maximum value of social welfare (25). Hence, the corresponding policy is optimal and the optimal monetary policy rule in this case follows from equation (28) if inflation and output deviations are set to zero.

$$\widehat{T}_t = -\frac{\theta q}{1 - \theta q}\left(\widehat{\theta}_t - \widehat{\theta}_{t-1} + \widehat{q}_t - \widehat{q}_{t-1}\right). \tag{31}$$

As a result, we obtain an example where the credit constraint, $\widehat{\theta}_t$, and the collateral price, $\widehat{q}_t$, are not directly targeted by the government. However, they are the only arguments in the government's reaction function. This is exactly the case discussed in Svensson (2009). The social planner does not care about the financial sector per se, but since it affects inflation and output volatilities, the policy maker must consider the change in the financial environment when it implements its monetary policy.

The optimal monetary policy rule (31) has a straightforward interpretation. Recalling the collateral constraint (3) and taking price and output stability into consideration, it may be written as

$$\widehat{T}_t = k\left(\widehat{B}_{t+1} - \widehat{B}_t\right), \tag{32}$$

where we define $k = \frac{\theta q}{1 - \theta q}$. Coefficient k has an important economic meaning. In the steady state, the collateral constraint (3) implies that $\theta q PY = B$, while the cash-in-advance constraint (2) implies that $PY = M + \theta q PY$. Combining those two expressions, the debt to money ratio can be computed

$$B = kM.$$

Therefore, k is the marginal effect on loans of a change in the base money. In other words, a 1 dollar expansion of the monetary base will create k dollars of loans: $k = \frac{dB}{dM}$. Equation (32) tells us how much the central bank should expand its monetary base. The expansion should just be sufficient to offset the reduction in debt capacity.

It is noteworthy that optimal policy in the simple framework without cost-push shocks can be reduced to two equations

$$\beta E_t \widehat{q}_{t+1} = \widehat{q}_t(1 - \theta(1 - \beta)) - \widehat{\theta}_t\theta(1 - \beta)); \tag{33}$$

$$\widehat{\theta}_{t+1} = \rho_\theta\widehat{\theta}_t - \xi_{\theta t+1}. \tag{34}$$

Blanchard and Kahn (1980) formulated the necessary conditions for the dynamic linear R.E. system to have a unique solution. It states that there must be the same number of eigenvalues larger than 1 in modulus as there are forward looking variables. To satisfy this condition, the following relation is necessary and sufficient, $\frac{1-\theta(1-\beta)}{\beta} > 1$, which is true if and only if $\theta < 1$.

In case $\theta > 1$, we will have indeterminacy and sunspot equilibria. In practice, $\theta > 1$ was observed in 2006-2007, when new mortgages were available with up to a 110% loan to value ratio. Such a high LTV might have been partly responsible for the house bubble and the subsequent financial crisis. Thus, according to Korteweg and Sorensen (2012), LTV significantly contributed to the probability of foreclosure sales. In this light, the suggestion of the Hong Kong Monetary Authority (2011) to use LTV as a policy tool for the macroprudential regulation seems to have some justification. For the sake of stability, we calibrate the steady-state value of θ to be less than 1.

3.2   Cost of inactive government

In this section, we will numerically assess the value of monetary policy. For this purpose, we will compare optimal policy generating (39) with a policy that neglects changes in the credit market. Our alternative policy is

$$\widehat{T}_t = 0. \tag{35}$$

If a negative credit shock hits the economy and the government does not provide any monetary response to that shock, $\widehat{T}_t = 0$, both deflation and a significant fall in GDP would be expected.

Figure 1. Inflation and output:

Impulse response to a negative credit shock, $\widehat{T}_t = 0$.
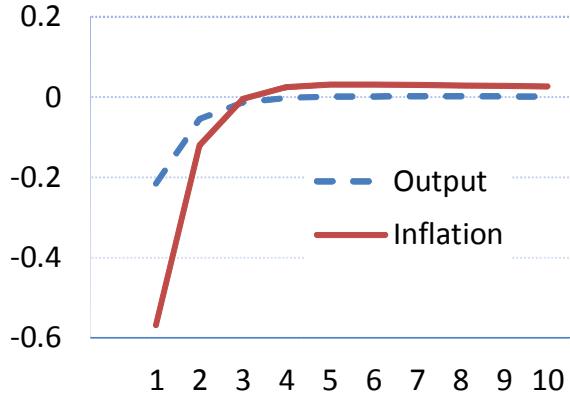
Figure 1 shows that a 1% drop in the loan to collateral value ratio reduces output by 0.2%. If our model is reasonably calibrated, a 20% drop in the mortgage LTV ratio will result in a 4% fall in GDP in the absence of quantitative easing. This would be even more damaging for consumer prices. In the absence of monetary transfers, the model economy will experience a 12% deflation.

3.3 Credit Shocks and the Money Multiplier

It is well known that the money multiplier fell dramatically after the recent financial crisis. Now, the monetary authorities have to expand the monetary base to a much larger extent in order to achieve the same expansion of broad money. The importance of the money multiplier is discussed in Goodhart (2009) who criticizes the macroeconomic literature for ignoring the money multiplier and for failing to formally model it. This criticism is not entirely fair, since the behavior of the money multiplier was a popular research topic in the 1990s. See, for example, Bernanke (1983), Bernanke and Blinder (1988), Beenstock (1989) and, more recently, Freeman and Kydland (2000). However, as the money multiplier was relatively stable for more than 20 years, it became a concern of second-order importance. Although the model we consider is very simple, it manages to identify two variables which may explain the fluctuation in the money multiplier as it is computed in equation (21). First there is θ_t, the household's borrowing constraint. If we simply consider mortgage contracts offered before the crisis, the loan to value ratio was up to 110% in the UK. After the crisis, it fell to 90% or by almost 20%.

The second variable, q_t, is defined as
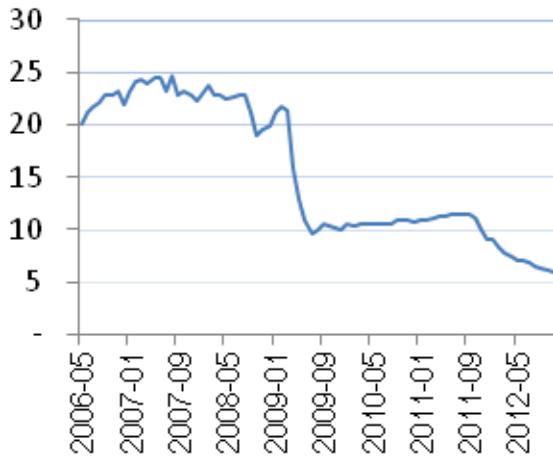
$$q_t = \frac{Q_t h_t}{P_t Y_t}. \tag{36}$$

In a relatively stable economy, where h and Y do not change, the proxy for q will be the real price for collateral. If we refer to the mortgage market, the collateral is houses and the real house price index will be a proxy for q. Therefore, the fall in house prices should reduce the money multiplier. As the money multiplier is significant for the transmission of monetary policy (Bernanke and Blinder, 1988; Goodhart, 2009; Abrams, 2011), its fluctuation should definitely be taken into account when monetary policy is designed. As house prices and the loan to value ratio affect the money multiplier, they cannot be neglected by the monetary authorities.
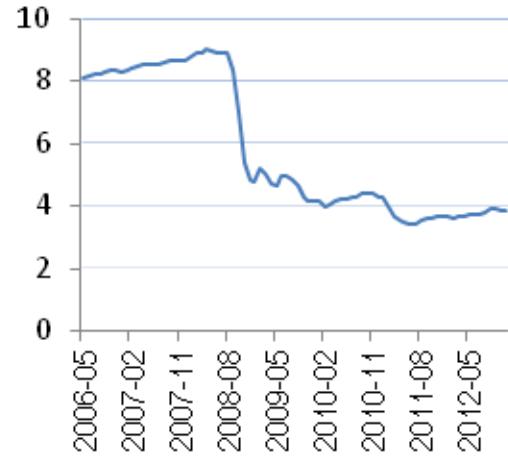
As we have previously noted, the money multiplier experienced a significant fall after the last financial crisis. Figure 2 shows the dynamics of the M2 multiplier in the US and the M4 multiplier in the UK.

