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The financial accelerator and monetary policy rules[*]

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ABSTRACT

The ability of financial frictions to amplify the output response of monetary policy, as in the financial accelerator model of Bernanke et al. (1999), is analyzed for a wider class of policy rules where the policy interest rate responds to both in inflation and the output gap. When policy makers respond to the output gap as well as in inflation, the standard financial accelerator model reacts less to an interest rate shock than does a comparable model without an operational financial accelerator mechanism. In recessions, when firm-specific volatility rises, financial acceleration due to financial frictions is further reduced, even under pure inflation targeting.

JEL Classification: E32 and E52.
Keywords: Financial acceleration, financial frictions, monetary policy.

1 Introduction

Bernanke et al. (1999) (henceforth BGG) show that credit market imperfections, when incorporated into a canonical New Keynesian macroeconomic model, significantly amplify the transmission of both real and nominal shocks. An unexpected decrease in the nominal interest rate, for example, raises the net worth of borrowers, which in BGG's financial accelerator framework lowers the external finance premium faced by borrowers. Hence, relative to models without credit market imperfections, the financial accelerator model yields a greater investment and thus output response to nominal interest rate shocks. In the original BGG contribution, monetary policy is modeled by an interest rate feedback rule where the contemporaneous policy rate reacts to its lag and to the lag of inflation.

In an early application of the financial accelerator model, Gilchrist and Leahy (2002) compare the transmission mechanism of anticipated and unanticipated real shocks under weak and strict inflation targeting as well as under a monetary policy rule that reacts to deviations in entrepreneurial net wealth. They do not, however, consider policy rules that react to the output gap. The principle aim of this paper is to examine the robustness of financial acceleration under alternative monetary policy rules where the policy rate reacts to both inflation and the output gap. Further, we investigate how an increase in financial frictions, which we model as a rise in firm-specific volatility and default rates, affects the transmission of monetary policy shocks under various monetary policy rules.

The amplification of monetary policy shocks by the financial accelerator mechanism can be readily overturned by assuming a more canonical Taylor-type interest rate rule where the policy rate reacts to both inflation and the output gap. Financial attenuation, as opposed to acceleration, becomes more pronounced the smaller the weight placed by the policy maker on inflation stabilization relative to the weight put on the output gap. One possible implication of this result is that, in the current conjuncture, where monetary authorities are increasingly moving away from inflation towards output stabilization (see arguments put forward by Taylor (2010) for the US and Martin and Milas (2011) for evidence for the UK), the presence of financial frictions actually makes monetary policy less effective. In the face of increasing financial frictions, our analysis suggests that the impact of financial frictions on the transmission of monetary policy shocks is reduced.

The remainder of this paper is structured as follows: Section 2 briefly outlines the financial

accelerator model of BGG, and section 3 describes the calibration of the model's parameters. In section 4, we examine the transmission mechanism of monetary policy shocks under strict inflation targeting as well as under a canonical Taylor rule. In section 5, we analyze how the transmission mechanism is altered by an increase in firm-specific volatility. Section 6 concludes.

2   Model

The model used is a variant of Bernanke et al. (1999), also described in Nolan and Thoenissen (2009). This is a relatively standard new Keynesian model with financial frictions. Agents have preferences defined separably over consumption and labour. Final goods markets are imperfectly competitive and there is Calvo-style stickiness in prices. Intermediate goods are produced by entrepreneurs who face financial frictions as in BGG.

The basic story underlying the financial frictions model of BGG is well understood, and so we will be brief. Entrepreneurs have insufficient funds to meet their investment needs. Hence, there is a demand for loanable funds, supplied by private agents via financial intermediaries. The financial intermediaries know that a fixed proportion of firms they lend to will go under. Furthermore, the returns from a particular investment are known with certainty only to the entrepreneur. The financial intermediary can only verify the return at some cost. It turns out that the optimal contract charges a premium on funds borrowed proportional to the entrepreneurs' net wealth. The higher the net wealth and the more funds sunk into a project by the entrepreneur, the more closely aligned are the incentives of the entrepreneur and the investor, and vice versa. Another way of putting this is that the expected gross return to holding a unit of capital, R^k, is linked to the risk free rate, R, through a risk premium as in:

$$\frac{ER_{t+1}^k}{R_{t+1}} = \chi \left(\frac{Q_t K_{t+1}}{NW_{t+1}} \right) \tag{1}$$

where Q is the price of capital and K is the capital stock. The greater the entrepreneur's net wealth, NW_{t+1} relative to the aggregate capital stock, the smaller will be the external finance premium. Entrepreneurial net wealth, NW_t evolves as follows:

$$NW_{t+1} = \gamma \left[R_t^k Q_{t-1} K_t - R_t \Upsilon_{t-1} - \frac{\mu \int_0^{\bar{\omega}_t} \omega dF(\omega) R_t^k Q_{t-1} K_t}{(Q_{t-1} K_t - NW_t)} \Upsilon_{t-1} \right] + w_t^e \tag{2}$$

where $\Upsilon_{t-1} \equiv (Q_{t-1} K_t - NW_t)$ and γ is the survival probability of the entrepreneur. Ag-

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gregate entrepreneurial net wealth, NW, is equal to the equity held by entrepreneurs at $t-1$ who are still in business at t, plus the entrepreneurial wage, w_t^e. All other terms are defined as in BGG.

Monetary policy is modeled by an interest rate feedback rule where the current period policy rate reacts to deviations in inflation from target (as in BGG) and to deviations of the output gap.

Table 1 summarizes the linearized equations of the model.

