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Abstract

Job creation and job destruction are investigated in an economy charaterised by search frictions in both labour and goods markets. We show that both the unemployment rate and the endogenous job destruction rate increase when the inflation rate rises, because demand declines due to the increase in the cost of holding money. Our numerical exercises suggest that the destruction of lower productivity jobs and the creation of higher productivity jobs may be inefficiently low under the zero nominal interest rate, which in turn causes the deviation of optimal long-run monetary policy from the Friedman rule.

Keywords: inflation, search frictions, money, welfare

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1 Introduction

Monetary policies can have profound impacts on the outcomes of the labour market and this argument has been investigated for a range of monetary models augmented by search frictions in the labour markets. Blanchard and Gali (2010) and Sveen and Weinke (2008) illustrate the new Keynesian perspectives on labour market dynamics. They find that introducing search frictions in the labour market hardly influences the new Keynesian optimal monetary policy analysis and confirm that the interest rate should be set to respond mainly to price changes, instead of to unemployment fluctuations. Other researchers examine the relationship between inflation and unemployment in an economy where exchanges in goods and labour markets both involve costly searches, in the framework of modern monetary search models[1]. Berentsen, Menzio and Wright (2011) investigate the labour market with search frictions using the new monetarist model of Lagos and Wright (2005). Their main findings are that a rise in inflation causes a higher unemployment rate, and that the long-run Phillips curve slopes upwards. To summarize, all the above studies focus on optimal monetary policy and the relationship between inflation and the unemployment rate.

The present study pursues a more detailed question: what are the impacts of monetary policy on job creation and job destruction? Furthermore, what is the implication for the optimal monetary policy in the long run if we introduce endogenous job destruction? Berentsen, Menzio and Wright (2011) assume the job destruction rate to be a constant, as in the standard search and matching labour market model, e.g. Pissarides (2000). Therefore, they cannot study the impacts of monetary policy on the job destruction rate. However, this question matters for at least two reasons. First, economists and the public not only care about the unemployment rate in the economy, but also about how many workers loss their jobs and the chances for unemployed workers to start a new job. Second, as will be shown later in this study, job creation and job destruction can have ambiguous effects on welfare and on the optimal long-run monetary policy. Because we model job creation and job destruction in the sense that a firm chooses to enter into or quit the business, our model also relates to the literature which investigates the monetary influence on economic welfare through the channel of endogenous firm entry and/or exit. Berentsen and Waller (2009) and Rocheteau and Wright (2005) study optimal long-run monetary policy in the presence of endogenous firm entry and monetary search frictions that make money essential in the economy:[2] Berentsen and Waller (2009) claim that the optimal policy would deviate from the Friedman rule if there is a congestion externality

[1]Search monetary models were pioneered by Kiyotaki and Wright (1991, 1993) for the case of indivisible commodities and money. They impose the assumption of indivisible commodities and money to guarantee the tractability of the model, because otherwise the distribution of money holding becomes too complicated to obtain analytical results. The search monetary frameworks which are suitable for macroeconomic and monetary policy analysis, i.e., the models that allow divisible commodities and money but circumvent the aforementioned difficulties concerning the distribution of money holdings, are developed along there lines: i) to introduce the assumption of large families and perfect risk sharing as in Shi (1997); ii) to introduce an extra centralized good market and the assumption of linear negative utility of labour input in this centralized good market, as in Lagos and Wright (2005); iii) to introduce complete financial markets among groups e.g. Faig (2006). For further details about the development of search monetary models, please refer to the review by Shi (2006).

[2]Lewis (2009), Bergin and Corsetti (2008) and Bilbiie et al. (2007) also study optimal monetary policy in the long run in the presence of endogenous firm entry, but they employ ingredients other than micro-founded search models to make money essential, say cash-in-advance constrains, money in utility function, wage rigidity, price rigidity and so on.

affecting firm entry; Rocheteau and Wright (2005) claim that the Friedman rule must be the optimal monetary policy in search equilibrium and competitive search equilibrium, but not necessarily in competitive equilibrium.[3]

This paper introduces endogenous job separation and firm heterogeneity à la Mortensen and Pissarides (1994) into Berentsen, Menzio and Wright (2011) to study job market flows in a stationary equilibrium. We also follow Mortensen and Pissarides (1994) by assuming that new jobs have the highest productivity in the economy and old jobs experience idiosyncratic shocks which damage their productivity. We confirm the conclusion concerning the long-run Phillips curve in Berentsen, Menzio and Wright (2011): a rise in inflation increases the unemployment rate; the long-run Phillips curve slopes upwards; the economy reaches the lowest unemployment rate when the central bank applies the Friedman rule. We also show that the endogenous job destruction rate rises when the inflation rate rises, because a higher inflation rate reduces the profits of all firms and make the less productive firms more likely to quit the business. The reasons lie in the fact that inflation is modelled as the cost of holding money in our micro-founded monetary exchange setting and a rise in inflation rate increases households' cost of holding money. Households then reduce their money holding in the decentralized market, which in turn reduces firms' selling and profits in this market. Therefore, high inflation encourages job destruction, which means that there are more jobs with the highest level of productivity replacing jobs with lower productivity levels in every period, at the cost of a higher job losing rate and a higher unemployment rate in the economy. Thus, in the steady state of the economy, the average productivity level of the economy is higher when inflation is higher, although total employment falls. This result shows that endogenous job separation and firm heterogeneity have major impacts on the long-run optimal monetary policy because the monetary authority can balance between the average productivity level and unemployment rate by setting interest rates. We claim that the destruction of lower productivity jobs and the creation of higher productivity jobs might be too low under the Friedman rule, which in turn implies that the optimal long-run monetary policy deviates from the Friedman rule.[4] We use numerical methods to verify this conjecture.

We then calibrate our theoretical model using parameter values commonly chosen in the relevant macro-labour and monetary economics literature. Our numerical exercises show that the destruction of lower productivity jobs and the creation of higher productivity jobs are indeed too low under the Friedman rule from the perspective of a welfare-maximizing monetary authority. The optimal interest rate implied by our numerical exercise is around 2% at quarterly level, which approximately equals the average quarterly interest rate of the US economy during 1955-2005. Moreover, we report that the maximal welfare gain of deviating from the Friedman rule is worth less that 1 per cent of consumption.

The remainder of the paper is organized as follows: the basic model is presented in section 2; the description of stationary equilibrium and its properties are given in section 3; the numerical experiments regarding optimal long-run monetary policy are performed in section 4; and the final section offers conclusion.

[3] Rocheteau and Wright (2005) define these three equilibria by their market structures of money-goods exchange: search equilibrium corresponds to bargaining; competitive equilibrium corresponds to price taking; competitive search equilibrium corresponds to price posting.

[4] This channel is absent in Berentsen, Menzio and Wright (2008), because of its exogenous constant job separation rate and homogenous firms setting. A positive nominal interest rate would only lead to a higher unemployment rate but not to productivity improvement. Therefore, the long run optimal monetary policy implied by their model is the Friedman rule.

2 The Basic Model

The present model is mostly based on Berentsen, Menzio and Wright (2011) and endogenous job separation and firm heterogeneity are introduced following Mortensen and Pissarides (1994).

2.1 The Environment

There are two types of private agents: firms and households. There exists a unit continuum of households; while the measurement of firms is endogenously determined. Households work, consume, and enjoy utility; firms create jobs, maximize profits and pay out dividends to households.

Time is discrete and continues forever. In each period, there are three markets that open sequentially. Market 1 is a labour market, Market 2 is a decentralized goods market and Market 3 is a centralized goods market. These three markets are indexed by $i = 1, 2, 3$, respectively.

The labour market is modelled in the spirit of Mortensen and Pissarides (1994), where firms create vacancies and households search for jobs. Firms and households meet each other bilaterally according to a matching technology. A pair consisting of a household and a firm then combine to create a job that produces a preliminary product, good x, and bargain over wages, w. Wages are chosen so as to share the surplus from a job match in fixed proportions at all times. The worker's share is $\eta \in (0, 1)$. Consequently, more productive jobs offer higher wages. We assume that the wage is paid in Market 3, thus it does not matter whether wages are paid in cash or goods. Each job is characterized by its productivity y and newly created jobs represent the highest productivity $\bar{y}$, following Mortensen and Pissarides (1994). Idiosyncratic shocks change the productivity of jobs according to a Poisson process with arrival rate δ. When a job is hit by such idiosyncratic shock, a new value of y is drawn from the fixed distribution $F(y)$. y has finite upper support $\bar{y}$ and no mass points. Thus, the productivity of any given job is a stochastic process with the initial condition of the upper support of the distribution and the terminal state of the reservation productivity that leads to job destruction. Filled jobs do not always exit when they are hit by shocks, because there is a cost involved in maintaining a vacancy, k. Existing filled jobs are destroyed only if their productivity falls below some critical number y_d. Therefore, the endogenous rate at which existing jobs are destroyed is $\delta F(y_d)$.