Figure 2. Money multiplier after the Crisis

UK: M4/MO   USA: M2 multiplier
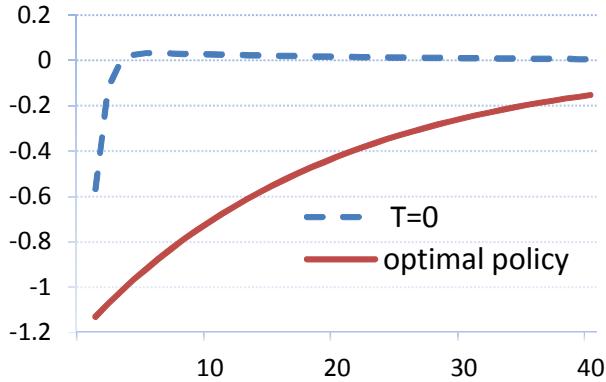
Source: FRED and BoE database

The slump of the money multiplier is consistent with our model. Although optimal monetary policy can stabilize output and price fluctuations, it causes an even stronger and much more persistent decrease in the money multiplier than a policy of inaction. Both policies imply a

16

dramatic fall in the money multiplier as we can see from Figure 3, but the stabilization policy almost doubles the size of the fall and causes a much slower recovery.

Figure 3. Money multiplier IRFs to negative credit shock

We draw Figure 3 based on a linear approximation to (21)

$$\widehat{m_t} = k(\widehat{\theta}_t + \widehat{q}_t). \tag{37}$$

It can be easily seen that the optimal policy rule in the absence of a cost-push shock (31) implies that optimal transfers should be equal to the negative of the change in the money multiplier

$$\widehat{T}_t = -(\widehat{m}_t - \widehat{m}_{t-1}). \tag{38}$$

Therefore, Figure 3 shows that optimal policy entails large expansion immediately after the reduction in LTV and then a gradual contraction in all subsequent periods.
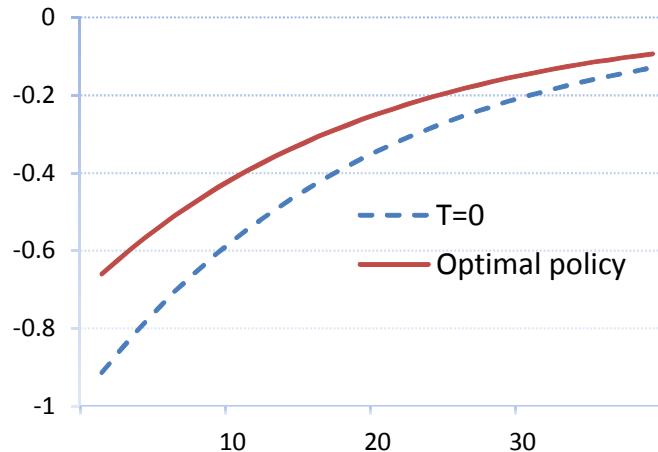
3.4 Credit shock and house prices

It is very intuitive that the relative price of collateral should react to the worsening in the loan to value ratio. The value of collateral in our model has two components. The first comes directly from the utility function (such as housing, for example). The second is indirect and associated with the use of collateral for borrowing purposes. The larger is LTV θ_t, the larger is the indirect component of the collateral value and therefore, the higher is the price of collateral.

17

Formally, this can be observed from equation (6). This is why the negative shock to θ_t should result in falling house prices.

Lower prices for collateral, in turn, further reduce the amount of available credit. As a consequence, households have less money to finance their consumption and purchase additional housing units. Figure 4 shows how house prices react to the tightening of the households' borrowing constraint in two different cases. The first case is when the government implements the optimal policy rule. The second case is when it keeps the monetary base constant, $\widehat{T}_t = 0$. When a negative credit market shock hits the economy, house prices decline in both cases, but optimal policy helps reduce the fall by approximately 20%.

Figure 4: House prices IRFs to negative credit shock.

3.5   Optimal policy with cost-push shock

Now we will consider an economy with a cost-push shock. The optimal policy in this case generates the same dynamics for the output gap and inflation as the basic New Keynesian model and is presented in Proposition 3.

Proposition 3 *Optimal policy implies the following inflation dynamics*

$$\widehat{\pi}_t = \rho \left(\widehat{Y}_{t-1} - \widehat{Y}_t - \alpha_\phi \left(\widehat{\phi}_{t-1} - \widehat{\phi}_t \right) \right), \tag{39}$$

where $\rho = \frac{\alpha_C}{\alpha_\pi} \frac{a}{(1-\alpha\beta)(v+2)(1-a)}$.

Proof. *Provided in the Appendix.* ∎

Proposition 3 shows that optimal inflation dynamics (39) should be the same as in the basic new Keynesian model of Clarida, Gali and Gertler (2000), which does not feature any credit constraint. However, to achieve these dynamics, the monetary policy has to react to the change in the credit conditions

$$\widehat{T}_t = -\widehat{\pi}_t - \left(\widehat{Y}_t - \widehat{Y}_{t-1} \right) - \frac{\theta q}{1 - \theta q} \left(\widehat{\theta}_t - \widehat{\theta}_{t-1} + \widehat{q}_t - \widehat{q}_{t-1} \right). \tag{40}$$

3.5.1   Cost-push shock, house prices and inflation

As expected, the optimal policy stabilises inflation. However it would be interesting to see how the cost-push shock will affect house prices and relative housing expenditure. When the loan to value ratio is constant and only cost-push shocks hit the economy, there is a trade off between inflation and house price stability. The policy of inaction, $\widehat{T}_t{=}0$, in the absence of credit shocks implies constant house prices as stated in Proposition 4.
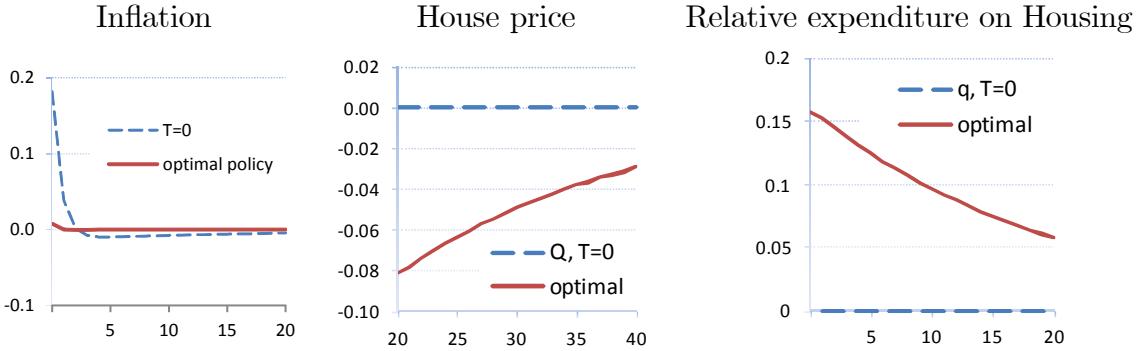
Proposition 4   *If $\widehat{\theta}_t = 0$, policy $\widehat{T}_t = 0$ results in house price stability and the stability of the relative collateral value, $\widehat{q}_t = 0$.*

Proof. *Provided in the Appendix.* ∎

Figure 5 shows that in the absence of credit shocks, the stabilization of house prices will result in a higher volatility of CPI inflation. This result is consistent with the findings of Iacoviello (2005).

It is interesting to see that the money multiplier, which is proportional to relative housing expenditure, increases with the cost push shock, when optimal policy is implemented. However, house prices decline in this case, but to a smaller extent than the corresponding decline in output.

Figure 5: Cost-push shock IRFs

4 Loan to value ratio

We have shown that the government ought to include the loan to value ratio (q_t) in its policy rule because it delivers the highest welfare measure. In that discussion, we considered θ_t to be exogenously given. Perhaps one of the most important questions is to identify the factors which explain the fluctuation in θ_t.