<center>Table 1: Linearized model equations</center>

Euler eqn	$\hat{\mu}_t = \hat{\mu}_{t+1} + i_t - E\pi_{t+1}$
Cons.- labour trade off	$\hat{\mu}_t + \phi\hat{n}_t - \hat{w}_t = 0$
MPL	$\hat{w}_t = \widehat{mc}_t + ((1-s_k)(1-s_e)-1)\,\hat{n}_t + s_k\hat{k}_t$
MPK	$\hat{\rho}_t = \widehat{mc}_t + (1-s_k)(1-s_e)\hat{n}_t - (1-s_k)\,\hat{k}_t$
Capital accumulation	$\hat{k}_{t+1} = \delta\hat{x}_t + \delta\hat{k}_t$
Price of capital	$\hat{r}_t^k = \frac{1-\delta}{r^k}\hat{q}_t - \hat{q}_{t-1} + \left(1 - \frac{1-\delta}{r^k}\right)\hat{\rho}_t$
Tobin's q	$\hat{q}_t = \phi''(x/k)\delta/\phi'(x/k)\left[\hat{x}_t - \hat{k}_t\right]$
GDP	$\hat{y}_t = s_k\hat{k}_t + (1-s_k)(1-s_e)\hat{n}_t$
Constraint	$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{x}{y}\hat{x}_t + \frac{DK}{Y}\Phi_t$
Philips curve prices	$\pi_t = \beta E\pi_{t+1} + \kappa_p\widehat{mc}_t$
Marginal utility	$\hat{\mu}_t = -\rho\hat{c}_t$
Fisher equation	$rr_t = i_t - E\pi_{t+1}$
Interest rate rule	$\hat{i}_t = \phi_i\hat{i}_{t-1} + (1-\phi_i)\phi_\pi\hat{\pi}_{t-1} + (1-\phi_i)\phi_y\hat{y}_t^{gap} + \hat{u}_t$
Entrepreneurial net wealth	$\frac{nw}{k}\widehat{nw}_{t+1} = \frac{\gamma}{\beta}\frac{nw}{k}\widehat{nw}_t + \gamma r^k\hat{r}_t^k + \frac{\gamma}{\beta}\left(\frac{nw}{k}-1\right)\hat{r}_{t-1}^k$
	$+\gamma\left(r^k - 1/\beta\right)\left[\hat{q}_{t-1} + \hat{k}_t\right] + \frac{(1-s_k)s_e}{s_k}\left(r^k - 1/\beta\right)\left[\hat{y}_t + \widehat{mc}_t\right]$
External finance premium	$E\left(\hat{r}_{t+1}^k - rr_t\right) = \chi\left(\hat{q}_t + \hat{k}_{t+1} - \widehat{nw}_{t+1}\right)$
Interest rate shock	$\hat{u}_t$
Parameters	$\Phi_t \equiv \ln\left[\frac{\mu\int_0^{\bar{\omega}_t}\omega R_t^k Q_{t-1}K_t\ f(\omega)d\omega}{\mu\int_0^{\bar{\omega}}\omega R^k\ f(\omega)d\omega K}\right]$
	$D \equiv \mu\int_0^{\bar{\omega}}\omega R^k\ f(\omega)d\omega$

All variables are per capita log-deviations from steady state. μ = marginal utility of consumption, mc = marginal cost, i = nom. interest π = inflation, n= hours worked, k = capital stock, x = investment, w = real wage, ρ = MPK, r^k= price of capital, q = Tobin's q, y = GDP, c = consumption, rr = risk free real interest rate, nw = entrepreneurial net wealth, u = monetary policy shock

3 Calibration

The model calibration is summarized in Table 2. The values chosen for structural parameters closely follow BGG, Gilchrist and Leahy (2002), Nolan and Thoenissen (2009) and are quite standard in the literature. Under our baseline calibration, monetary policy is assumed to respond only to lagged inflation and the coefficient on inflation is set to 1.1, as in BGG. When we allow the monetary authority to react to output gap, we set the weight on the output gap to 0.2 which is in line with estimates from Christensen and Dib (2008).

Table 2: Parameters of the models

Parameter	Description	Value
Parameters in utility function		
β	Discount factor	0.9902
ρ	Consumption	1
η	Labour	1/3
Parameters in production of goods		
s_k	Capital share	0.35
s_e	Share of entrepreneurial labour	0.01
δ	Depreciation rate	0.025
$\frac{-\phi''(x/k)\delta}{\phi'(x/k)}$	Curvature of adjustment cost fn.	4
Parameters in retail sector		
X	Steady state markup (prices)	1.1
α^p	Calvo parameter prices	0.75
κ_p	$(1-\alpha^p)(1-\alpha^p\beta)/\alpha^p$	
Parameters in the monetary policy rule		
ϕ_i	Weight on lagged policy rate	0.9
ϕ_π	Weight on inflation	1.1
ϕ_y	Weight on output gap	0.2
Parameters in financial accelerator		
μ	Proportion of output lost to monitoring	0.12
σ	Volatility of firm-specific shock	0.28
γ	Survival probability of entrepreneurs	0.978
$\frac{R^k}{R}$	External finance premium (annualized bps)	226
$\frac{QK}{NW}$	Capital stock to net worth (leverage) ratio	1.982
$F(\omega)$	Quarterly business failure rate	0.0086
ω	Cut-off rate for default	0.494
χ	Elasticity	0.039

4   Monetary policy shocks

In this section, we analyze the transmission of monetary policy shocks in the presence of financial frictions for alternative policy rules.

Figure 1 shows the response of output, investment, the Fisher equation or risk free real interest rate, rr_t (as in Table 1), and the external finance premium (in annualized basis points) to an unanticipated 100 basis point decline in the nominal interest rate in the financial accelerator (FA) model and an alternative model with the same steady state but a constant external finance premium (No FA).

Under the pure inflation targeting rule, financial frictions considerably amplify the effects of monetary policy shocks. A negative interest rate shock lowers the costs of borrowing for the entrepreneur and leads to an increase in investment and output. In the FA model, the increase in the relative price of capital (Tobin's q) associated with the investment boom raises the net wealth of entrepreneurs which lowers the external finance premium, because it allows entrepreneurs to self-finance a greater proportion of new investment. A lower external finance premium further increases investment and output. A monetary expansion, which lowers the policy rate, raises inflation. The real interest rate applicable to the consumer, therefore, declines by more than the initial reduction in the nominal interest rate. Monetary policy reacts to the ensuing inflation with a lag, returning the policy rate to its steady state. The adjustment paths of the real interest rate, and therefore the paths of consumption, are very similar in the models. As a result, the difference in the responses of output is mainly driven by the different responses of investment.

Figure 2 shows the response of the models assuming an interest rate feedback rule where the policy rate responds to both lagged inflation and the current output gap. For this set of impulse responses, we set the coefficient on the output gap in the Taylor rule, ϕ_y, to 0.2. Unlike in the original BGG specification, under this monetary policy rule there is no financial acceleration relative to the constant external finance premium case. The external finance premium declines, but does so by less than in the previous case (17 as opposed to 37 bps). Investment is still subject to the financial accelerator mechanism, albeit to a lesser extent.