Market 2 (decentralized goods market) is modelled in the spirit of the price-taking version of the decentralized goods market in Berentsen, Menzio and Wright (2011). Unlike the day-time market in Lagos and Wright (2005), where goods are traded bilaterally through bargaining between pairs formed by anonymous matching, there is a Walrasian auctioneer in the market so the participating buyers and sellers take the market price as given. However, Market 2 is not a centralized goods market because there are restrictions for the buyers (households) and sellers (firms) who participate in the market. In specific, we assume that the buyers and sellers must meet bilaterally[5] according to a matching

[5] This assumption is made to make sure that the measure of sellers equals that of buyers, so there is only one term of trade (selling equals buying) in the economy. This is a slightly stronger restriction than other price-taking versions of the decentralized goods markets, such as, Rocheteau and Wright (2005) or Berentsen, Camera and Waller (2007). However, there is no loss of any generality and the analysis becomes easier.

technology before entering into the decentralized market where good q is traded. The decentralized market here is less decentralized than that of Lagos and Wright (2005), but it does not make money inessential as long as we maintain the assumption of anonymity.[6] The good q is transformed by the preliminary product, i.e. good x, without any labour input, following a transforming technology which is specified below.[7] Unused good x is taken by the firm from Market 2 to Market 3, without any depreciation or cost.

Market 3 (centralized goods market) is modelled in the spirit of the night-time market in Lagos and Wright (2005), where good x is traded multilaterally. All the private agents take the market prices as given. Unsold good x then vanishes between two periods. Also, without loss of generality we assume that agents discount at rate β between Market 3 and the next Market 1, but not between the other markets.

We assume a central bank exists and controls the supply of fiat money. We denote the growth rate of the money supply by ϖ, so that $\hat{M} = (1+\varpi)M$, where M denotes the per capita money stock in Market 3 and the variable with a circumflex indicates the value of variable over the next period. Therefore, in steady states, ϖ is the inflation rate. The central bank implements its inflation goal by providing deterministic lump-sum injections of money, ϖM, to the household at the end of each period.

Throughout the discussion in the text, we assume policy and the productivity distribution to be constant, and focus only on steady states.

2.2 Households

We now consider optimal decisions of the household. Let $s = e, u$ index employment status: e indicates that a household is employed; u indicates that a household is unemployed. We adopt the convention of Berentsen, Menzio and Wright (2011) for measuring real balances z, i.e. the nominal balances of households in current Market 3, next Market 1 and Market 2, m, are deflated by P, the price level in the current centralized goods market (Market 3), which is the latest price known for that market. Thus, when an agent brings $\hat{m}$ fiat money to the next period, we let $\hat{z} = \hat{m}/P$ denote his or her real balance. When he then takes this $\hat{m}$ fiat money to the next Market 3, his or her real balance is then given by $\hat{m}/\hat{P} = \hat{z}\hat{\rho}$, where $\hat{\rho} = P/\hat{P}$ converts $\hat{z}$ into the units of the numeraire in that market. Notice $\hat{\rho} = 1/(1+\varpi)$, where ϖ is the inflation rate between this and the next centralized goods market.

2.2.1 Centralized Goods Market

We now consider optimal decisions of the household, starting with Market 3. A household, who is employed in a firm of productivity y and with a real balance z, solves

$$\begin{aligned} W_{3,e}(z,y) &= \max_{x,\hat{z}} \{x + \beta W_{1,e}(\hat{z},y)\} \\ \text{s.t. } x + \hat{z} &= w(y) + \Delta + \frac{\pi M}{p} + z, \end{aligned} \tag{1}$$

[6]Rocheteau and Wright (2005), Berentsen, Camera and Waller (2007) and Berentsen, Menzio and Wright (2011) make similar points.

[7]Berentsen, Menzio and Wright (2011) assume that firms first produce the goods traded in market 2 while the goods traded in market 3 are made from the goods traded in market 2. Here we assume the transformation techonolodgy to be of a different type. The reason is explained in footnote 9.

where W denotes the value function of an employed household, x is the consumption in Market 3, $w(y)$ is the wage paid by the firm of productivity y, and Δ is dividend income.[8] We here assume that utility is linear in x, as in Lagos and Wright (2005) and Berentsen, Menzio and Wright (2011), to ensure that all agents in the centralized market choose the same real balance to enter into the next period. Substituting x from the budget constraint into (1) yields

$$W_{3,e}(z,y) = w(y) + \Delta + \frac{\pi M}{p} + z + \max_{\hat{z}}\{-\hat{z} + \beta W_{1,e}(\hat{z},y)\}. \tag{2}$$

Therefore, $W_{3,e}(z,y)$ is linear in z.

Similarly, the problem for an unemployed household with a real balance z reads

$$\begin{aligned} W_{3,u}(z) &= \max_{x,\hat{z}}\{x + \beta W_{1,u}(\hat{z})\} \qquad (3) \\ \text{s.t. } x + \hat{z} &= b + \Delta + \frac{\pi M}{p} + z, \end{aligned}$$

where b is the home production of an unemployed household. We have $0 < b < \bar{y}$, which means that home production is lower than the highest productivity level. Substituting x from the budget constraint into (3) yields

$$W_{3,u}(z) = b + \Delta + \frac{\pi M}{p} + z + \max_{\hat{z}}\{-\hat{z} + \beta W_{1,u}(\hat{z})\}. \tag{4}$$

$W_{3,u}(z)$ is also linear in z.

2.2.2 Decentralized Goods Market

We now move to Market 2. The value functions of an employed household and an unemployed household, respectively, with real balance z, read

$$W_{2,e}(z,y) = \alpha_h \max_q\{v(q) + W_{3,e}[\rho(z-dq),y]\} + (1-\alpha_h)W_{3,e}(\rho z,y) \tag{5}$$

$$W_{2,u}(z) = \alpha_h \max_q\{v(q) + \alpha_h W_{3,u}[\rho(z-dq)]\} + (1-\alpha_h)W_{3,u}(\rho z) \tag{6}$$

where α_h is the probability of a household trading in the decentralized goods market, and d is the real price of good q in the second market and is taken as given by both households and firms. The matching technology in the goods market will be discussed later. $v(\cdot)$ is the household's utility function of consuming goods. $v(\cdot)$ is twice differentiable with $v(0) = 0$, $v' > 0$, $v'' < 0$, $\lim_{q\to 0} v'(q) = +\infty$, and $\lim_{q\to+\infty} v'(q) = 0$.

Using the linearity of W_3, implied by (2) and (4)), equations (5) and (6) become

$$W_{2,e}(z,y) = \alpha_h \max_q[v(q) - \rho dq] + W_{3,e}(\rho z,y), \tag{7}$$

$$W_{2,u}(z) = \alpha_h \max_q[v(q) - \rho dq] + W_{3,u}(\rho z). \tag{8}$$

[8] We assume the representative household holds the representative portfolio and therefore receives same amount of dividend. So the equilibrium dividend Δ equals the average profit of all the firms.

2.2.3 Labour Market

The value function of an employed household with real balance z reads

$$W_{1,e}(z,y) = (1-\delta)W_{2,e}(z,y) + \delta \int_{-\infty}^{\bar{y}} \max\{W_{2,e}(z,l), W_{2,u}(z)\} dF(l). \tag{9}$$

The second term of the right-hand side of (9) shows that a household would leave the firm (job separation) if the value of working in a firm with productivity l is smaller than the value of being unemployed.

The value function of an unemployed household with real balance z reads

$$W_{1,u}(z) = (1-\lambda_h)W_{2,u}(z) + \lambda_h W_{2,e}(z,\bar{y}), \tag{10}$$

where λ_h is the probability for an unemployed household to find a vacancy. The matching technology in the labour market will be discussed in section 3.2. The second term on the right-hand side of (10) shows that the job which an unemployed worker takes has the highest productivity.