Endogenising the loan to value ratio can have a number of very important policy implications, for example if the LTV ratio equals the effective recovery rate of mortgages, so that direct lending to households compromises the balance sheet of the central bank. In that case, Help to Buy and Start Up loans will result in budget losses and a Funding for Loans scheme could in this case result in yet more non-performing loans on the central bank's balance. We are not aware of any model which can assess these consequences.[4]

Although the recovery rate is highly correlated with default risk (Mora, 2012), there are some other explanatory factors which can be influenced by the government.

4.1 Expected collateral inflation

One of the explanatory variables of changes in LTV may be expected house price inflation (Q_t) as in Iacoviello (2005). This can easily be modelled by substitution of (29).

$$\widehat{\theta}_t = \delta E_t \left(\widehat{Q}_{t+1} - \widehat{Q}_t \right) + u_t, \tag{41}$$

---

[4]In our model, the central bank simply increases its liabilities in the form of outstanding cash without any back-up on the asset side.

where $\delta < 1$, and u_t is a persistent shock unrelated to expected changes in house prices.

Shocks u_t can in this case be regarded as a shock to expected future house prices which is another factor for consideration for the monetary authorities. In some cases, shocks to expectations do not reflect the changes in fundamentals. The central bank will not compromise its balance sheet by buying collateral and keep it for a longer time period until the negative shock dies out.

In this modified model, we received a very similar impulse response function to the unexpected change in u_t and still we found that an optimal policy can completely stabilize output and inflation when the credit shock affects the economy. Similarly, in response to cost-plush shocks, policy $\widehat{T}_t = 0$ stabilizes house prices but causes a positive and relatively large response of the consumer price inflation.

4.2 Liquidity, transaction costs and the value of collateral

An increase in the collateral value is an effective way to raise the money multiplier. The attractiveness of collateral may increase with liquidity. A positive relation between the liquidity of collateral and the availability of funding is discussed in Brunnermeier and Pedersen (2009). Securization of collateral is one way of enhancing liquidity. Different types of securitization were used for the American housing market. Thus, according to Frame and White (2005), the Federal Home Loan Mortgage Corporation – commonly known as Freddie Mac – was created to support the mortgage markets by securitizing mortgages. That arrangement worked successfully for at least 30 years before the last crisis. The Funding for Lending scheme is in line with that reasoning. The possibility of a collateral swap for T-Bills should increase the value of collateral which can be used by commercial banks to secure liquidity.

However, collateral securitization could also add to risk and even generate additional moral hazard problems (Ashcraft and Schuermann, 2008). In this context, the Freddie Mac arrangements were safer than the new measures proposed in the UK. The mortgages that were eligible for securitization with Freddie Mac usually required a 20% downpayment, while the Funding for Lending Scheme does not specify the quality of the loans that can be used in the scheme (Bank of England, 2012). British "Help to Buy" may be even riskier, such that the government will "loan up to 20% of the value of your new build home and "mortgage guarantee" where

lenders will be incentivised to make more mortgages available for people with small deposits."
(HM Treasury, 2013). It could mean that the government intends to provide for the risk margin
charged by the lender as an insurance against construction risks.

Apart from the expected value of collateral, the loan to value ratio should depend on the
recovery rate for non-performing loans. The recovery rate negatively depends on the transaction
costs associated with selling the repossessed assets. Any taxes collected during that process
would negatively contribute to LTV. One straightforward recommendation can be to abolish
stamp duties for repossessed properties.

The other way of increasing the value of collateral is to encourage the construction and
consumption of housing. According to Frame and White (2005), the US government uses a
tax deduction of the mortgage interest and accelerated depreciation on rental housing for that
purpose.

5 Preference for Money

Michael Woodford (2012) has suggested that the assumption about perfect substitutability be-
tween financial instruments could exclude from the model some very important economic real-
ities. One way to make money and loans imperfectly substitutable is to model real money as a
part of the preference function[5]. The intuition is standard: people may prefer to hold money
as it is a liquid asset that can be used for certain transactions where it may be difficult to
use credit; i.e. rent payment or home repairs (See Telyukova and Wright, 2008). This would
justify an additional positive value of cash money compared to credit. However, there are other
occasions when money may be less desired. For example, in electronic transactions, especially
when payment is made to an unknown supplier, credit cards are safer to use than debit cards[6].
This is just one example to demonstrate that as financial market develops the preference for
cash may decline.

To capture imperfect substitutability between cash and credit, we modify household prefer-

---

[5]We are grateful to the referee for that interesting idea.

[6]Section 75 of the Consumer Credit Act 1974 states that a cardholder's credit card company jointly liable
with the merchant for any purchases made on a credit card between £100 and £30,000.

ence (1) in the following way

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t u(Y_t, h_t, L_t) = E_t \sum_{t=0}^{\infty} \beta^t \left(\log Y_t + \eta \log h_t + \xi \log \left(\frac{M_t}{P_t} \right) - \lambda \frac{L_t^{v+1}}{v+1} \right), \qquad (42)$$

where parameter ξ reflects the preference of using money. As before, the household maximises its utility subject to constraints (2, 3 and 5). The first order conditions will be slightly different.

5.1 Labour supply

First, the labour supply equation (9) of the main model will become (43)

$$-U'_{L,t} = \frac{W_t}{P_t} \left(U'_{m,t} + U'_{c,t} \right), \qquad (43)$$

where U'_m is the marginal utility from real money. That can not be derived from a conventional money in utility model without credit constraint[7], where labour supply equation (9) is usual. The difference is that in our model households can not borrow cash. The household can only increase money holding by supplying more labour. As money is used for consumption, it gives double benefits: through utility from consuming good and through pleasure from possessing money. Therefore, households supply more labour when they have a greater preference for cash[8].

5.2 Credit constraint

It is even more interesting see how the condition when the credit constraint is binding will change. Equation (7) is becomes

$$\mu_{2t} P_t = U'_{ct} - \frac{\beta(1+r_t)}{\pi_{t+1}} \left(U'_{m,t+1} + U'_{c,t+1} \right) > 0, \qquad (44)$$

---

[7]See for example Walsh (2010), ch 2.

[8]For a particular preference form (42) equation (43)

$$\lambda L_t^v = w_t \left(\xi/(M_t/P_t) + 1/Y_t \right)$$

and the labour supply increases with ξ and declines with real money.

23

where μ_{2t} is the Lagrange multiplier to constraint (3). The probability that the constraint is binding declines with the value of U'_m. In particular, if U'_m is large the constraint can be non-binding, in contrast to the case when $U'_m = 0$.

Now we will investigate when the credit constraint is binding in a steady state. Condition (44) becomes

$$\left(1 - \frac{\beta(1+r)}{\pi}\right) U'_c - \frac{\beta(1+r)}{\pi} U'_m > 0. \tag{45}$$

Consider the case when r is sufficiently small, $1 - \frac{\beta(1+r)}{\pi} > 0$. Then, as we noticed above, inequality (45) is satisfied if $U'_m = 0$. As U'_m increases, the constraint may became non-binding. We can compute a threshold such that if preferences towards money grow stronger than that value, the credit constraint becomes non-binding.

Proposition 5 *If the real interest rate is smaller than the inverse discount factor, $\frac{(1+r)}{\pi} < \frac{1}{\beta}$, There exists a value ξ^*, such that inequality (45) is satisfied for preferences as in (42), when $0 \leq \xi < \xi^*$, where*

$$\xi^* = \left(1 - \frac{\theta\eta}{1-\beta}\right) \frac{1 - \frac{\beta(1+r)}{\pi}}{\frac{\beta(1+r)}{\pi}}. \tag{46}$$

Proof. *See appendix.* ∎

In Proposition 5 we define the value of parameters under which the credit constraint is binding in steady state. The larger is ξ^* the higher is the probability that the constraint is binding. From formula (46) we directly obtain Corollary 6.