With investment being more responsive to monetary policy shocks, an interest rate rule that responds to the output gap adjusts the real interest rate more rapidly. As a result, the real interest rate falls by less and returns to its steady state value quicker in the FA

Figure 1: Monetary shock - pure inflation targeting
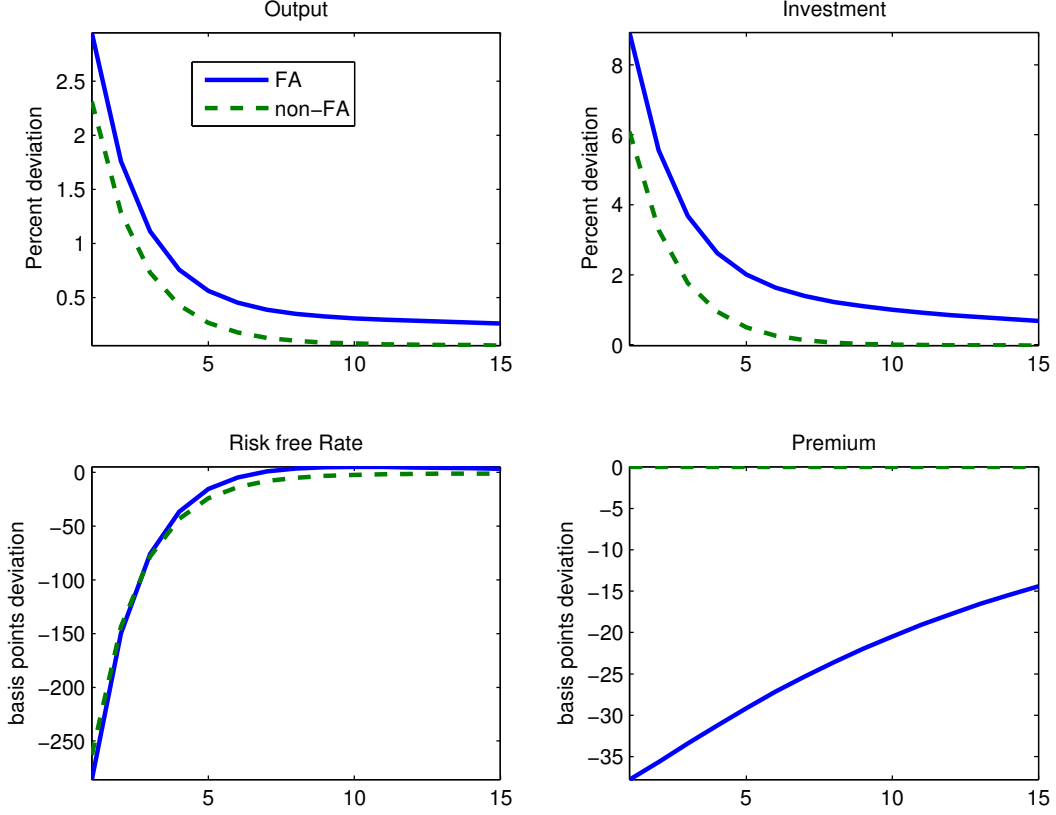
Note: $\hat{\imath}_t = \phi_i \hat{\imath}_{t-1} + (1 - \phi_i)\phi_\pi \hat{\pi}_{t-1}$ where $\phi_i = 0.9, \phi_\pi = 1.1$

model than in the non-FA model. The dynamics of the real interest rate are linked to the dynamics of consumption via the Euler equation. Because the real interest rate falls by more in the non-FA model, consumption rises by more. The greater response of consumption in the non-FA model helps to explain why we do not observe financial acceleration in output despite the financial acceleration in investment.

We analyze the robustness of this result for various coefficient values on inflation and output in the monetary policy rule. Figure 3 plots the ratio of the initial output responses of the FA and the non-FA models to a negative interest rate shock. A value of this ratio greater than 1 implies that monetary policy shocks are amplified, while a value below 1 indicates that the effect of monetary policy shocks is attenuated. Figure 3 shows that for sufficiently large values of ϕ_y relative to ϕ_π, the output response to monetary policy shocks is attenuated, rather than accelerated, by the presence of financial frictions. The greater the weight that the policy maker puts on output stabilization, as opposed to inflation stabilization, the less

Figure 2: Monetary shock - Taylor-type feedback rule
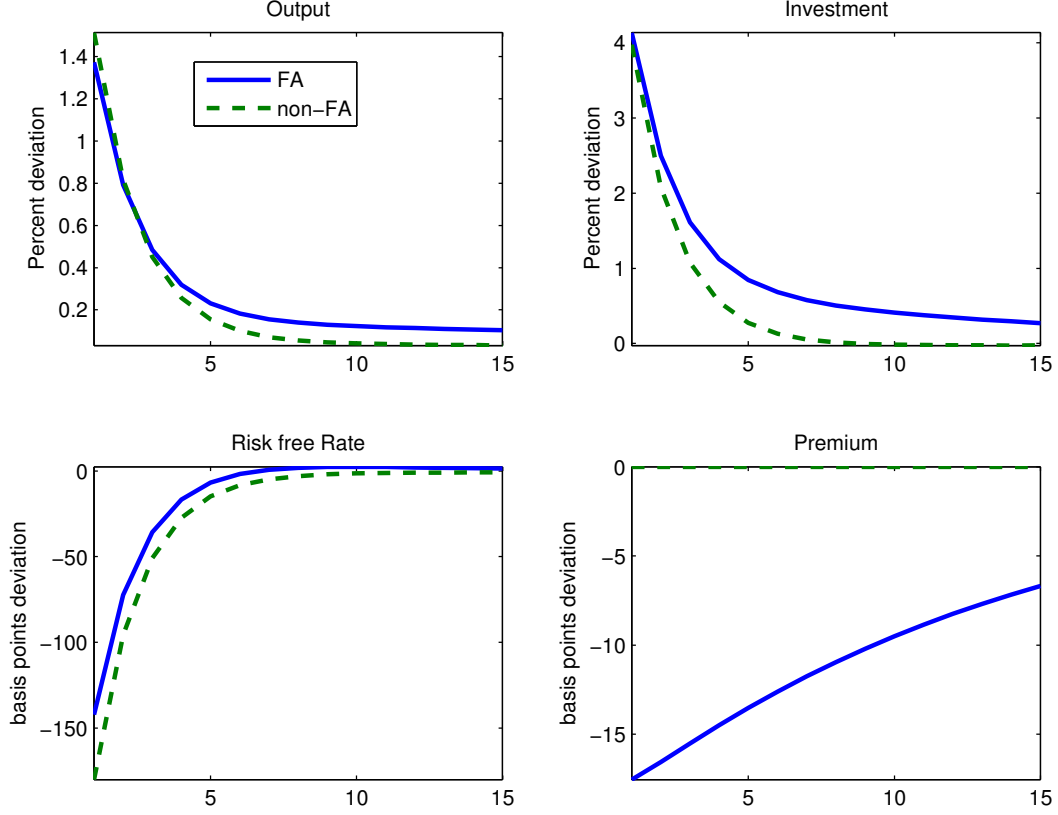
Note: $\hat{\imath}_t = \phi_i \hat{\imath}_{t-1} + (1 - \phi_i)\phi_\pi \hat{\pi}_{t-1} + (1 - \phi_i)\phi_y \hat{y}_t^{gap}$ where $\phi_i = 0.9, \phi_\pi = 1.1, \phi_y = 0.2$

effective discretionary monetary policy becomes[1].