It is convenient to summarize the three markets in a single equation. Substituting $W_{2,e}$ and $W_{2,u}$ from (7) and (8) into (9) and using the linearity of $W_{3,e}$ and $W_{3,u}$ yields

$$W_{1,e}(z,y) = \alpha_h \max_q [v(q) - \rho d q] + \rho z + (1-\delta)W_{3,e}(0,y) + \delta \int_{-\infty}^{\bar{y}} \max\{W_{3,e}(0,l), W_{3,u}(0)\} dF(l). \tag{11}$$

Substituting $W_{2,e}$ and $W_{2,u}$ from (7) and (8) into (10) and using the linearity of $W_{3,e}$ and $W_{3,u}$, we get

$$W_{1,u}(z) = \alpha_h \max_q [v(q) - \rho d q] + \rho z + (1-\lambda_h)W_{3,u}(0) + \lambda_h W_{3,e}(0,\bar{y}). \tag{12}$$

Then, substituting $W_{1,e}$ and $W_{1,u}$ from (11) and (12) of the next period into (2) yields our equation for the value function of households in Market 3 only, namely

$$\begin{aligned} W_{3,e}(z,y) &= w(y) + \Delta + \frac{\pi M}{p} + z + \max_{\hat{z}}\{-\hat{z} + \beta\hat{\alpha}_h \max_{\hat{q}}[v(\hat{q}) - \hat{\rho}\hat{d}\hat{q}] + \beta\hat{\rho}\hat{z}\} \\ &\quad + \beta[\delta \int_{-\infty}^{\bar{y}} \max\{W_{3,e}(0,l), W_{3,u}(0)\} dF(l) + (1-\delta)W_{3,e}(0,y)]. \end{aligned} \tag{13}$$

Similarly, substituting $W_{1,e}$ and $W_{1,u}$ from (11) and (12) of next period into (4) yields

$$\begin{aligned} W_{3,u}(z) &= b + \Delta + \frac{\pi M}{p} + z + \max_{\hat{z}}\{-\hat{z} + \beta\hat{\alpha}_h \max_{\hat{q}}[v(\hat{q}) - \hat{\rho}\hat{d}\hat{q}] + \beta\hat{\rho}\hat{z}\} \\ &\quad + \beta[(1-\hat{\lambda}_h)W_{3,u}(0) + \hat{\lambda}_h W_{3,e}(0,\bar{y})]. \end{aligned} \tag{14}$$

This completes the description of the problem faced by households.

2.3 Firms

We assume that at the beginning of each period, firms can post a vacancy at a fixed cost k. If the vacancy is matched with a household in the labour market, the firm enters into the decentralized market at this time; otherwise, the vacancy is destroyed. Let V denote the value function of a filled job and U the value function of a vacancy. We now consider optimal decisions of the firm, again starting with Market 1. The value function of a job with productivity y reads

$$V_1(y) = (1-\delta)V_2(y) + \delta \int_{-\infty}^{\bar{y}} \max\{V_2(l), 0\} dF(l). \tag{15}$$

The second term of the right-hand side of (15) shows that the firm terminates the job if its value after the productivity shock is smaller than zero. The value function for a vacancy reads

$$U = -k + \lambda_f V_2(\bar{y}), \tag{16}$$

where λ_f is the probability for a vacancy to be filled by an unemployed household.

The free entry condition reads

$$U = 0. \tag{17}$$

(16) and (17) then imply

$$k = \lambda_f V_2(\bar{y}). \tag{18}$$

We now consider Market 2. We assume that the technology transforming good q into good x is characterized by $c(q)$, which is assumed to be twice differentiable and to satisfy $c(0) = 0$, $c' > 0$, $c'' > 0$. We claim that $c(\cdot)$ is the opportunity cost of trade in the decentralized goods market in terms of real balance in Market 3.[9]

Then the value function of a firm in Market 2 reads

$$V_2(y) = \alpha_f \max_q V_3[y, y - c(q), \rho q d] + (1-\alpha_f)V_3(y, y, 0), \tag{19}$$

where α_f is the probability for a firm to trade in Market 2. $V_3(\cdot,\cdot,\cdot)$ is the value function of a firm in Market 3, in which the first argument is the productivity of the firm, the second argument is good x that the firm takes to Market 3 and the third argument is the real money balance that the firm takes to Market 3.

The value function of the firm in Market 3 then reads,

$$V_3(y, x, z) = x + z - w(y) + \beta V_1(y). \tag{20}$$

Substituting V_3 from (20) into (19) yields

$$V_2(y) = y + \alpha_f \max_q [\rho q d - c(q)] - w(y) + \beta V_1(y). \tag{21}$$

Define $R \equiv \max_q [\rho q d - c(q)]$, which is a firm's profit in Market 2 if it gets the chance to trade. Substituting V_1 from (15) into (21) yields

$$V_2(y) = y + \alpha_f R - w(y) + \beta(1-\delta)V_2(y) + \beta\delta \int_{-\infty}^{\bar{y}} \max\{V_2(l), 0\} dF(l). \tag{22}$$

[9] We want the opportunity cost to be independant of the firm's productivity and to be identical across the economy to ensure tractability of the model. Therefore, unlike Berentsen, Menzio and Wright (2011), I assume that the preliminary product of the firm is traded in Market 3 instead of Market 2.

This completes the description of the firms' problem.

3 Equilibrium

3.1 Goods Market

We first describe the matching technology in the decentralized goods market. We assume that the probability for a household to trade in the decentralized goods market α_h is exogenous and constant.[10] Then, the probability for a firm to trade in Market 2 α_f is determined in equilibrium. The measurement of firms must equal the measurement of the employed households $1-u$, where u is the measurement of the unemployed households, i.e. the unemployment rate. The measurement of firms matched in the decentralized goods market must equal the measurement of households matched in the decentralized goods market, which yields $1 \cdot \alpha_h = \alpha_f(1-u)$. Therefore, we have

$$\alpha_f = \frac{\alpha_h}{1-u}. \tag{23}$$

(23) means that the more firms there are in the market, the harder it is for a firm to be matched in the decentralized goods market and make a profit. This is called congestion externality in Berentsen and Waller (2009). We will return to equation (23) in section 4.3 when explaining how the congestion externality in the present model differs from that in Berentsen and Waller (2009).

We now establish the equilibrium conditions for Market 2. The optimal selling problems are identical for all the firms in the economy and read[11]

$$\max_q [\rho q d - c(q)].$$

The FOC then reads

$$\rho d = c'(q). \tag{24}$$

The optimal buying problems are identical for all the households in the economy and read

$$\max_q [v(q) - \rho d q]$$
$$\text{s.t. } dq \leq z.$$

The FOC then reads

$$\begin{array}{lll} v'(q) = \rho d & \text{if} & z > z^* \\ dq = z & \text{if} & z \leq z^*, \end{array} \tag{25}$$

where z^* is defined so as to satisfy

$$v'(\frac{z^*}{d}) = \rho d.$$

[10] I make this assumption for simplicity. My assumption here results in a unique equilibrium. Berensten, Menzio and Wright (2011) assume α_h to be endogenous and to depend on the unemployment rate, which brings much complexity into the analysis and results in the possibility of multiple equilibria. However, Berensten, Menzio and Wright (2011) only analyse the case of a unique equilibrium. Therefore, making α_h exogenous does not change the analytical conclusion for the properties of the equilibrium unemployment rate and the terms of trade.

[11] In fact, there is a feasibility constraint $q \leq y$. For simplicity, we assume that this condition is always slack, following Berensten, Menzio and Wright (2011).

Comparing (13) and (14) implies that the problems of optimal real balance taken into the next period are identical for all household in the economy and read:

$$\max_z\{-z + \beta\alpha_h \max_q[v(q) - \rho dq] + \beta\rho z\}. \tag{RB}$$

Assumption 1 *We have* $1 + \pi \geq \beta$, *i.e.* $\rho \leq \frac{1}{\beta}$.

Assumption 1 means that the money growth rate is higher than the Friedman rule would require. To see its application more clearly, we use the Fisher equation, which links the nominal interest rate and inflation in the long run. The Fisher equation reads

$$1 + i = \frac{1 + \pi}{\beta}, \tag{26}$$

where i is the nominal interest rate. Therefore, Assumption 1 simply means that $i \geq 0$, while the Friedman rule requires $i = 0$, i.e. $\pi = \beta - 1$. Assumption 1 makes sure that optimal problem (RB) has a meaningful solution, as Lagos and Wright (2005) point out. Then we only consider the equilibrium of the economy either in the case $1 + \pi > \beta$, or the case $1 + \pi = \beta$, but equilibrium is the limit as $1 + \pi \to \beta$ from above.

Proposition 1 *Under Assumption 1, in equilibrium,* q *is the solution to*

$$\frac{v'(q)}{c'(q)} = \frac{i}{\alpha_h} + 1, \tag{27}$$

the households set their next period real balance at

$$z = c'(q)q, \tag{28}$$

and the firm's profit in Market 2 (if they get the chance to trade) equals

$$R = c'(q)q - c(q). \tag{29}$$

Proof: see appendix.[12]

(27) fully characterizes the equilibrium in the decentralized goods market. We then have the following proposition for the goods market:

Proposition 2 *(27) is the equilibrium condition for the goods market. With Assumption 1, the solution exists and is unique. Furthermore,* q *is decreasing in* i*; and the firms' revenue in Market 2,* R*, is increasing in* q *and decreasing in* i*.*

Proof: see appendix.