Corollary 6 ξ^* *increases with i) inflation, π; and*
ξ^* *declines with ii) nominal interest rate, r; iii) real interest rate, $\frac{(1+r)}{\pi}$; iv) household patience, β; v) Loan to value ratio, θ; vi) intristic value of housing, η.*

Corollary 6 helps to understand the evolution of the importance of the credit constraint over time. It could have been that the LTV ratio were not so important in the past because the real interest rate was high and thus ξ^* was low. At the same time, the financial market was less developed and borrowing practice was less socially acceptable, hence ξ was high. Perhaps, it was a time when ξ was larger than ξ^* and the borrowing constraint was non-binding. As a consequence the loan to value ratio was not so important. According to Peñaloza and Barnhart

(2011) the culture of debt/credit payment became more acceptable overtime and, with further development of the financial market, hence ξ might have fallen below ξ^* making the LTV ratio so vital.

6   Conclusion

We have shown that in contrast to some other findings presented in the literature, such as Bernanke and Gertler (2001) and Iacoviello (2005), a simple stylized macro model could yield results which are in favor of including a credit market variables into the optimal monetary policy rule. In particular, monetary authorities should adjust their rule to unfavorable changes in loan conditions, such as a fall in the loan to value ratio or the relative price of collateral, i.e. the real house price index.

We derived our results in a model where social welfare consists of output and inflation gaps and thus, credit market variables are not part of the government direct objective. However, as mentioned in Svensson (2009), credit capacities affect output and inflation through households' behavioral constraints. A lower households' borrowing rate reduces demand and, as a result, causes deflation. We have shown that an exogenous decrease in the loan to value ratio can be offset by expansionary monetary policy in such a way that credit tightening will neither affect output nor the consumer price inflation.

We connected our results to the money multiplier which is the most important variable in propagation of monetary policy, as discussed in Bernanke and Blinder (1988) and recently in Abrams (2011). Indeed, it is very intuitive that the expansion of the monetary base should be larger when the money multiplier falls. And since the multiplier depends on the loan to value ratio and real house prices, optimal monetary policy should react to their fluctuations. Finally, we have shown that optimal policy generates a large and persistent fall in the money multiplier in response to credit shocks.

Although our model is helpful in providing some justification for quantitative easing and explaining the fall in the money multiplier, there are a number of important extensions that should be addressed. First, the volatility of the loan to value ratio requires an economic explanation. This would allow for a better assessment of a number of currently proposed or adopted policy measures. Second, the assets of the central bank are not modelled directly and,

therefore, it is impossible to see how the risk taken on the central bank balance sheet will affect the economy. If assets are risky, the central bank will face difficulties when it decides to implement a monetary contraction.

Finally, the money multiplier per se does not generate any value in our model, and the steady-state output value does not depend on any financial variables. This is not the case according to King and Levine (1993) and Freeman and Kydland (2000) who found that total borrowing by the non-financial sector has a positive effect on economic growth.

References

[1] Abrams, B. A., 2011. Financial-sector shocks in a credit-view model, *Economics Letters*, Elsevier, vol. 112(3), pp. 256-258.

[2] Adam, Klaus & Billi, Roberto M., 2007. "Discretionary monetary policy and the zero lower bound on nominal interest rates," Journal of Monetary Economics, Elsevier, vol. 54(3), pages 728-752, April.

[3] Ashcraft, B., Schuermann, T., 2008. Understanding the Securitization of Subprime Mortgage Credit, Staff Reports 318, Federal Reserve Bank of New York.

[4] Bank of England, 2012. Funding for Lending Scheme, Market Notice.

[5] Beenstock M., 1989. The Determinants of the Money Multiplier in the United Kingdom, *Journal of Money, Credit and Banking*, vol. 21, No. 4, pp. 464-480.

[6] Benigno, P. and Nisticò, S. 2013. 'Safe Assets, Liquidity and Monetary Policy'. London, Centre for Economic Policy Research, DP9767.

[7] Benigno, P. and Woodford, M. 2005. Inflation Stabilization And Welfare: The Case Of A Distorted Steady State, *Journal of the European Economic Association*, vol. 3, pp. 1185-1236.

[8] Benigno, P. and Woodford, M. 2012. Linear-quadratic approximation of optimal policy problems, *Journal of Economic Theory*, vol. 147, pp. 1-42.

[9] Bernanke, B., S., 1983. Nonmonetary Effects of the Financial Crisis in Propagation of the Great Depression, *American Economic Review*, vol. 73(3), pp. 257-76.

[10] Bernanke, B., S., Reinhart, R., V., Sack, B., P., 2004. Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment, *Brookings Papers on Economic Activity*, vol. 35(2), pp. 1-100.

[11] Bernanke, Ben S & Blinder, Alan S, 1988. Credit, Money, and Aggregate Demand, *American Economic Review*, vol. 78(2), pages 435-39, May.

[12] Bernanke, B. and Gertler, M. 1989. Agency Costs, Net Worth and Business Fluctuations, *American Economic Review,* vol. 79, No. 1, pp. 14-31.

[13] Bernanke, B. and Gertler, M. 2001. Should Central Banks Respond to Movements in Asset Prices?, *American Economic Review*, vol. 91, pp. 253-257.

[14] Bernanke, B. and Mishkin, F. 1997. Inflation Targeting: A New Framework for Monetary Policy?, *Journal of Economic Perspectives*, vol. 11, pp. 97-116.

[15] Blanchard, O. and Kahn, C., M. 1980. The Solution of Linear Difference Models under Rational Expectations, *Econometrica*, vol. 48, pp. 1305-1313.

[16] Brunnermeier M., K., Eisenbach, T., M. and Sannikov, Y., 2012, Macroeconomics with Financial Frictions: A Survey, *NBER Working Papers* 18102.

[17] Brunnermeier, M., K. and Pedersen, L., H., 2009, Market Liquidity and Funding Liquidity, *The Review of Financial Studies*, vol. 22, pp. 2201-2238.

[18] Calvo, G., A. 1983. Staggered Prices in an Utility-Maximizing Framework, *Journal of Monetary Economics*, vol. 12, pp. 383-398.

[19] Carlstrom, C., T., Fuerst, T., S., 1995. Interest Rate Rules vs. Money Growth Rules a Welfare Comparison in a Cash-in-advance Economy, *Journal of Monetary Economics*, vol. 36, pp. 247-267.

[20] Clarida, R., H. 2012. What Has - And Has Not - Been Learned About Monetary Policy in a Low-Inflation Environment?, *Journal of Money, Credit and Banking,* vol. 44, pp. 130-140.

[21] Clarida, R., G., J., Gertler, M. The Science of Monetary Policy: A New Keynesian Perspective, *Journal of Economic Literature,* vol. 37, pp. 1661-1707.

[22] Coibion, Olivier, Yuriy Gorodnichenko & Johannes Wieland, 2012. "The Optimal Inflation Rate in New Keynesian Models: Should Central Banks Raise Their Inflation Targets in Light of the Zero Lower Bound?," Review of Economic Studies, Oxford University Press, vol. 79(4), pages 1371-1406.

[23] Dell Ariccia, G., Detragiache, E., Rajan, R., 2008. The Real Effect of Banking Crises, *Journal of Financial Intermediation*, vol. 17, pp. 89-112.

[24] Dixit, A. and Stiglitz, J. 1977. Monopolistic Competition and Optimum Product Diversity, *American Economic Review,* vol. 67, pp. 297-308.

[25] Fisher, I., 1933. The Debt-Deflation Theory of Great Depressions, *Econometrica*, vol. 1, pp. 337-357.

[26] Frame, W., S, Lawrence J., W., 2005. Fussing and Fuming over Fannie and Freddie: How Much Smoke, How Much Fire?, *Journal of Economic Perspectives*, vol. 19, pp. 159-184.

[27] Freeman, S., Kydland, F., E. 2000. Monetary Aggregates and Output, *American Economic Review*, vol. 90(5), pages 1125-1135, December.

[28] Friedman, M., 2000. "Canada and Flexible Exchange Rates", the keynote speech on "Revisiting the Case for Flexible Exchange Rates - 2000 conference", Bank of Canada.

[29] Friedman, M., 2006. Why Money Matters", Wall Street Journal, November 17th.