5   Financial Acceleration and Firm-Specific Volatility

This section analyzes how an increase in financial frictions affects the transmission of monetary policy shocks under various monetary policy rules. It is well established that economic downturns are associated with an increase in firm-specific volatility and default rates, see for example Bruche and Gonzlez-Aguado (2010) or Kollmann et al. (2011). In the BGG framework there is a direct mapping between firm-specific volatility, σ, and the business failure rate. Firm-specific volatility also affects the leverage ratio, the steady state external finance premium as well as the elasticity of the external finance premium to the net wealth to loans ratio, χ.

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[1]Appendix A provides further sensitivity analysis. Appendix A1 considers an alternative policy rule where monetary policy is allowed to respond to output gap growth. Appendix A2 examines the robustness of our results when agents are assumed to be more risk averse

Figure 3: Financial acceleration of monetary policy shocks - various policy rules
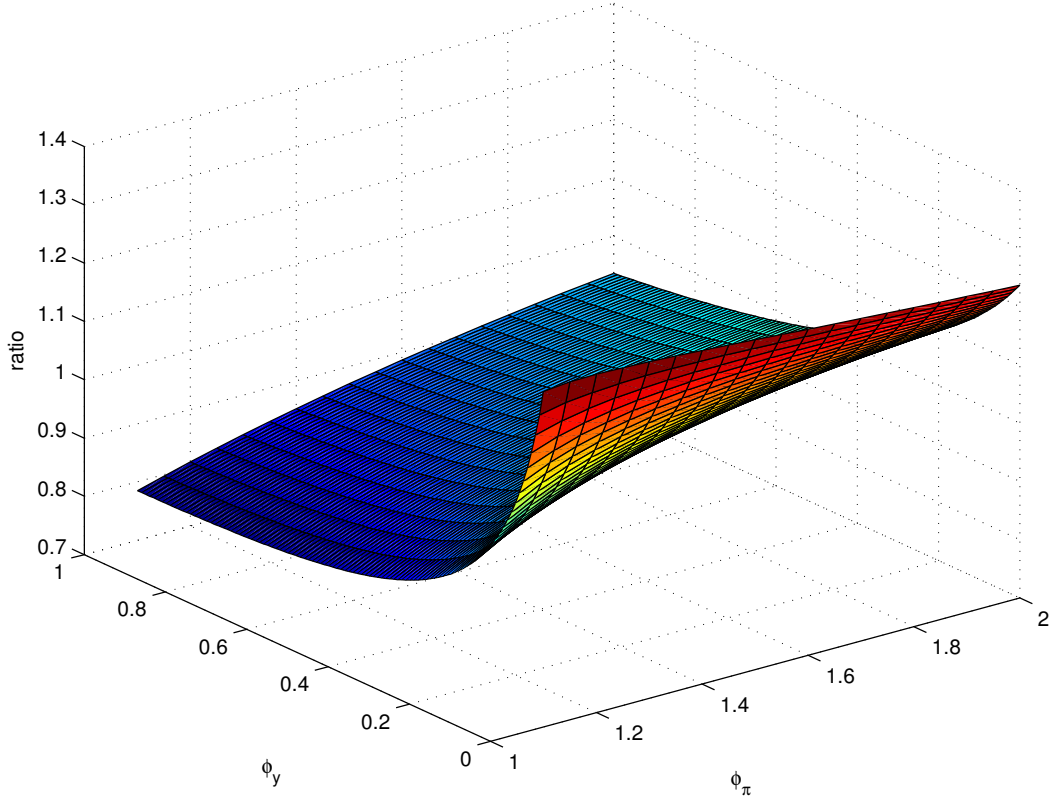

Note: The surface shows the initial impulse response of output for the FA model relative to that of the non-FA model for values of ϕ_π from 1.1 to 2 and for ϕ_y from 0 to 1. A value of more than 1.0 signifies financial acceleration.

Figure 4 shows how these model parameters are affected by a rise in firm-specific volatility. A rise in volatility raises the business failure rate, the external finance premium and the elasticity of the external finance premium, χ. As the default rate and the external finance premium rise, the leverage ratio falls. Appendix B shows how to derive the parts of the steady state of the model pertaining to the financial accelerator mechanism.

The steady state leverage ratio and the elasticity of the external finance premium, χ, play a key role in determining whether an increase in firm-specific volatility accentuates the differences between the standard New Keynesian and the financial accelerator models (for a common steady state). An increase in χ makes the external finance premium more sensitive to changes in net wealth, *ceteris paribus*, amplifying the effects highlighted in Figures 1 and 2. A reduction in the leverage ratio, on the other hand, dampens the response of net wealth to a monetary policy shock, thus reducing the effect of monetary policy shocks on the external