Because $R = c'(q)q - c(q)$ is a function of q, we will denote R as $R(q)$ later. Note that, when $i = 0$, q and $R(q)$ reach their maxima and $v'(q) = c'(q)$ holds.

3.2 Labour Market

We now describe the matching technology in the labour market. As in the standard labour market literature, the total match M is a constant return to scale function of the measurement of unemployed households, u and of vacancies, ϕ,

[12] Similar proofs can also be found in Rocheteau and Wright (2005).

$$M = m(\phi, u),$$

where $m(\phi, u)$ is a CRS function and increasing in both ϕ and u. M is also equal to the job destruction and job construction in equilibrium. Then, the probability for an unemployed household to find a job is

$$\lambda_h = \frac{m(\phi, u)}{u} = m(\mu, 1), \tag{30}$$

where $\mu = \phi/u$ is the labour market tightness. The probability for a vacancy to be filled is

$$\lambda_f = \frac{m(\phi, u)}{\phi} = m(1, \frac{1}{\mu}). \tag{31}$$

Furthermore, the functional form of $m(\phi, u)$ is assumed to ensure that $\lambda_h,\ \lambda_f \in (0, 1)$.

We assume that wages are determined at the end of the labour market, although they are not paid until Market 3, as mentioned earlier. Therefore, it does not matter whether wages are paid in money or goods. There is no commitment to maintain the same wage level across periods, so wages are newly bargained in every period. We also use the generalized Nash solution, in which the household has bargaining power η and threat points are given by continuation values. The surplus for a household that meets a firm of productivity y is

$$S_h(y) = W_{2,e}(0, y) - W_{2,u}(0) = W_{3,e}(0, y) - W_{3,u}(0) \tag{32}$$

by virtue of (7) and (8) and the linearity of W_2 in z. The surplus for a firm of productivity y is

$$S_f(y) = V_2(y). \tag{33}$$

The total surplus for such a job match $S(y)$ is

$$S(y) \equiv S_h(y) + S_f(y) = W_{3,e}(0, y) - W_{3,u}(0) + V_2(y) \tag{34}$$

by definition. Wage bargaining divides the surplus from a job match in fixed proportions, i.e.,

$$\frac{S_h(y)}{\eta} = \frac{S_f(y)}{1-\eta} = S(y). \tag{35}$$

Since $S(y)$ is monotonically increasing in y, job destruction satisfies the reservation property. (35) implies that there is a unique reservation productivity y_d that solves

$$\frac{S_h(y_d)}{\eta} = \frac{S_f(y_d)}{1-\eta} = S(y_d) = 0$$

such that jobs that get a shock $l < y_d$ are destroyed. Therefore, jobs are destroyed at rate $\delta F(y_d)$. There are two group of jobs: the first group includes newly created jobs and old jobs which have not experienced idiosyncratic shocks; the second group consists of old jobs which have experienced idiosyncratic shocks. Jobs in the first group have productivity $\bar{y}$ and we denote the measurement of jobs in this group as γ; while the productivity of jobs in the second group follows a truncated distribution of $F(x)$, truncated at y_d, and we denote its distribution function as $G(x)$. In the steady state, the unemployment rate, new job matching, job destruction, job destruction rate and the measures of two groups of jobs are all constant. Therefore, this requirs that, firstly, new job matching equals job destruction, i.e.

$$\lambda_h u = \delta F(y_d)(1-u), \tag{36}$$

and secondly, job flows into the first group equal job flows out of the first group, i.e.,

$$\lambda_h u = \delta\gamma. \tag{37}$$

We then have the following proposition for the labour market:

Proposition 3 *The equilibrium condition in the labour market is characterized by the following two equations for the labour market tightness μ and the reservation productivity y_d:*

$$\frac{[1-\beta(1-\delta)]k}{(1-\eta)m(1,\frac{1}{\mu})} = \bar{y} - y_d \tag{38}$$

$$y_d + \alpha_h\left[1 + \frac{\delta F(y_d)}{m(\mu,1)}\right] R(q) - b + \frac{\beta\delta}{1-\beta(1-\delta)}\int_{y_d}^{\bar{y}}[1-F(l)]dl - \frac{\eta\beta k}{1-\eta}\cdot\mu = 0. \tag{39}$$

The other labour market variables can be expressed by μ and y_d as follows:

$$u = \frac{\delta F(y_d)}{m(\mu,1)+\delta F(y_d)}, \tag{40}$$

$$\gamma = \frac{m(\mu,1)F(y_d)}{m(\mu,1)+\delta F(y_d)}, \tag{41}$$

$$M = \frac{\delta m(\mu,1)F(y_d)}{m(\mu,1)+\delta F(y_d)}. \tag{42}$$

Proof: see appendix.

3.3 Steady State Equilibrium

Propositions 1 and 2 show that goods market equilibrium is solely defined by q and equilibrium condition (27), independently of the situation in the labour market. Firm's revenue in Market 2, $R(q)$ is the link between the labour market and the goods market. This leads to the following definition:

Definition 1 *The steady state equilibrium of the economy is a triple $\{q, \mu, y_d\}$ that satisfies equilibrium conditions (27), (38) and (39).*

Before moving on to the existence and uniqueness of the equilibrium, we need to prove the following Lemma about the properties of the equilibrium conditions in the labour market.

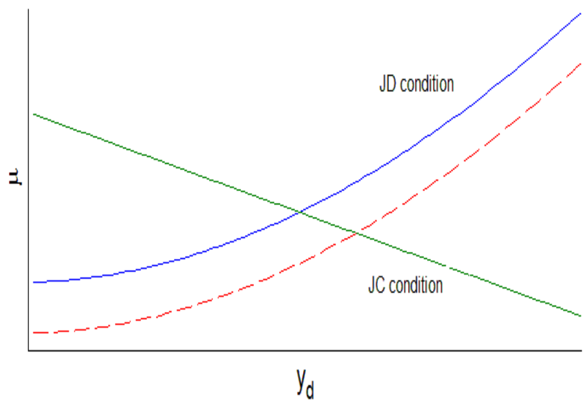
Lemma 1 *(38) slopes downward in (y_d, μ) space. Given $R(q)$ constant, (39) slopes upward in (y_d, μ) space and (39) shifts down if $R(q)$ goes down.*

Proof: see appendix.

(38) states how job creation depends on the reservation productivity y_d in the economy and we refer this as the job creation condition (JC). The higher reservation productivity

implies lower value of the jobs, which in turn discourages job creation and hence results in a lower vacancy-unemployment ratio. Thus, the JC curve is downward sloping in the (y_d, μ) space. (39) describes how the vacancy-unemployment ratio determines reservation productivity y_d and we refer to (39) as the job destruction condition (JD). The higher vacancy-unemployment ratio implies that there are more firms with state-of-the-art technology and thus jobs less profitable to maintain suffered idiosyncratic shocks. In turn, the reservation productivity is higher. Thus, the JD curve is upward sloping in the (y_d, μ) space.

To establish the existence and uniqueness of the equilibrium, we only need to consider the labour market with a given $R(q)$. By virtue of the above Lemma, we know that y_d and μ are also uniquely determined by (38) and (39). Henceforth, we have established the existence and uniqueness of the steady state equilibrium. Now we check the property of equilibrium when the interest rate changes. The effect of interest rate changes on q has been shown in Proposition 2: q and $R(q)$ become smaller when i goes up. By virtue of Lemma 1, we can draw the curves of (38) and (39) in a (y_d, μ) space.



Graph 1: joint determination of the labour market tightness and the reservation productivity by job creation (JC) and destruction conditions (JD)

It has been shown in Lemma 1 that, $R(q)$ becomes smaller because i increases. Then curve (38) does not move and curve (39) goes down. We then conclude that μ becomes smaller, i.e. $\frac{\partial \mu}{\partial i} < 0$, and y_d becomes larger i.e. $\frac{\partial y_d}{\partial i} > 0$. It is also useful to show that the unemployment rate becomes larger. (40) implies that

$$\frac{\partial u}{\partial i} = \frac{\partial u}{\partial \mu}\frac{\partial \mu}{\partial i} + \frac{\partial u}{\partial y_d}\frac{\partial y_d}{\partial i} = -\frac{\delta F(y_d)}{[m(\mu,1)+\delta F(y_d)]^2}\frac{dm(\mu,1)}{d\mu}\frac{\partial \mu}{\partial i} + \frac{\delta F'(y_d)m(\mu,1)}{[m(\mu,1)+\delta F(y_d)]^2}\frac{\partial y_d}{\partial i} > 0.$$

So we confirm the conclusion of Berentsen, Menzio and Wright (2011) that a higher interest rate or a higher inflation rate leads to a higher unemployment rate in the long run; the Phillips Curve should slope upward.