[30] Gambacorta, L., Hofmann, B. and G. Peersman, 2012, The Effectiveness of Unconventional Monetary Policy at the Zero Lower Bound: A Cross-Country Analysis , *BIS Working Papers* 384, Bank for International Settlements.

[31] Goodhart C. A. E. , 2009, The Continuing Muddles of Monetary Theory: A Steadfast Refusal to Face Facts, *Economica*, vol. 76, pp. 821-830.

[32] Hildebrand, M., P. 2006. Monetary Policy and Financial Markets, *Financial Markets and Portfolio Management*, vol. 20, pp. 7-18.

[33] HM Treasury, 2013, Budget 2013.

[34] Hong Kong Monetary Authority, 2011, Loan-to-value ratio as a macroprudential tool - Hong Kong SAR's experience and cross-country evidence, BIS Papers chapters, in: Bank for International Settlements (ed.), Capital flows, commodity price movements and foreign exchange intervention, volume 57, pages 163-178.

[35] Iacoviello, M. 2005. House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle, *American Economic Review,* vol. 95, pp. 739-764.

[36] King, R., G. and Levine, R., 1993, Finance and Growth: Schumpeter Might Be Right, *The Quarterly Journal of Economics*, vol. 108, pp. 717-37.

[37] Korteweg, A. and Sorensen, M., 2012, Estimating Loan-to-Value and Foreclosure Behavior, NBER Working Papers 17882.

[38] Kosuke, A., Proudman, J. and Vliegbe, J. 2004. House Prices, Consumption and Monetary Policy: A Financial Accelerator Approach, *Journal of Financial Intermediation*, vol. 13, pp. 414-435.

[39] Levine, Paul & Pearlman, Joseph & Pierse, Richard, 2008. "Linear-quadratic approximation, external habit and targeting rules," Journal of Economic Dynamics and Control, Elsevier, vol. 32(10), pages 3315-3349, October.

[40] Monacelli, Tommaso, 2009, New Keynesian models, durable goods, and collateral constraints, Journal of Monetary Economics, Elsevier, vol. 56(2), pages 242-254, March.

[41] Mora, Nada, 2012., What determines creditor recovery rates?, Economic Review, Federal Reserve Bank of Kansas City, issue Q II.

[42] McCallum, B., T., 1985, On Consequences and Criticisms of Monetary Targeting, *Journal of Money, Credit and Banking*, vol. 17, pp. 570-97.

[43] Midrigan, V., and Philippon, T. 2011. Household Leverage and the Recession, *NYU Working Paper*, No FIN-11-038.

[44] Mishkin, S., F. 2011. Monetary Policy Strategy: Lessons from the Crisis, *NBER Working Paper,* No. 16755.

[45] Peersman, G., Smets, F. 1999. The Taylor Rule: A Useful Monetary Policy Benchmark for the Euro Area?, *International Finance,* vol. 2, pp. 85-116.

[46] Peñaloza Lisa and Michelle Barnhar, Living U.S. Capitalism: The Normalization of Credit/Debt, Journal of Consumer Research, Vol. 38, No. 4 (December 2011), pp. 743-762

[47] Telyukova, Irina A. & Randall Wright, 2008. A Model of Money and Credit, with Application to the Credit Card Debt Puzzle, Review of Economic Studies, vol. 75(2), pages 629-647.

[48] Taylor, J., B. 1993. Discretion versus Policy Rules in Practice, *Carnegie-Rochester Conference Series on Public Policy*, December 1993, vol. 39, pp. 195-214.

[49] Thorbecke, W. 2012. On Stock Market Returns and Monetary Policy, *The Journal of Finance,* vol. 52, pp. 635-654.

[50] Svensson L. 2009. "Flexible Inflation Targeting: Lessons from the Financial Crisis" Speech Given at the Workshop "Towards a New Framework for Monetary Policy? Lessons from the Crisis." Amsterdam: Organized by De Nederlandsche Bank.

[51] Walsh, C. 2010, , Monetary Theory and Policy, MIT Press, 3d edition.

[52] Woodford, M. 2001. The Taylor Rule and Optimal Monetary Policy, *American Economic Review*, vol. 91, pp. 232-237.

7 Appendix

7.1 First-order conditions for household optimization

The household maximises the expected discounted sum of future utility (1) subject to constraints (2, 3, 5). The corresponding Lagrangian is:

$$
\begin{aligned}
\mathcal{L} = \ & E_t \sum_{t=0}^{\infty} \beta^t [(U(C_t, L_t, h_t) \\
& + \mu_{1t}(-P_t C_t - Q_t(h_t - h_{t-1}) + B_t + M_t) \\
& + \mu_{2t}(-B_t + \theta_t Q_t h_t,) \\
& + \mu_{3t}(-M_t + W_t L_t + \Pi_t + T_t - (1 + r_{t-1}) B_{t-1})];
\end{aligned}
$$

where μ_{it} are Lagrange multipliers. The first-order conditions with respect to consumption (C_t), housing quantity (h_t), debt (B_t) and money (M_{t+1}) are:

$$
\frac{\partial \mathcal{L}}{\partial C_t} = U'_c(C_t) - \mu_{1t} P_t = 0; \tag{47}
$$

$$
\frac{\partial \mathcal{L}}{\partial h_t} = U'_h - \mu_{1t} Q_t + \beta \mu_{1t+1} Q_{t+1} + \mu_{2t} \theta_t Q_t = 0; \tag{48}
$$

$$
\frac{\partial \mathcal{L}}{\partial L_t} = U'_L + \mu_{3t} W_t \tag{49}
$$

$$
\frac{\partial \mathcal{L}}{\partial B_t} = \mu_{1t} - \mu_{2t} - \beta(1 + r_t)\mu_{3t+1} = 0; \tag{50}
$$

$$
\frac{\partial \mathcal{L}}{\partial M_t} = \mu_{1t} - \mu_{3t} = 0. \tag{51}
$$

First we use (47) and (49) to claim that constraints (2) and (5) are binding and for computing Lagrange multipliers μ_{1t} and μ_{3t}. That and (51) immediately give

$$
\frac{U'_c(C_t)}{P_t} = -\frac{U'_L}{W_t},
$$

as the labour supply equation. Furthermore, equation (50) becomes

$$
\mu_{2t} = \frac{U'_c(C_t)}{P_t} - (1 + r_t)\beta \frac{U'_c(C_{t+1})}{P_{t+1}}.
$$

We need to check if μ_{2t} is strictly positive. Consider a deterministic steady state with price

stability and zero interest rate. In this case,

$$\mu_2 = \frac{U_c'(C)}{P}(1-\beta) > 0.$$

That is exactly the steady state that we will investigate; assuming that the shocks are not too large, the Lagrange multiplier should be positive and therefore constraint (3) should be binding.

Finally, combining (48) and (50) we get

$$U_h' - U_c'(C_t)\frac{Q_t}{P_t} + \beta U_c'(C_{t+1})\frac{Q_{t+1}}{P_{t+1}} + U_c'(C_t)\frac{Q_t}{P_t}\theta_t - \beta\frac{U_c'(C_{t+1})}{P_{t+1}}(1+r)\theta_t Q_t = 0. \quad (52)$$

This is the same as (6) in the main text.