Figure 4: Firm-specific volatility and the determinants of χ
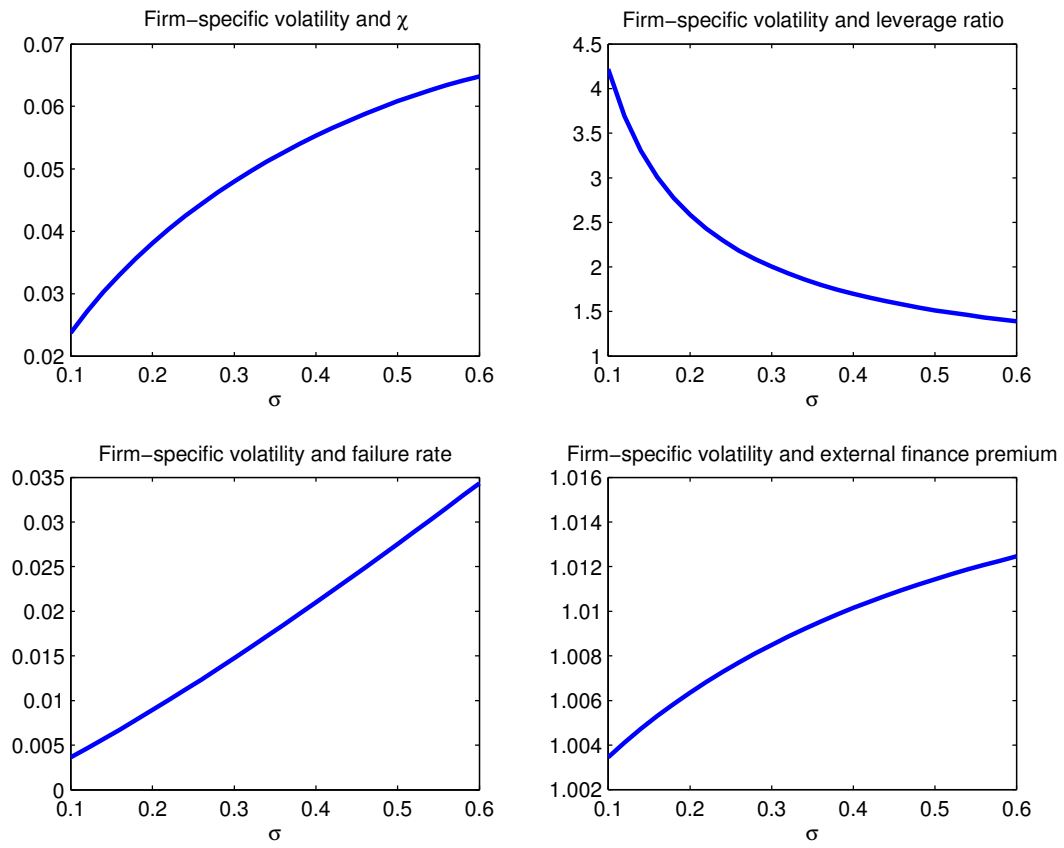
Steady state effects of varying the volatility of the firm-specific productivity shock, σ.

finance premium. As can be seen in equation (3), the log-linearized net wealth equation, a lower leverage ratio, $\frac{k}{nw}$ lowers volatility of net wealth (by reducing the responsiveness of net wealth to its determinants):

$$
\begin{aligned}
\widehat{nw}_{t+1} &= \frac{\gamma}{\beta}\widehat{nw}_t + \frac{k}{nw}\gamma r^k \hat{r}_t^k + \frac{\gamma}{\beta}\left(1 - \frac{k}{nw}\right)\hat{r}_{t-1}^k + \frac{k}{nw}\gamma\left(r^k - 1/\beta\right)\left[\hat{q}_{t-1} + \hat{k}_t\right] \quad (3) \\
&+ \frac{k}{nw}\frac{(1-s_k)s_e}{s_k}\left(r^k - 1/\beta\right)\left[\hat{y}_t + \widehat{mc}_t\right]
\end{aligned}
$$

In Figures 5 and 6, we analyze the ratio of the initial responses of the FA and the non-FA model to a unit monetary policy shock for various levels of firm-specific volatility. As before, in order to keep the models comparable, we impose $\chi = 0$ on the non-FA model, but only after the steady state has been computed, thus guaranteeing a common steady state across both models. Under pure inflation targeting, Figure 5 suggests that as firm-specific volatility increases, the difference in the response of the two models to a monetary policy shock diminishes. There is less financial acceleration of output in the FA model as firm-specific volatility rises.

A monetary policy rule that reacts to both inflation and the output gap, a traditional Taylor rule, can result financial *deceleration* in the FA model. Figure 6 suggests that just as under pure inflation targeting, the differences between the FA and the non-FA model diminish as firm-specific volatility increases.

There are two opposing effects on the transmission mechanism of monetary policy shocks when firm-specific volatility increases. On the one hand, the external finance premium becomes more sensitive to changes in entrepreneurial net wealth, *i.e* χ rises. This increases financial acceleration. On the other hand, leverage, defined as net borrowing relative to net wealth, declines and reduces the volatility of net wealth - thus reducing financial acceleration. Given our calibration, the latter effect outweighs the former so that an increase in the volatility of firm-specific shocks attenuates the differences between the financial accelerator and the standard new Keynesian models.

6   Conclusion

We revisit the transmission mechanism of monetary policy shocks in the financial accelerator model of Bernanke et al. (1999) when, as in the current conjuncture, monetary policy makers place a greater emphasis on output as opposed to inflation stabilization. Even for modest

Figure 5: Financial acceleration and firm-specific volatility
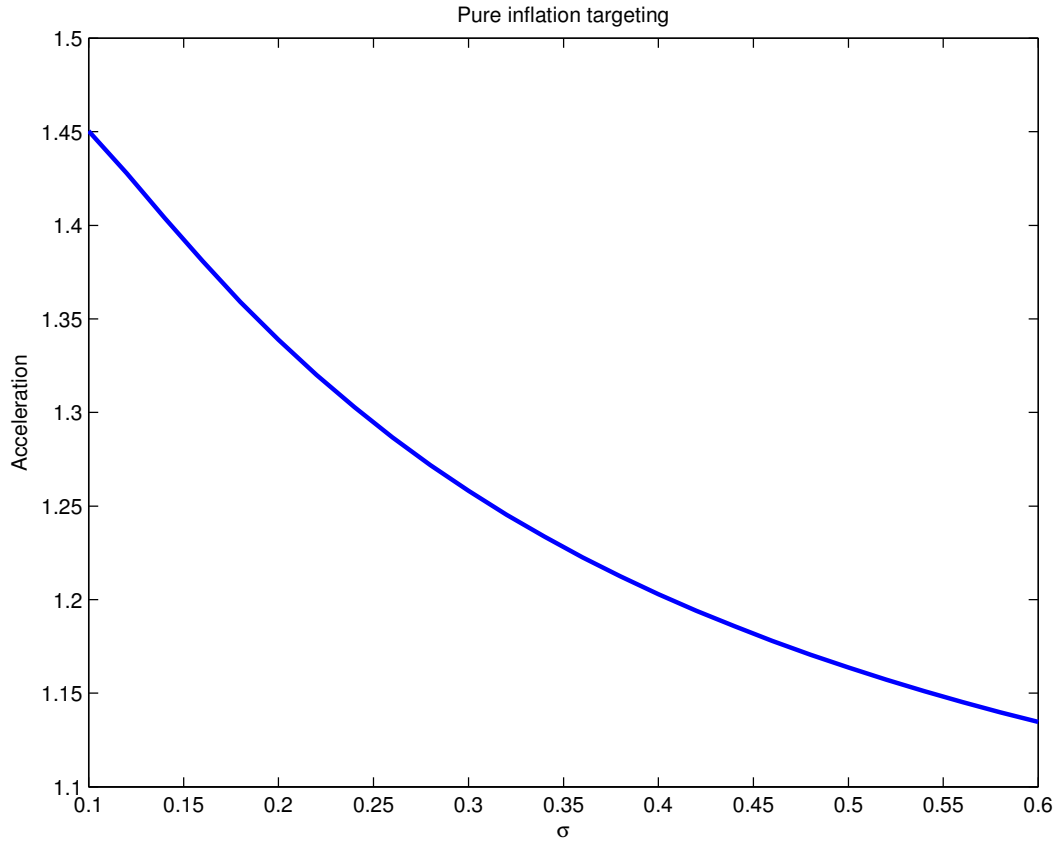
Note: Acceleration is defined as the ratio of the initial response of output to a monetary policy shock of the FA model relative to the standard new Keynesian model. The NK model has the same steady state as the FA model, differing only through setting $\chi = 0$ ex post. Policy rule: $\hat{\imath}_t = \phi_i \hat{\imath}_{t-1} + (1 - \phi_i)\phi_\pi \hat{\pi}_{t-1}$ where $\phi_i = 0.9, \phi_\pi = 1.1$