Summarizing, we have established the following results:

Proposition 4 *With Assumption 1, the steady state equilibrium always exists and is unique. Furthermore, in this equilibrium the amount of goods traded in the decentralized* *goods market, q, and labour market tightness μ are decreasing in the interest rate i, while the reservation productivity level y_d and the unemployment rate u are increasing in i.*

The above proposition is the most important result of the present study. We could also study the job losing rate and job finding rate in the labour market. The job losing rate for the employed is $\delta F(y_d)$, which is also the job destruction rate and an increasing function of y_d; the job finding rate for the unemployed is $m(\mu, 1)$, which is an increasing function of μ. We then conclude from Proposition 4 that when the inflation rate increases, it is more likely that the employed will lose their jobs, because the job destruction rate $\delta F(y_d)$ will rise. The reason is that the jobs become less profitable for firms (R goes down) and the rise of trading opportunities (α_f goes up because there are fewer firms) can not compensate for the loss in profits. Furthermore, when the inflation rate increases, it is harder for the unemployed to find a new job, because the job finding rate $m(\mu, 1)$ will drop.

4 Optimal Monetary Policy: Numerical Experiments

When we introduce endogenous job separation and firm heterogeneity into the model of Berentsen, Menzio and Wright (2011), we find that a higher rate of inflation causes more job separations and a higher unemployment rate. An important feature of the present model is that higher inflation also causes a higher rate of job destruction $\delta F(y_d)$. Intuitively, higher inflation makes firms less profitable in the decentralized goods market as well as less profitable in general, so that firms with lower productivity are more likely to quit the market, i.e. y_d rises. Because job destruction is modelled as a process where higher productivity jobs replace jobs with lower productivity in every period, there is a role for monetary policy to affect welfare by adjusting the job destruction rate.

We notice that Berentsen, Menzio and Wright (2011) assume all firms to have the same productivity level and the loss of jobs to take place at an exogenous constant rate. Thus job destruction in Berentsen, Menzio and Wright (2011) simply takes the form of some firms replacing others with the very same productivity, and higher unemployment must consequently generate a welfare loss. A positive interest rate would only a incur higher unemployment rate but no improvement in productivity. Therefore, the optimal monetary policy implied by their model is the Friedman rule.

However, in our model, higher inflation encourages job destruction, which means that more jobs with higher productivity replace jobs with lower productivity in every period, at the cost of a higher job losing rate and a higher unemployment rate. Thus, assuming the steady state of the economy, the average productivity level of the economy is higher when inflation is higher, although the total employment falls. This result shows that endogenous job separation and firm heterogeneity may have a major impact on optimal monetary policy because the monetary authority can balance the average productivity level and the unemployment rate by setting interest rates.

In this section, the optimal monetary policy of the central bank is formally depicted. However, we cannot give analytical proof for our conjecture that the destruction of lower-productivity jobs and the creation of higher-productivity jobs might be too low under the Friedman rule. Therefore, we resort to numerical methods to justify our conjecture.

4.1 Optimal Monetary Policy

The total output of the economy Y is the sum of firm production and home production, i.e.

$$Y = \gamma \bar{y} + (1 - u - \gamma) \int_{y_d}^{\bar{y}} y dG(y) + ub. \tag{43}$$

We assume that the central bank treats all households equally when it designs the optimal monetary policy. Then, by virtue of the linearity of the households' utility function in Market 3, we can define the periodic welfare function L as,

$$L = \alpha_h v(q) + Y - \alpha_h c(q) - k\phi, \tag{44}$$

where the first term is the total utility households get from Market 2 every period and the remaining terms are the total utility households get from Market 3. Because all the exogenous parameters are assumed to be constant, the central bank's optimal monetary policy design problem degenerates into a static problem, in which the central bank chooses the optimal interest rate i to maximize the periodic welfare function L subject to the equilibrium conditions (27), (38) and (39). The central bank's problem is,

$$\max_{i,q,u,\gamma,y_d,\mu} \alpha_h[v(q) - c(q)] + \gamma \bar{y} + (1 - u - \gamma) \int_{y_d}^{\bar{y}} y dG(y) + ub - k\mu u$$

subject to (27), (38) and (39), where u and γ are defined by (40) and (41).

We know that for a given interest rate, terms of trade in the decentralized goods market q is uniquely determined from Proposition 2; and the endogenous labour market variables $(\mu, y_d, u, \phi, \gamma)$ are functions of q from Proposition 3. So the central bank's optimal monetary policy design problem can be treated as the choice of decentralized goods market allocation q to maximize the periodic welfare function L subject to the labour market equilibrium conditions (38) and (39). So the central bank's problem is equivalent to

$$\max_q L(q) = \Psi(q) + \Phi(q)$$

where

$$\Psi(q) \equiv \alpha_h[v(q) - c(q)]$$

$$\Phi(q) \equiv \gamma(q)\bar{y} + [1 - u(q) - \gamma(q)] \int_{y_d(q)}^{\bar{y}} y dG(y) + u(q)b - k\mu(q)u(q).$$

$\mu(q)$ and $y_d(q)$ are solutions to (38) and (39); $u(q)$ and $\gamma(q)$ follow (40) and (41).

It is optimal for the central bank to set the interest rate at i_o such that the corresponding q_o, which is the solution to (27) for given i_o, satisfies

$$L'(q_o) = 0.$$

However, it is impossible to determine the signs of the above terms analytically. Thus, we answer this question using numerical methods in a reasonably calibrated model.

4.2 Parameters and Calibration Targets

Similar to the calibration in Berentsen, Menzio and Wright (2011), we choose a quarter as one period and look at the United States economy during the period 1955-2005.

Some specific functional forms are first set:

i) The matching function in the labour market is standard,

$$m(\mu, 1) = Z\mu^{\alpha}$$

where the labour market matching efficiency is $Z > 0$ and the labour market matching elasticity is $\alpha \in (0, 1)$.

ii) The distribution from which the productivity level is drawn after idiosyncratic shock is a uniform distribution[13] within the support $[0, \bar{y}]$. The distribution function $F(y)$ then reads

$$F(y) = \begin{cases} 0 & \text{if } y < 0, \\ \frac{y}{\bar{y}} & \text{if } 0 \leq y < \bar{y}, \\ 1 & \text{if } \bar{y} \leq y. \end{cases}$$

Thereafter, $G(y)$, the distribution function of a truncated distribution of $F(y)$ truncated at y_d becomes

$$G(y) = \begin{cases} 0 & \text{if } y < y_d, \\ \frac{F(y)-F(y_d)}{1-F(y_d)} = \frac{y-y_d}{\bar{y}-y_d} & \text{if } y_d \leq y < \bar{y}, \\ 1 & \text{if } \bar{y} \leq y. \end{cases}$$

iii) The household's utility function in Market 2 is

$$v(q) = \frac{Aq^{1-a}}{1-a},$$

where a, the elasticity of the utility function, is a positive number belonging to $(0, 1)$ and A, the weight of Market 2, is positive.

vi) The firm's cost function is assumed to be

$$c(q) = \frac{Aq^{1+\sigma}}{1+\sigma},$$

where σ, the elasticity of the cost function is positive. Therefore, the functional forms $c(\cdot)$ and $v(\cdot)$ satisfy the aforementioned properties. We set the cost function to have the same scale parameter as Market 2's utility function. This is because we measure the total utility in terms of Market 3's goods, we want the scale of the cost of Market 3's goods to reproduce Market 2's and its utility level to be comparable.[14]

Thus, there are twelve parameters to be set:

[13] A uniform productivity distribution is also used by Mortensen and Pissarides (1994).

[14] In our comparable study, Berentsen, Menzio and Wright (2008) set the scales of both the cost function and market 2's utility function to be 1.

- preferences as described by β and a;
- technology as described by b, δ, $\bar{y}$, Z, α, α_h, σ and k;
- market structure as described by A, B and η.

We now calibrate this model such that the numerical prediction for the steady state equilibrium fits the 1955-2005 United States economy on average.