7.2   Optimal steady state

Following Benigno and Woodford (2012), we will find the best steady state for the optimal commitment policy from a timeless perspective. The policy maker will maximize household utility (12) subject to constraints (13-20). It is easy to see that constraints (13) and (18-20) are only used to define q_t, M_t, P_t and T_t. Therefore, we write a Lagrangian to reduced model (14-17).

$$
\begin{aligned}
\mathcal{L}_t \;=\; & E_t\sum_{t=0}^{\infty}\beta^t\{\log Y_t - \lambda\frac{Y_t^{v+1}\Delta_t^{v+1}}{v+1} && (53)\\
& +\Lambda_{2t}(-X_t + Y_t + a\beta E_t\pi_{t+1}^{\varepsilon-1}X_{t+1}) && (54)\\
& +\Lambda_{3t}\left(-Z_t + \phi_t\lambda Y_t^{v+2}\Delta_t^v + a\beta E_t\pi_{t+1}^{\varepsilon}Z_{t+1}\right) && (55)\\
& +\Lambda_{4t}\left[X_t\left(\frac{1-\alpha\pi_t^{\varepsilon-1}}{1-a}\right)^{\frac{1}{1-\varepsilon}} - \frac{\varepsilon}{\varepsilon-1}Z_t\right] && (56)\\
& +\Lambda_{5t}\left[-\Delta_t + \alpha\Delta_{t-1}\pi_t^{\varepsilon} + (1-\alpha)\left(\frac{1-\alpha\pi_t^{\varepsilon-1}}{1-a}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]\}. && (57)
\end{aligned}
$$

The first-order conditions are

$$\frac{\partial \mathcal{L}_t}{\partial \pi_t} \pi_t = + (\varepsilon - 1) \Lambda_{2t-1} a \pi_t^{\varepsilon-1} X_t$$

$$+ \varepsilon \Lambda_{3t-1} a \pi_t^{\varepsilon-1} Z_t + \Lambda_{4t} X_t \left(\frac{1 - \alpha \pi_t^{\varepsilon-1}}{1 - a} \right)^{\frac{1}{1-\varepsilon}} \frac{\alpha \pi_t^{\varepsilon-1}}{1 - \alpha \pi_t^{\varepsilon-1}}$$

$$+ \Lambda_{5t} \varepsilon \Delta_{t-1} \alpha \left(\pi_t^{\varepsilon} - \pi_t^{\varepsilon-1} \left(\frac{1 - \alpha \pi_t^{\varepsilon-1}}{1 - a} \right)^{\frac{1}{\varepsilon-1}} \right); \qquad (58)$$

$$\frac{\partial \mathcal{L}_t}{\partial Y_t} Y_t = 1 - \lambda Y_t^{v+1} \Delta_t^{v+1} + \Lambda_{2t} Y_t + \Lambda_{3t} \phi_t \lambda Y_t^{v+2} \Delta_t^{v} (v+2); \qquad (59)$$

$$\frac{\partial \mathcal{L}_t}{\partial \Delta_t} = -\lambda Y_t^{v+1} \Delta_t^{v} + \Lambda_{3t} v \phi_t \lambda Y_t^{v+2} \Delta_t^{v-1} - \Lambda_{5t} + \beta \alpha \Lambda_{5t+1} \pi_t^{\varepsilon}; \qquad (60)$$

$$\frac{\partial \mathcal{L}_t}{\partial X_t} = -\Lambda_{2t} + a \pi_t^{\varepsilon-1} \Lambda_{2t-1} + \Lambda_{4t} \left(\frac{1 - \alpha \pi_t^{\varepsilon-1}}{1 - a} \right)^{\frac{1}{1-\varepsilon}}; \qquad (61)$$

$$\frac{\partial \mathcal{L}_t}{\partial Z_t} = -\Lambda_{3t} + a \pi_t^{\varepsilon} \Lambda_{3t-1} - \Lambda_{4t} \frac{\varepsilon}{\varepsilon - 1}. \qquad (62)$$

We can easily verify that prices are stable in steady state, that is $\pi = 1$. From constraints (13-20) and first-order conditions (58-62), we compute the steady-state values for endogenous variables and Lagrange multipliers

$$\begin{array}{ll}
1 = \frac{\varepsilon}{\varepsilon-1} \phi \lambda Y^{v+1}; & \Lambda_2 = \frac{1 - \lambda Y^{v+1}}{Y(v+1)}; \\
X = \frac{Y}{1 - a\beta}; & \Lambda_3 = -\Lambda_2 \frac{\varepsilon}{\varepsilon-1}; \\
Z = \frac{\varepsilon-1}{\varepsilon} X; & \Lambda_4 = \Lambda_2 (1 - a); \\
\Delta = 1; & \Lambda_5 (1 - \beta\alpha) (v+1) = -\lambda Y^{v+1} - v.
\end{array} \qquad (63)$$

7.3 Linear approximation to the constraints of private behavior

To make our model more tractable and comparable to the standard New Keynesian version, we log linearise equations (13)-(20) around the zero inflation steady state[9].

---

[9]All variables with hats **are here** expressed in terms of percentage deviations from **the** steady state.

$$\beta\widehat{q}_{t+1} = \widehat{q}_t(1 - \theta(1 - \beta(1 + r))) - \widehat{\theta}_t\theta(1 - \beta(1 + r)) + r\beta\theta\widehat{r}_t + \beta\theta(1 + r)\left(\widehat{Y}_t - E_t\widehat{\pi}_{t+1} - E_t\widehat{Y}_{t+1}\right) \quad (64)$$

$$\widehat{X}_t = (1 - a\beta)\widehat{Y}_t + a\beta E_t\left(\widehat{X}_{t+1} + (\varepsilon - 1)\widehat{\pi}_{t+1}\right) \quad (65)$$

$$\widehat{Z}_t = (1 - \alpha\beta)\left[(v + 2)\widehat{Y}_t + \widehat{\phi}_t\right] + \alpha\beta E_t\left(\varepsilon\widehat{\pi}_{t+1} + \widehat{Z}_{t+1}\right) \quad (66)$$

$$\widehat{Z}_t = \widehat{X}_t + \frac{a}{1 - a}\widehat{\pi}_t \quad (67)$$

$$\widehat{\Delta}_t = \alpha\widehat{\Delta}_{t-1} \quad (68)$$

$$\widehat{M}_t = \widehat{P}_t + \widehat{Y}_t - \frac{\theta q}{1 - \theta q}\left(\widehat{\theta}_t + \widehat{q}_t\right) \quad (69)$$

$$\widehat{M}_t = \widehat{M}_{t-1} + \widehat{T}_t \quad (70)$$

$$\widehat{\pi}_t = \widehat{P}_t - \widehat{P}_{t-1}. \quad (71)$$

We start with the log linear approximation to constraints (13-20) around the optimal steady state. As shown in Benigno and Woodford (2005), constraint (17) implies that $\widehat{\Delta}_t = \alpha\widehat{\Delta}_{t-1} + O2$, and the log deviation of the relative price dispersion is of second-order importance when price stability is optimal. Therefore, the log linearisation of (13-16) gives (64-67). In turn, (65-67) can be combined into one equation which represents the Phillips Curve. First, combine equations (66) and (67):

$$\widehat{X}_t + \frac{a}{1 - a}\widehat{\pi}_t = (1 - \alpha\beta)\left[(v + 2)\widehat{Y}_t + \widehat{\phi}_t\right] + \alpha\beta\left(\varepsilon\widehat{\pi}_{t+1} + \widehat{X}_{t+1} + \frac{a}{1 - a}\widehat{\pi}_{t+1}\right). \quad (72)$$

We subtract expression (65) and simplify to obtain the New Keynesian Phillips curve

$$\widehat{\pi}_t = \frac{1 - a}{a}(1 - \alpha\beta)\left[(v + 2)\widehat{Y}_t + \widehat{\phi}_t\right] + \beta E_t\widehat{\pi}_{t+1}. \quad (73)$$

7.4   Second-order approximation

Applying the Benigno and Woodford (2012) algorithm, we will get the social welfare function which consists of the sum of squares of the output and inflation gaps. In particular, Benigno and

Woodford (2012) show that the second-order approximation to social welfare can be computed as a sum of pure second-order terms.