Figure 6: Financial acceleration and firm-specific volatility
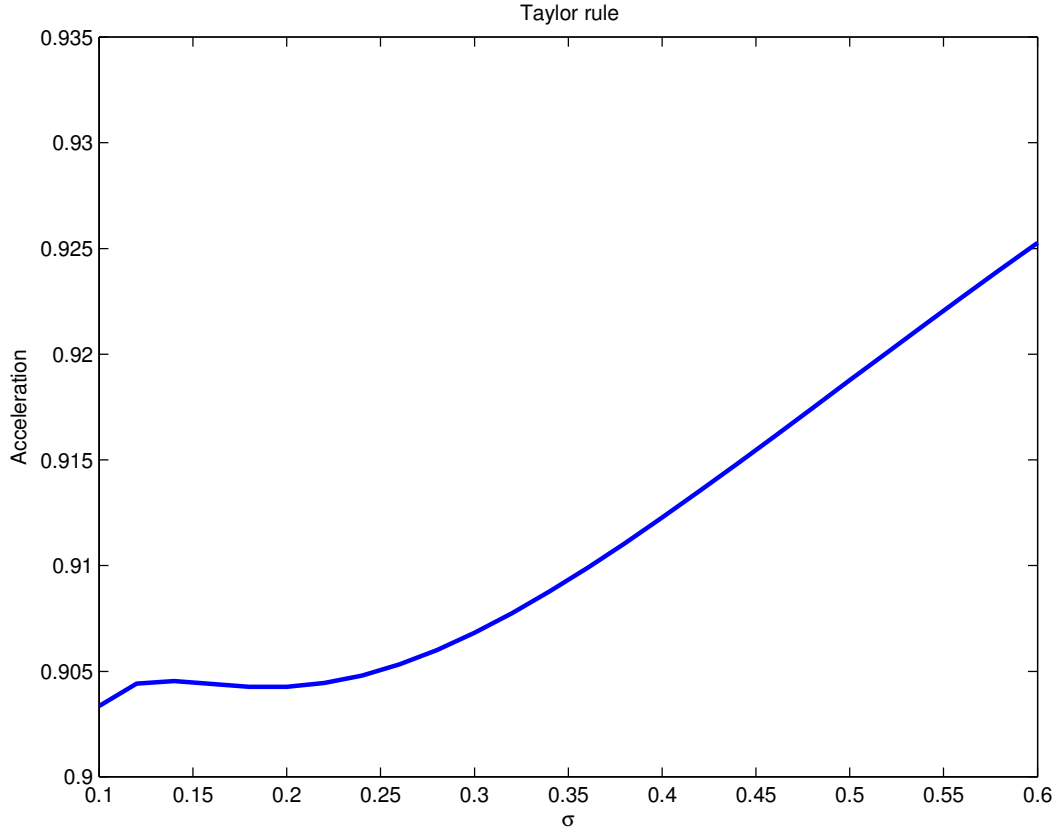
Note: Acceleration is defined as the ratio of the initial response of output to a monetary policy shock of the FA model relative to the standard new Keynesian model. The NK model has the same steady state as the FA model, differing only through setting $\chi = 0$ ex post. Policy rule: $\hat{\imath}_t = \phi_i \hat{\imath}_{t-1} + (1 - \phi_i)\phi_\pi \hat{\pi}_{t-1} + (1 - \phi_i)\phi_y \hat{y}_t^{gap}$ where $\phi_i = 0.9, \phi_\pi = 1.1, \phi_y = 0.2$

values of the coefficient on the output gap in the interest rate feedback rule, the response of GDP to a monetary policy shock is muted by the presence of the financial accelerator mechanism. We observe financial *dec*eleration following a monetary policy shock.

Furthermore, we investigate how an increase in financial frictions, which we model as a rise in firm-specific volatility and default rates, affects the transmission of monetary policy shocks under various monetary policy rules. We find that a rise in firm-specific volatility attenuates the differences between the financial accelerator and the standard new Keynesian models.

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A Sensitivity Analysis

A.1 Taylor rules with growth

The estimated Taylor-type rules of Smets and Wouters (2007) and De Graeve (2008) suggest that monetary policy makers react to both the output gap and to changes in the output gap. Here, we repeat the exercise of Figure 3 while keeping $\phi_{dy} = 0.28$, the weight on the change in the output gap at 0.28, the value suggested by De Graeve (2008). The central tenet of our argument is unaffected by this change.

A.2 Risk averse households

Figure 8 repeats the analysis of Figure 3 for the case where agents are more risk averse. Increased risk aversion results in a smoother consumption profile which should reduce the amount of deceleration in the Taylor rule example. Compared to Figure 3, we see more financial acceleration for small output gap coefficients and less deceleration for large values of ϕ_y when the coefficient of relative risk aversion is set to 6, a relatively high value in the literature. Nevertheless, the basic tenet of our argument is unaffected by this change.

B The optimal contracting problem - not for publication

Contracting problem - draws heavily on the appendices in Gertler et al. (2007) and Meier and Müller (2006).