We set β to match the average quarterly real interest rate, measured as the difference between the nominal interest rate and inflation. We normalize the home production b to be 1. We also set $\bar{y} = 3$.[15] The job-specific technology shock arrival rate, δ, is chosen to be 0.081, as in Mortensen and Pissarides (1994). The parameters Z, α and k are fixed to be consistent with the US labour market feature which is described in Shimer (2005). Thus, α is set at 0.028, the labour market matching elasticity with respect to vacancy. Z and k are chosen to match the average unemployment rate 0.06 and average UE (unemployment to employment) transition rate (λ_h), respectively, given the average vacancies are normalized to be 1 as in Berentsen, Menzio and Wright (2011). Notice that although Shimer (2005) claims that the monthly average UE transition rate is 0.45, we need to compute the quarterly rate: $\lambda_h = 1 - (1 - 0.45)^3 = 0.834$.

We then set A, B, a, α_h and σ as in the relevant monetary economics literature. First, we set $\alpha_h = 0.9$ without further explanation, because α_h matters little for our quantitative conclusions. We also set $\sigma = 1$ for reasons of simplicity. We still need two conditions to pin down A and a. The first condition is that the equilibrium condition (27), which links the goods and labour markets, must hold numerically. Therefore, we have to use the numerical value of the average interest rate when computing the equilibrium condition (27). The average annual nominal interest rate is 0.074. Then we claim that the average quarterly nominal interest rate is $\frac{0.074}{4} = 0.019$. The second condition is that the money demand (real balance) predicted by our model must be consistent with the average US money demand, 0.179. This condition is commonly used in the monetary economics literature, for instance by Lucas (2000) and Lagos and Wright (2005) among others. In our model, the money demand M/pY equals

$$\frac{M}{pY} = \frac{M/p}{Y} = \frac{c'(q)q}{\alpha_h[v(q) - c(q)] + \gamma\bar{y} + (1 - u - \gamma)\frac{\bar{y}+y_d}{2}}.$$

The targets discussed above are summarized in Table 1. These targets are sufficient to pin down all but one parameter, η, the wage bargaining power of the households. η is assumed to equal α, by the Hosios (1990) rule from the labour-macro literature, although it is not a necessary condition for reasonable calibrations.[16] We first set η to equal α and we will also check the robustness of our quantitative conclusions later when η varies.

[15] Berentsen, Menzio and Wright (2008) assume the average productivity to be twice as high as home production. So it is fairly reasonable to set the maximal productivity to be 3 here.

[16] See Shi (1998), among others.

Table 1: Calibration Targets

Description	Value
average unemployment u	0.060
average vacancies v (normalization)	1
average UE rate λ_h	0.834
elasticity of λ_h wrt μ	0.280
job-specific technology shock rate δ	0.081
household's trade probability in Market 2 α_h	0.900
average money demand M/pY	0.179
elasticity σ of cost function	1
average nominal interest rate i	0.019
average real interest rate r	0.008
home production b (normalization)	1
ratio $\bar{y}/b$	3

Table 2 summarizes the calibrated parameter values. However, we first have to verify these values satisfy our assumption made in section 3.1. When determining the terms of trade, we claim that the feasibility constraint $c(q) < y$ is always slack. The maximal q and minimal y_d occur when $i = 0$. With the functional form $c(q)$ and $v(q)$ we have $q^* = 1$ and the maximal $c(q)$ is $\frac{A(q^*)^{1+\sigma}}{1+\sigma} = \frac{1.826}{2} = 0.913$, while the minimal y is $y_d = 2.032$. Therefore, we confirm our assumption that the feasibility constraint $c(q) < y$ is always slack.

Table 2: Parameter Values

	Description	Value
β	discount factor	0.008
b	home production	1
a	elasticity of utility function	0.799
δ	job-specific technology shock rate	0.081
$\bar{y}$	maximal productivity	3
Z	labour market matching efficiency	0.390
α	labour market matching elasticity	0.280
α_h	household's trade probability in Market 2	0.900
σ	elasticity of cost function	1
k	vacancy posting cost	0.443
A	Market 2's weight	1.826
η	wage bargaining power of household	0.280

4.3 Results

Using the calibrated parameters from above, we can compute the steady state equilibrium of the model for $i \in [0, 0.1]$ using our equilibrium conditions (27), (38) and (39). Figure 1 plots the steady state unemployment rate when the central bank sets different interest rate values. We confirm the conclusion of Berentsen, Menzio and Wright (2011) that higher inflation leads to a higher unemployment rate in the long run.

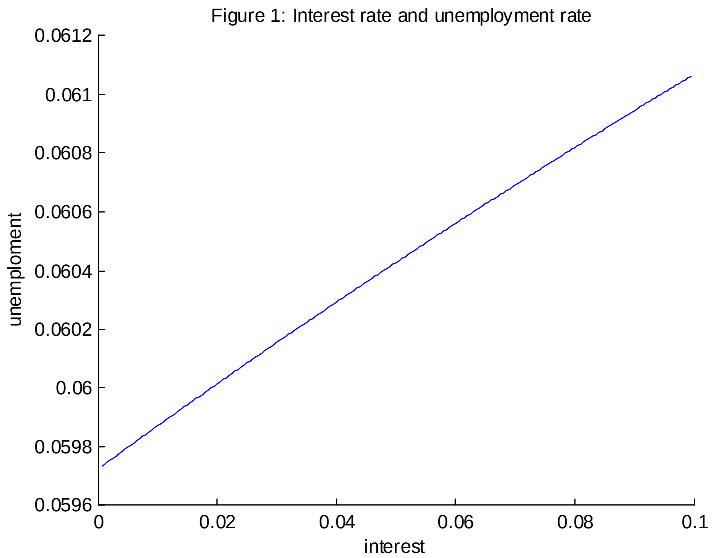


Figure 2 plots the steady state welfare level as defined in (44) when the central bank sets different interest rate values. Normalizing the welfare at $i = 0$ to be 100, we can directly read off the relative loss or gain of welfare when the central bank sets different interest rate levels. Figure 2 is the main quantitative finding in our numerical experiments. Our calibrations show that the highest welfare level is achieved when the central bank sets $i = 0.027$ (quarterly level), which is slightly above the long-run interest rate level of the United States ($i = 0.019$ as mentioned before). The gain derived from deviating from the Friedman rule can also be seen from the plot. Figure 2 shows that the welfare improvement when the central bank sets $i = 0.027$ relative to the welfare level when the central bank sets $i = 0$ is less than one percent (0.5%, to be more precise). Because the welfare is measured in terms of the consumption of goods x and there is no disutility of work[17], we claim that the welfare gain made by deviating from the Friedman Rule is worth 0.5 per cent of consumption. Furthermore, when the central bank sets i higher than the optimal interest rate level, there is much danger that the economy will experience significant welfare loss, given that the right side of the curve in Figure 2 becomes increasingly steep.

[17] We assume that there is home production, which could be enjoyed as consuming goods x. That means we measure the utility of leisure in term of goods x in the model.

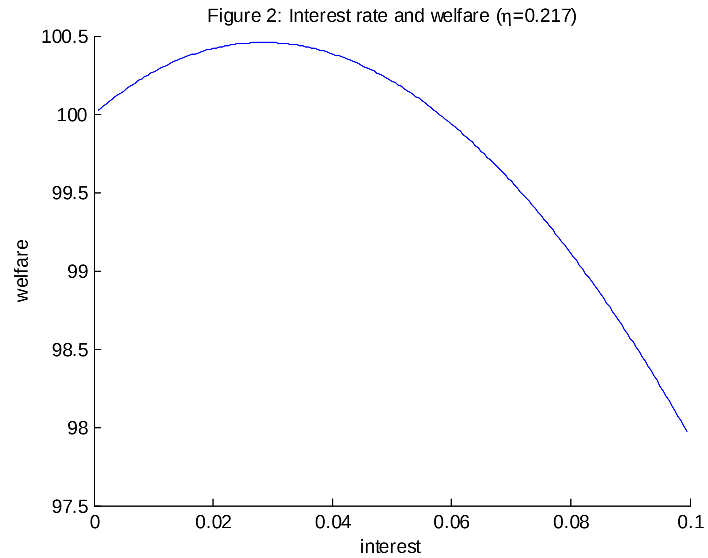
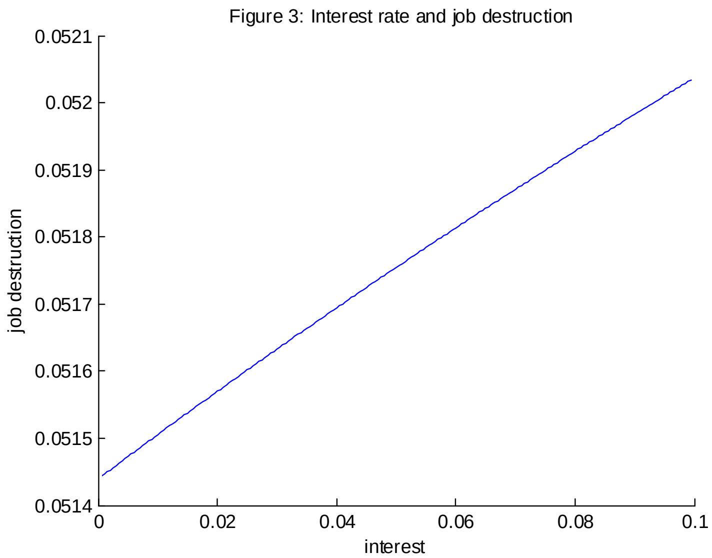


Figure 3 plots the steady state job destruction when the central bank sets different interest rate values, because we are also interested in whether higher inflation can cause higher job destruction. Figure 3 confirms this relationship.