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t u_t = E_t \sum_{t=0}^{\infty} \beta^t \left[S(u_t) + \sum_i \Lambda_i S(F_i) \right], \tag{74}$$

where F_i is the dynamic constraint imposed by household behavior and Λ_i is the value of the corresponding Lagrange multiplier in steady state. Furthermore, $S(\cdot)$ is a functional defined on twice differentiable functions of multiple arguments $F(X_t)$, $X_t = [X_{1t}, ...X_{nt}]$ as follows

$$S(F(X_t)) = \widehat{X}_t' X' \nabla^2 F(X) X \widehat{X}_t = \sum_{jk} \frac{\partial^2 F(X)}{\partial X_j \partial X_k} X_k X_j \widehat{X}_{kt} \widehat{X}_{jt},$$

where $\widehat{X}_{kt}$ is the log deviation of variable X_{kt} from its steady-state value X_k. To implement that algorithm, we need to apply functional S to constraints (14-17) since all other constraints are not binding and the corresponding Lagrange multipliers have zero values in the optimal steady state

$$S_u = S(\log Y_t - \lambda \frac{Y_t^{v+1} \Delta_t^{v+1}}{v+1}) = -\widehat{Y}_t^2 - v\lambda Y^{v+1} \widehat{Y}_t^2; \tag{75}$$

$$S_2 = S(-X_t + Y_t + a\beta E_t \pi_{t+1}^{\varepsilon-1} X_{t+1}) = a\beta(\varepsilon-1)(\varepsilon-2)X E_t \widehat{\pi}_{t+1}^2 + 2a\beta(\varepsilon-1)X E_t \widehat{\pi}_{t+1} \widehat{X}_{t+1}; \tag{76}$$

$$S_3 = S(-Z_t + \phi_t \lambda Y_t^{v+2} \Delta_t^v + a\beta E_t \pi_{t+1}^{\varepsilon} Z_{t+1})$$

$$= (v+2)(v+1)\phi\lambda Y^{v+2}\widehat{Y}_t^2 + 2(v+2)\phi\lambda Y^{v+2}\widehat{Y}_t \phi_t + 2a\beta Z\varepsilon\widehat{\pi}_{t+1}\widehat{Z}_{t+1} + a\beta Z\varepsilon(\varepsilon-1)\widehat{\pi}_{t+1}^2; \tag{77}$$

$$S_4 = S\left(X_t \left(\frac{1-\alpha\pi_t^{\varepsilon-1}}{1-a} \right)^{\frac{1}{1-\varepsilon}} - \frac{\varepsilon}{\varepsilon-1} Z_t \right)$$

$$= 2\frac{a}{1-a} X\widehat{\pi}_t \widehat{X}_t + \frac{a}{1-a} X \left(\varepsilon\frac{a}{1-a} + \varepsilon - 2 \right) \widehat{\pi}_{t+1}^2; \tag{78}$$

$$S_5 = S\left(-\Delta_t + \alpha\Delta_{t-1}\pi_t^{\varepsilon} + (1-\alpha) \left(\frac{1-\alpha\pi_t^{\varepsilon-1}}{1-a} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right)$$

$$= \frac{a}{1-a} \left((1-a)\varepsilon(\varepsilon-1) + \varepsilon\frac{a}{1-a} + (\varepsilon-2) \right) \widehat{\pi}_t^2. \tag{79}$$

Using steady-state values (63), we can compute the welfare approximation to (74)

$$W = S_u + \Lambda_2 S_2 + \Lambda_3 S_3 + \Lambda_4 S_4 + \Lambda_5 S_5. \tag{80}$$

First, we simplify S_3. We will use (67) to substitute for the $\widehat{\pi}_{t+1}\widehat{Z}_{t+1}$ term:

$$\widehat{Z}_{t+1}\widehat{\pi}_{t+1} = \widehat{X}_{t+1}\widehat{\pi}_{t+1} + \frac{a}{1-a}\widehat{\pi}_{t+1}^2 + O3.$$

$$S_u = -\left(1 + v\lambda Y^{v+1}\right)\widehat{Y}_t^2; \tag{81}$$

$$\Lambda_2 S_2 = \Lambda_2 a\beta(\varepsilon - 1)\frac{Y}{1 - a\beta}\left((\varepsilon - 2)E_t\widehat{\pi}_{t+1}^2 + 2E_t\widehat{\pi}_{t+1}\widehat{X}_{t+1}\right); \tag{82}$$

$$\Lambda_3 S_3 = -\Lambda_2\left(v + 2\right)\left(v + 1\right)Y\widehat{Y}_t^2 + -\Lambda_2 2(v + 2)Y\widehat{Y}_t\widehat{\phi}_t;$$

$$-\Lambda_2 2a\beta\frac{Y}{1 - a\beta}\varepsilon\left(\widehat{X}_{t+1}\widehat{\pi}_{t+1} + \frac{a}{1 - a}\widehat{\pi}_{t+1}^2\right) - \Lambda_2 a\beta\frac{Y}{1 - a\beta}\varepsilon(\varepsilon - 1)\widehat{\pi}_{t+1}^2; \tag{83}$$

$$\Lambda_4 S_4 = 2\Lambda_2 a\frac{Y}{1 - a\beta}\widehat{\pi}_t\widehat{X}_t + \Lambda_2 a\frac{Y}{1 - a\beta}\left(\varepsilon\frac{a}{1 - a} + \varepsilon - 2\right)\widehat{\pi}_{t+1}^2; \tag{84}$$

$$\Lambda_5 S_5 = -\frac{a}{1 - a}\frac{\lambda Y^{v+1} + v}{(1 - \beta\alpha)(v + 1)}\left((1 - a)\varepsilon(\varepsilon - 1) + \varepsilon\frac{a}{1 - a} + (\varepsilon - 2)\right)\widehat{\pi}_t^2. \tag{85}$$

Now we use that for any dynamic variable x_t,

$$\sum_{t=0}^{+\infty}\beta^t x_{t+1} = \frac{1}{\beta}\sum_{t=0}^{+\infty}\beta^t x_t - \frac{1}{\beta}x_0 = \frac{1}{\beta}\sum_{t=0}^{+\infty}\beta^t x_t + tip,$$

where x_0 is considered a "precommitted" variable which cannot be changed because of commitment and therefore, it is regarded as "term independent of policy (tip)".

Finally, we can compute the welfare approximation to (74).

$$U_t + O3 = E_t\sum_{t=0}^{+\infty}\beta^t W = -E_t\sum_{t=0}^{+\infty}\beta^t\left[\alpha_C\left(\widehat{Y}_t + \alpha_\phi\widehat{\phi}_t\right)^2 + \alpha_\pi\widehat{\pi}_t^2\right], \tag{86}$$

where the parameters are defined as follows

$$
\begin{aligned}
\alpha_C &= \left(1 + v - 2\lambda Y^{v+1}\right); \\
\alpha_\phi &= \frac{1 - \lambda Y^{v+1}}{1 + v - 2\lambda Y^{v+1}} \frac{v+2}{v+1}; \\
1 &= \frac{\varepsilon}{\varepsilon - 1} \phi \lambda Y^{v+1}; \\
\alpha_\pi &= \frac{a\left[\left(1 - \lambda Y^{v+1}\right)\varepsilon + \left(\lambda Y^{v+1} + v\right)\left((1-a)\varepsilon(\varepsilon-1) + \frac{\varepsilon - 2 + 2a}{1-a}\right)\right]}{(1-a)(1-\beta\alpha)(v+1)}.
\end{aligned}
$$

7.5   Solution to social planner LQ problem

The social planner maximises (86) subject to constraint (73) only, since all other constraints are non-binding,

$$
J_t = -\frac{1}{2} E_t \sum_{t=1}^{\infty} \beta^t \left[\alpha_C \left(\widehat{Y}_t - \alpha_\phi \widehat{\phi}_t \right)^2 + \alpha_\pi \widehat{\pi}_t^2 + \lambda_t \left(-\widehat{\pi}_t + \frac{1-a}{a}(1-\alpha\beta)\left[(v+2)\widehat{Y}_t + \widehat{\phi}_t\right] + \beta\widehat{\pi}_{t+1} \right) \right]. \quad (87)
$$

The first-order conditions imply the optimal inflation dynamics

$$
\widehat{\pi}_t = \rho\left(\widehat{Y}_{t-1} - \widehat{Y}_t - \alpha_\phi\left(\widehat{\phi}_{t-1} - \widehat{\phi}_t\right)\right), \quad (88)
$$

where $\rho = \frac{\alpha_C}{\alpha_\pi} \frac{a}{(1-\alpha\beta)(v+2)(1-a)}$.