Figure 7: Sensitivity Analysis: Financial acceleration of monetary policy shocks - various policy rules with $\phi_{dy} = 0.28$
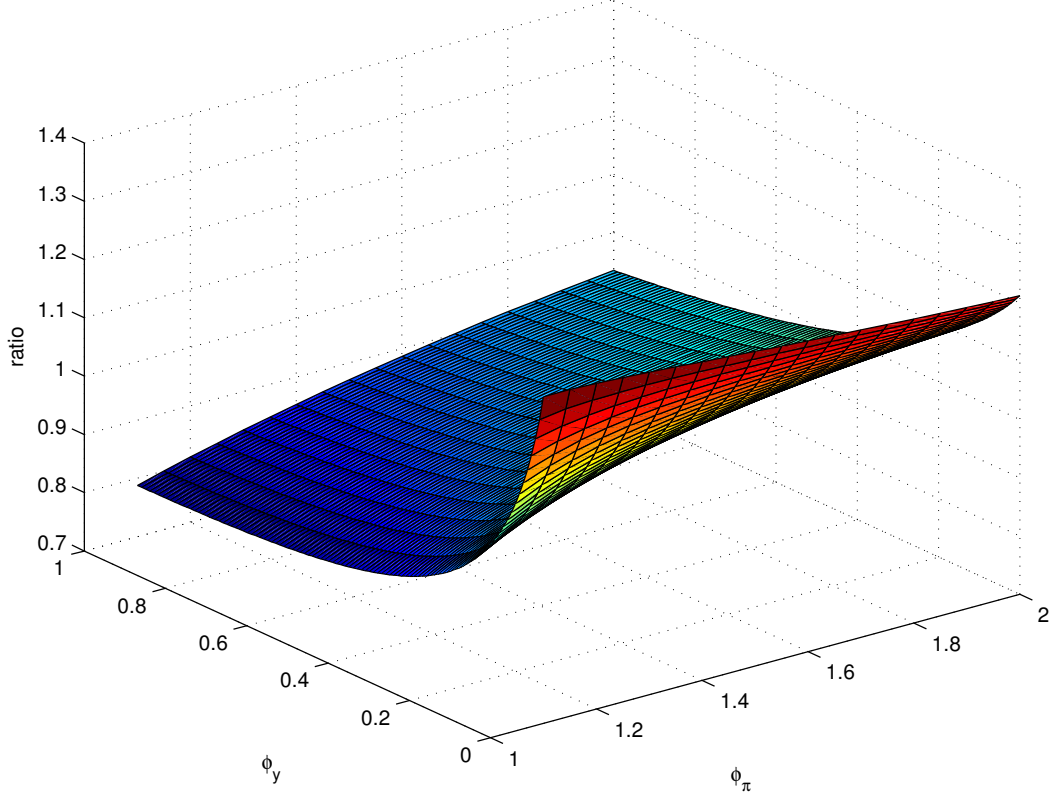
Note: The surface shows the initial impulse response of output for the FA model relative to that of the non-FA model for values of ϕ_π from 1.1 to 2 and for ϕ_y from 0 to 1 for a given weight of 0.28 on the growth in the output gap. A value of more than 1.0 signifies financial acceleration.

Let profits per unit of capital equal:

$$\omega R^k$$

where $\ln(\omega)$ is lognormally distributed so that $E(\omega) = 1$. Let N be the steady state level of net worth, Q the price of capital (Tobin's q, which is 1 in the steady state). K is the steady state value of the capital stock. Each period, the entrepreneur has to buy the entire capital stock, to do so, the entrepreneur borrows $B = QK - N$ from the financial intermediary to invest K units of capital in a project. The total return on capital is $\omega R^k QK$, where R^k is the steady state return on capital. It is assumed that ω is unknown to both lender and borrower prior to the investment decision. After the investment decision, the lender can observe ω by paying a monitoring cost of $\mu \omega R^k QK$, where μ is a parameter between 0 and 1. The

Figure 8: Sensitivity Analysis: Financial acceleration of monetary policy shocks - various policy rules assuming risk averse households (CRRA=6)
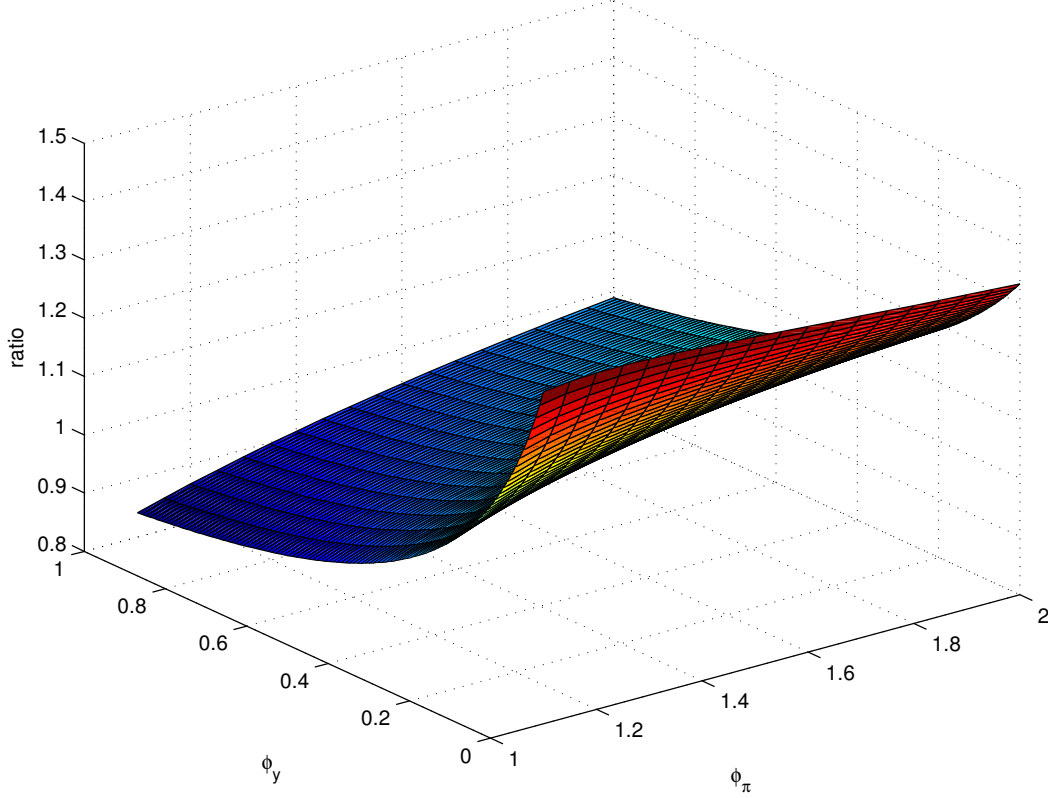
Note: The surface shows the initial impulse response of output for the FA model relative to that of the non-FA model for values of ϕ_π from 1.1 to 2 and for ϕ_y from 0 to 1 when households are risk averse. A value of more than 1.0 signifies financial acceleration.

opportunity cost of funds to the lender is the risk free rate, i.e. $R = 1/\beta$.