Therefore, Figures 2 and 3 jointly suggest that it is very likely that job destruction and the creation of higher productivity jobs are too low under the Friedman rule, which in turn means that a zero interest rate is too low from a welfare-maximizing point of view and drives the optimal long-run monetary policy away from the Friedman rule. Then it is optimal for the central bank to set the interest rate strictly above zero.

To summarize, our numerical exercises confirm that the Friedman rule is not optimal in a calibrated model with endogenous job separation and firm heterogeneity.

4.4 Robustness: η

Now it is necessary to check whether our quantitative conclusion that the optimal interest rate is above zero is robust for different η values. As mentioned earlier, there is no justified reason for assuming that the wage bargaining power of households, η, satisfies the Hosios (1990) rule. We try four different values for η and the results are summarized in Figure 4 and Table 3, namely, $\{0.15; 0.5; 0.72; 0.8\}$.

Figure 4.1: Interest rate and welfare (η=0.15)

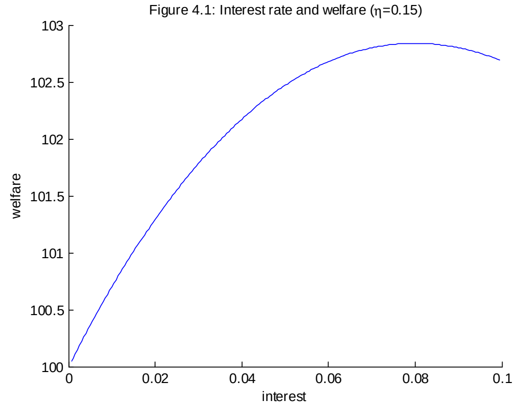


Figure 4.2: Interest rate and welfare (η=0.50)

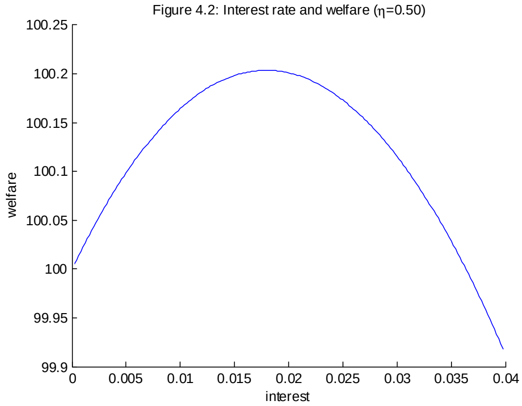


Figure 4.3: Interest rate and welfare (η=0.72)

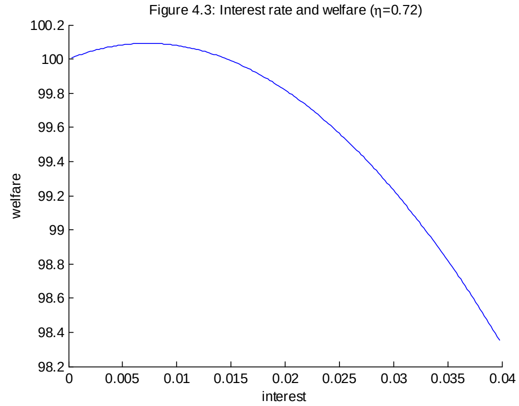
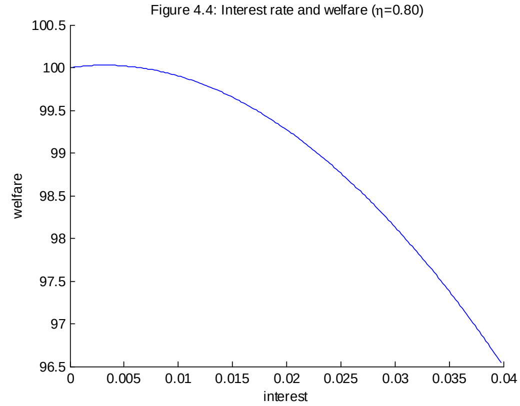


Figure 4.4: Interest rate and welfare (η=0.80)



Figures 4.1 to 4.4 plots the steady-state welfare level when the central bank sets different interest rates $i \in [0, 0.4]$ in four cases with different η, the wage bargaining power of households in the labour market. First, it is clear that our conclusion that the optimal interest rate is above zero is fairly robust for different η values; second, the lower the wage bargaining power of the households, the more likely it is that the central bank will gain from setting the interest rate above zero and the higher the welfare gain from this derivation; third, when η is sufficiently high, say $\eta \geq 0.80$, the Friedman rule approximates or indeed equals the optimal policy; last but not least, the resulting welfare improvements are all quite small, less than 1 per cent. The optimal interest rate for each case and its welfare improvement relative to the Friedman rule are shown in Table 3.

Table 3: Optimal interest rate for different η

η	Optimal interest rate	Welfare improvement
0.15	0.080	2.03%
0.22	0.027	0.50%
0.50	0.018	0.20%
0.72	0.007	0.08%
0.80	0.002	less than 0.01%

4.5 Further Comments

As mentioned before, (23) shows that congestion externality exists in the decentralized goods market, which means that the more firms there are in the market, the harder it is for a firm to be matched. Berentsen and Waller (2009) study optimal monetary policy using a modified version of Lagos and Wright (2005) with endogenous firm entry.[18] They claim that congestion externality is a reason for the Friedman rule to be regarded as suboptimal, but their mechanism is different from mine. In their model, firms need to pay certain costs before entering into the market every period. They then show that there is too much firm entry in the market under the Friedman rule and the congestion externality makes entry less efficient. They suggest that the central bank should set the interest rate strictly above zero to reduce firm entry in equilibrium. In the present study, we emphasize the role of a positive interest rate in prompting less productive firms to quit by reducing their profit in the decentralized market. However, a higher interest rate reduces a firm's profit, whether with or without congestion externality. Furthermore, we claim that under the Friedman rule, new job creations are too few instead of too many in the presence of this congestion externality, which is suggested by Berentsen and Waller (2009). Therefore, our explanations of why the Friedman rule is not optimal go beyond the congestion externality present in the decentralized goods market.

We also want to compare our results with Rocheteau and Wright (2005), who also reach the conclusion that the Friedman rule may not be the optimal long-run monetary policy in a "competitive equilibrium"[19] (price-taking) in the presence of endogenous firm entry. The mechanism suggested by Rocheteau and Wright (2005) is essentially the same as that in Berentsen and Waller (2009): due to the "congestion externality in the decentralized goods market", firm entry is too high under the Friedman rule. Furthermore, both Rocheteau and Wright (2005) and Berentsen and Waller (2009) assume an exogenous firm exit rate equal to 1, while our study assumes an endogenous firm exit rate.

5 Conclusion

We have presented a model of search frictional goods and labour markets while the job separate rate is endogenously determined. We confirm the conclusion reached in some related literature that the unemployment rate increases with the inflation rate and that the long-run Phillips curve slopes upwards. The economy achieves the lowest unemployment rate when the central bank applies the Friedman rule. We also show that the endogenous job destruction rate becomes higher when the inflation rate rises, because higher inflation reduces the profits of all firms and makes the less productive firms more likely to quit the business. The reasons for this lie in that the inflation is modelled as the cost of holding money in our micro-founded monetary exchange setting and a rise in inflation increases households' cost of holding money. Households then reduce their money holding in the decentralized market, which in turn reduces firms' selling and profits in this market. This paper also indicates the possibility that the optimal long-run monetary policy should deviate from the Friedman rule if a higher inflation rate can

[18] The workhorse model that Berensten and Waller (2009) is based on Berentsen, Camera and Waller (2007), which is a version of Lagos and Wright (2005) with a banking sector.

[19] We borrow the terminalogy "competitive equilibrium" from Rocheteau and Wright (2005) to show that the decentralized goods market pricing mechenism is price taking. Also see footnote 3.

promote enough high-productivity job creation.