7.6   Proof of Proposition 1

In this appendix we will show that a moderate fall in the LTV ratio can be neutralised by lowering the interest rate and without quantitative easing. Consider model (13)-(20) and a policy which aims to achieve a complete stabilization of the credit shock by the means of interest rate. That is $T_t = T_{t+1} = 0$, $\pi_t = \pi_{t+1} = 1; Y_{t+1} = Y_t = Y_{t-1}$. That implies constant velocity of cash and from (18) it follows that product $\theta_t q_t$ is also constant.

$$
\theta_{t+1}q_{t+1} = \theta_t q_t = \theta_{t-1}q_{t-1} = A; \quad (89)
$$

To compute the interest rate dynamics for stabilising policy we will combine (13) for periods t and $t-1$ with (89) and on the assumption that prices and output are stable, $\frac{Y_t}{\pi_{t+1}Y_{t+1}} = 1$, we derive

$$r_t = r_{t-1} + \frac{1}{\beta}\left(\frac{1}{\theta_{t-1}} - \frac{1}{\theta_t}\right) - \left(E_{t-1}\frac{1}{\theta_t} - E_t\frac{1}{\theta_{t+1}}\right)$$

7.7   Proof of Proposition 4

Proposition 4 *If $\widehat{\theta}_t = 0$, a policy of inaction, $\widehat{T}_t = 0$, results in house price stability and the stability of the relative collateral value.*

Proof. Consider dynamics (27)-(28) with $\widehat{T}_t = 0$, $\widehat{\theta}_t = 0$.

$$\widehat{\pi}_t = \frac{1-a}{a}(1-\alpha\beta)\left[(v+2)\widehat{Y}_t + \widehat{\phi}_t\right] + \beta E_t\widehat{\pi}_{t+1}; \tag{90}$$

$$\beta E_t\widehat{q}_{t+1} = \widehat{q}_t(1-\theta(1-\beta)) + \beta\theta(\widehat{Y}_t - E_t\widehat{\pi}_{t+1} - E_t\widehat{Y}_{t+1}); \tag{91}$$

$$\widehat{\pi}_t = \widehat{Y}_{t-1} - \widehat{Y}_t + \frac{\theta q}{1-\theta q}\left(\widehat{q}_t - \widehat{q}_{t-1}\right). \tag{92}$$

Plugging Equation (92) with one lead into (91), we would get that the dynamic for relative house expenditure does not depend on a shock or any other variable in the system.

$$\beta E_t\widehat{q}_{t+1} = \widehat{q}_t(1-\theta(1-\beta)) - \beta\theta\frac{\theta q}{1-\theta q}\left(E_t\widehat{q}_{t+1} - \widehat{q}_t\right),$$

which implies complete stability, $\widehat{q}_t = 0$. Moreover, by definition, $Q_t = q_t P_t Y_t$ inflation of house prices is

$$\widehat{\pi}_{Qt} := \widehat{Q}_t - \widehat{Q}_{t-1} = \widehat{\pi}_t + \widehat{Y}_t - \widehat{Y}_{t-1} + \left(\widehat{q}_t - \widehat{q}_{t-1}\right).$$

Combining it with (92) and $\widehat{q}_t = 0$, we get zero house price inflation in every period. ∎

7.8 Money in the Utility Function

The Lagrangian to the household problem when there is a preference over the means of payment is giving by:

$$
\begin{aligned}
\mathcal{L} &= E_t \sum_{t=0}^{\infty} \beta^t \Big[\Big(U(C_t, \frac{M_t}{P_t}, L_t, h_t \Big) \\
&+ \mu_{1t}(-P_t C_t - Q_t(h_t - h_{t-1}) + B_t + M_t) \\
&+ \mu_{2t}(-B_t + \theta_t Q_t h_t,) \\
&+ \mu_{3t}(-M_t + W_t L_t + \Pi_t + T_t - (1 + r_{t-1}) B_{t-1})];
\end{aligned}
$$

where μ_{it} are Lagrange multipliers. The first-order conditions with respect to consumption (C_t), housing quantity (h_t), debt B_t and money (M_{t+1}) are:

$$
\frac{\partial \mathcal{L}}{\partial C_t} = U_c^{'}(C_t) - \mu_{1t} P_t = 0; \tag{93}
$$

$$
\frac{\partial \mathcal{L}}{\partial h_t} = U_{h,t}^{'} - \mu_{1t} Q_t + \beta \mu_{1t+1} Q_{t+1} + \mu_{2t} \theta_t Q_t = 0; \tag{94}
$$

$$
\frac{\partial \mathcal{L}}{\partial L_t} = U_{L,t}^{'} + \mu_{3t} W_t = 0; \tag{95}
$$

$$
\frac{\partial \mathcal{L}}{\partial B_t} = \mu_{1t} - \mu_{2t} - \beta(1 + r_t)\mu_{3t+1} = 0; \tag{96}
$$

$$
\frac{\partial \mathcal{L}}{\partial M_t} = \frac{U_m^{'}}{P_t} + \mu_{1t} - \mu_{3t} = 0. \tag{97}
$$

It changes the labor supply equation

7.9 Proof of Proposition 5

Proof. Condition (45) for preference function (42) implies

$$
\frac{M}{PC} > \xi \frac{\frac{\beta(1+r)}{\pi}}{1 - \frac{\beta(1+r)}{\pi}}. \tag{98}
$$

From the first order conditions (93), (94), (96) and (97), we get the Euler equation for investment in the housing market

$$U'_{h,t} - U'_{c,t}\frac{Q_t}{P_t} + \beta U'_{c,t+1}\frac{Q_{t+1}}{P_{t+1}} + \theta_t\frac{Q_t}{P_t}\left[U'_{ct} - \frac{\beta(1+r_t)}{\pi_{t+1}}\left(U'_{m,t+1} + U'_{c,t+1}\right)\right] = 0;$$

which in steady state is the same as (99)

$$\eta - \frac{Q}{PC}(1-\beta) + \theta\frac{Q}{PC}\left(\left[1 - \frac{\beta(1+r)}{\pi}\right] - \xi\frac{\beta(1+r)}{\pi}\frac{PC}{M}\right) = 0. \tag{99}$$

Recall that if the constraint is binding, then $\theta\frac{Q}{CP} = 1 - \frac{M}{CP}$. We use letter u to denote the velocity of money, $u = \frac{M}{CP}$; equation (99) becomes

$$\eta - \frac{(1-u)}{\theta}(1-\beta) + (1-u)\left(\left[1 - \frac{\beta(1+r)}{\pi}\right] - \xi\frac{\beta(1+r)}{\pi}\frac{1}{u}\right) = 0.$$

Consider the function $f(u) := \eta - \frac{(1-u)}{\theta}(1-\beta) + (1-u)\left(\left[1 - \frac{\beta(1+r)}{\pi}\right] - \xi\frac{\beta(1+r)}{\pi}\frac{1}{u}\right)$. It is easy to see that when $\xi > 0$, there exists a unique solution $u^* \in [0,1]$ such that $f(u^*) = 0$. To prove it we need to note that, $f(u)u$ is a quadratic function with positive first coefficient and $\lim_{u\to 0} f(u) = -\infty$, $f(1) = \eta > 0$. That implies that for any $u \in [0, u^*)$, $f(u) < 0$; and for any $u > u^*$, it follows that $f(u) > 0$. To prove that the credit constraint is binding, we simply need to show that $u^* > \xi\frac{\frac{\beta(1+r)}{\pi}}{1-\frac{\beta(1+r)}{\pi}}$, which in the case of positive ξ and small interest rate, $\frac{\beta(1+r)}{\pi} < 1$, is equivalent to $f(\xi\frac{\frac{\beta(1+r)}{\pi}}{1-\frac{\beta(1+r)}{\pi}}) < 0$. That is the same as condition $\xi < \left(1 - \frac{\theta\eta}{1-\beta}\right)\frac{1-\frac{\beta(1+r)}{\pi}}{\frac{\beta(1+r)}{\pi}}$. ∎

7.10   Parameter values

We use the following parameter values: $q = 0.5$, $v = 2$, $\beta = 0.95$, $\theta = 0.8$, $a = 0.5$, $\varepsilon = 6$, $\phi = 1.1$, $\rho_\theta = 0.95$, $\rho_\phi = 0.95$, $\eta = 0.005$.