Our numerical exercises confirm the conjecture that the job destruction is too low under the Friedman rule for a set of parameters calibrated using data from the United States economy. The optimal interest rate implied by our numerical exercise is around 2.7% at the quarterly level, which is slightly higher than the average quarterly interest rate of the US economy from 1955-2005. Moreover, we also report that the maximal welfare gain of deviating from the Friedman rule is less that 1 per cent of consumption, which is a relatively small number.

Appendix: Proofs

Proof of Proposition 1

Proof. Denote the objective function as

$$O(z) \equiv -z + \beta\alpha_h \max_q [v(q) - \rho dq] + \beta\rho z$$

Case I, $\rho < \frac{1}{\beta}$.
(25) implies

$$\text{if } z > z^*, \frac{\partial q}{\partial z} = 0 \tag{a.1}$$

$$\text{if } z < z^*, \frac{\partial q}{\partial z} = \frac{1}{d}. \tag{a.2}$$

Then (a.1) implies,

$$\text{if } z > z^*, O'(z) = \beta\rho - 1 < 0, \tag{a.3}$$

which means that the objective function is decreasing in z for all $z > z^*$. Furthermore, the second condition in (25) implies,

$$\text{if } z < z^*, O(z) = -z + \beta\alpha_h[v(q) - \rho z] + \beta\rho z. \tag{a.4}$$

Computation then shows that

$$\text{if } z < z^*, O'(z) = \beta\rho - 1 + \beta\alpha_h[v'(q)\frac{1}{d} - \rho]. \tag{a.5}$$

Furthermore,

$$\lim_{z \to z^{*-}} O'(z) = \beta\rho - 1 + \beta\alpha_h[v'(\frac{z^*}{d})\frac{1}{d} - \rho] = \beta\rho - 1 < 0. \tag{a.6}$$

Therefore, (a.3) and (a.6) imply that the optimal $O(z)$ is reached when $z < z^*$ and z satisfies

$$\beta\rho - 1 + \beta\alpha_h[v'(q)\frac{1}{d} - \rho] = 0. \tag{a.7}$$

Substituting d from (24) into (a.7) gets

$$\beta\rho - 1 + \beta\alpha_h[\frac{\rho v'(q)}{c'(q)} - \rho] = 0. \tag{a.8}$$

Substituting ρ from $\rho = \frac{1}{1+\pi}$ and the Fisher equation (26) into (a.3) and rearranging yields (27). Given the knowledge of q, (24) and the FOC of (25) for $z \leq z^*$ implies (28). Similarly, substituting (24) into the definition of R yields (29).

Case II, $\rho = \frac{1}{\beta}$.

As mentioned above, we only consider the equilibrium of the economy in the case $1 + \pi = \beta$ but as a limit as $1 + \pi \to \beta$ from above. By continuity of all functions, this case can be proved using the same arguments as above. ■

Proof of Proposition 2

Proof. Computation shows that

$$\frac{d[\frac{v'(q)}{c'(q)}]}{dq} = \frac{v''(q)c'(q) - v'(q)c''(q)}{[c'(q)]^2} < 0,$$

$$\lim_{q \to 0} \frac{v'(q)}{c'(q)} = +\infty,$$

$$\lim_{q \to +\infty} \frac{v'(q)}{c'(q)} = 0.$$

Then, given $i \geq 0$, the solution of q for (27) exist and is unique. Furthermore, q is decreasing in i, i.e., $\frac{\partial q}{\partial i} < 0$. To see that R is also decreasing in i, we have, by the virtue of (29) in Proposition 1,

$$\frac{\partial R}{\partial i} = \frac{\partial R}{\partial q}\frac{\partial q}{\partial i} = c''(q)q\frac{\partial q}{\partial i} < 0.$$

■

Proof of Proposition 3

Proof. Firstly, (36) implies (40) directly. Then (40) and (37) give us (41). The total matching amount M is $\lambda_h u$ by definition, which is (42).

Subtracting (14) from (13) and using the steady state condition yields

$$[1 - \beta(1 - \delta)]S_h(y) = w(y) - b - \beta\lambda_h S_h(\bar{y}) + \beta\delta \int_{-\infty}^{\bar{y}} \max\{S_h(l), 0\}dF(l). \tag{a.9}$$

(22) can be rewritten as

$$[1 - \beta(1 - \delta)]S_f(y) = y + \alpha_f R(q) - w(y) + \beta\delta \int_{-\infty}^{\bar{y}} \max\{S_f(l), 0\}dF(l). \tag{a.10}$$

Adding (a.9) and (a.10) and using the definition of y_d implies

$$[1 - \beta(1 - \delta)]S(y) = y + \alpha_f R(q) - b - \beta\lambda_h S_h(\bar{y}) + \beta\delta \int_{y_d}^{\bar{y}} S(l)dF(l). \tag{a.11}$$

Using the fact that $S_h(y) = \eta S(y)$, (a.11) becomes

$$[1 - \beta(1 - \delta)]S(y) = y + \alpha_f R(q) - b - \eta\beta\lambda_h S(\bar{y}) + \beta\delta \int_{y_d}^{\bar{y}} S(l)dF(l). \tag{a.12}$$

Taking the derivative of (a.12) with respect to y yields

$$S'(y) = \frac{1}{1-\beta(1-\delta)} \tag{a.13}$$

(a.12) and (a.13) imply after integration by parts that

$$\begin{aligned}[1-\beta(1-\delta)]S(y) &= y + \alpha_f R(q) - b - \eta\beta\lambda_h S(\bar{y}) + \beta\delta \int_{y_d}^{\bar{y}} S'(l)[1-F(l)]dl \\ &= y + \alpha_f R(q) - b - \eta\beta\lambda_h S(\bar{y}) + \frac{\beta\delta}{1-\beta(1-\delta)} \int_{y_d}^{\bar{y}} [1-F(l)]dl.\end{aligned} \tag{a.14}$$

Setting $y = y_d$ and $y = \bar{y}$ in (a.14) implies

$$\eta\beta\lambda_h S(\bar{y}) = y_d + \alpha_f R(q) - b + \frac{\beta\delta}{1-\beta(1-\delta)} \int_{y_d}^{\bar{y}} [1-F(l)]dl, \tag{a.15}$$

$$[1-\beta(1-\delta)]S(\bar{y}) + \eta\beta\lambda_h S(\bar{y}) = \bar{y} + \alpha_f R(q) - b + \frac{\beta\delta}{1-\beta(1-\delta)} \int_{y_d}^{\bar{y}} [1-F(l)]dl. \tag{a.16}$$

Using (18) and (35) to eliminate $S(\bar{y})$ from (a.15) yields

$$\frac{\eta\beta k}{1-\eta} \cdot \mu = y_d + \alpha_f R(q) - b + \frac{\beta\delta}{1-\beta(1-\delta)} \int_{y_d}^{\bar{y}} [1-F(l)]dl. \tag{a.17}$$

Substituting α_f from (23) into (a.17) and using the expression of u in (40), we get (39). Subtracting (a.15) from (a.16) yields

$$[1-\beta(1-\delta)]S(\bar{y}) = \bar{y} - y_d \tag{a.18}$$

Substituting $S(\bar{y})$ from (18) into (a.18) implies (38) ■

Proof of Lemma 1

Proof. It is obvious that the left-hand side of (38) is an increasing function of μ and independent of y_d; while the right-hand side of (38) is a decreasing function of y_d and independent of μ. Therefore, (38) slopes downward in (y_d, μ) space.

Denote the left-hand side of (39) by $\Gamma(y_d, \mu, R)$. Computation shows that

$$\frac{\partial\Gamma}{\partial y_d} = 1 - \frac{\beta\delta}{1-\beta+\beta\delta} + \frac{\alpha_h \delta R(q)}{m(\mu,1)} F'(y_d) + \frac{\beta\delta}{1-\beta(1-\delta)} F(y_d) > 0,$$

$$\frac{\partial\Gamma}{\partial\mu} = -\frac{\eta\beta k}{1-\eta} - \frac{\alpha_h \delta F(y_d)}{[m(\mu,1)]^2} R(q) \frac{dm(\mu,1)}{d\mu} < 0.$$

This implies,

$$-\frac{\frac{\partial\Gamma}{\partial y_d}}{\frac{\partial\Gamma}{\partial\mu}} > 0.$$

Therefore, (39) slopes upward in (y_d, μ) space. Furthermore, computation shows that

$$\frac{\partial \Gamma}{\partial R} = \alpha_h \left[1 + \frac{\delta F(y_d)}{m(\mu, 1)} \right] > 0$$

such that

$$-\frac{\frac{\partial \Gamma}{\partial R}}{\frac{\partial \Gamma}{\partial \mu}} > 0.$$

This implies that, for any given y_d, μ become smaller when $R(q)$ goes down, i.e. (39) shifts down if $R(q)$ goes down. ■

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