The optimal contract specifies a cutoff value of $\bar{\omega}$ such that if $\omega > \bar{\omega}$, the borrower pays the fixed amount $\bar{\omega}R^kQK$ to the lender and keeps the equity $(\omega - \bar{\omega})R^kQK$. If ω turns out to be less than $\bar{\omega}$, the borrower receives nothing and the lender monitors the borrower, incurring the monitoring cost, and receives $(1 - \mu)\omega R^kQK$, i.e. the realised return less the monitoring cost. In equilibrium, the lender earns an expected return equal to the safe rate R implying

$$\left(\int_0^{\bar{\omega}} \omega f(\omega)d\omega + \bar{\omega} \int_{\bar{\omega}}^\infty f(\omega)d\omega - \mu \int_0^{\bar{\omega}} \omega f(\omega)d\omega \right) R^kQK = R(QK - N) \qquad (4)$$

where $f(\omega)$ is the pdf of ω. The term $\int_0^{\bar{\omega}} \omega f(\omega)d\omega = E(\omega|\omega < \bar{\omega})\Pr(\omega < \bar{\omega})$ and $\bar{\omega} \int_{\bar{\omega}}^\infty f(\omega)d\omega = \bar{\omega}\Pr(\omega \geq \bar{\omega})$

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The optimal contract maximizes the payoff to the entrepreneur subject to the lender earning the required return:

$$\max_{K,\bar{\omega}} \left(\int_{\bar{\omega}}^{\infty} \omega f(\omega) d\omega \right) R^k Q K$$

subject to (4).

BGG define the following terms to simplify the maximization problem. $\Gamma(\bar{\omega})$ is defined as the expected gross share of profits going to the lender. Likewise, $1 - \Gamma(\omega)$ is the share going to the entrepreneur.

$$\Gamma(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega f(\omega) d\omega + \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega$$

Let us define expected monitoring costs as:

$$\mu G(\bar{\omega}) \equiv \mu \int_0^{\bar{\omega}} \omega f(\omega) d\omega$$

We can now re-write the steady state contracting problem (in terms of the Lagrange), but first define the following: $s = R^k/R$ and $k = QK/N$

$$\max L = (1 - \Gamma(\bar{\omega}))sk + \lambda \left\{ [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] sk - (k - 1) \right\}$$

$$\frac{\partial L}{\partial k} : ((1 - \Gamma(\bar{\omega})) + \lambda [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]) s - \lambda = 0$$

$$\frac{\partial L}{\partial \bar{\omega}} : -\Gamma'(\bar{\omega})sk + \lambda [\Gamma'(\bar{\omega}) - \mu G(\bar{\omega})] sk = 0$$

$$\frac{\partial L}{\partial \lambda} : [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] sk = (k - 1)$$

Next, we rearrange these equations to yield expressions for λ, s and k as functions of ω

$$\lambda(\bar{\omega}) = \frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G(\bar{\omega})}$$

$$s = \frac{\lambda(\omega)}{1 - \Gamma(\bar{\omega}) + \lambda(\omega) [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}$$

$$s = \frac{\lambda(\omega)}{\Psi(\omega)} \text{ where } \Psi(\omega) = 1 - \Gamma(\bar{\omega}) + \lambda(\omega) [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]$$

$$
\begin{aligned}
\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] sk &= (k-1) \\
\frac{\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] \lambda(\omega)}{\Psi(\omega)} &= \frac{(k-1)}{k} \\
-\frac{\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] \lambda(\omega)}{\Psi(\omega)} + 1 &= 1/k \\
\frac{\Psi(\omega)}{1 - \Gamma(\bar{\omega})} &= k
\end{aligned}
$$

so we have

$$
\begin{aligned}
\lambda(\bar{\omega}) &= \frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})} & (5) \\
s(\bar{\omega}) &= \frac{\lambda(\bar{\omega})}{\Psi(\bar{\omega})} & (6) \\
k(\bar{\omega}) &= \frac{\Psi(\bar{\omega})}{1 - \Gamma(\bar{\omega})} & (7)
\end{aligned}
$$

We need one more steady state relationship:

$$
\begin{aligned}
N_{t+1} &= \gamma \left[R_t^k Q_{t-1} K_t - R_t \left(Q_{t-1} K_t - N_t \right) - \kappa \int_0^{\bar{\omega}^j} \omega R_t^k Q_{t-1} K_t dF(\omega) \right] + W_t^e \\
N &= \gamma \left[R^k Q K - R Q K + R N - \kappa \int_0^{\bar{\omega}^j} \omega R_t^k Q_{t-1} K_t dF(\omega) \right] + W^e
\end{aligned}
$$

Now, according to GGN, the final expression reduces to:

$$
N = \gamma(1 - \Gamma(\bar{\omega})) R^k Q K + W^e
$$

$$
\frac{N}{K} = \gamma(1 - \Gamma(\bar{\omega})) R^k + \frac{W^e}{K}
$$

$$
\begin{aligned}
R^k &= \alpha Y/K + (1 - \delta) \\
W^e &= (1 - \alpha)\Omega Y
\end{aligned}
$$

Thus

$$
\begin{aligned}
\frac{W^e}{K} &= (1 - \alpha)\Omega Y/K = \frac{(1 - \alpha)\Omega}{\alpha}(R^k - (1 - \delta)) \\
\frac{N}{K} &= \gamma(1 - \Gamma(\bar{\omega})) R^k + \frac{(1 - \alpha)\Omega}{\alpha}(R^k - (1 - \delta)) \\
\beta\left(k(\bar{\omega})\right)^{-1} &= \gamma(1 - \Gamma(\bar{\omega})) s(\bar{\omega}) + \frac{(1 - \alpha)\Omega}{\alpha}(s(\bar{\omega}) - (1 - \delta)\beta) & (8)
\end{aligned}
$$

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We now substitute (5), (6) and (7) into (8) to solve for $\bar{\omega}$. To do so we need to deal with a few 'awkward' terms in $\Psi(\bar{\omega})$:

$$\Psi(\bar{\omega}) = 1 - \Gamma(\bar{\omega}) + \Gamma'(\bar{\omega})\frac{[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})}$$

F $\qquad\qquad '(\bar{\omega}) = 1 - F(\bar{\omega})$ is the c.d.f of ω at $\bar{\omega}$ and $\mu G'(\bar{\omega}) = \mu \bar{\omega} f(\bar{\omega})$ where $f(\bar{\omega})$ is the p.d.f of ω at $\bar{\omega}$. This leaves us with $\Gamma(\bar{\omega})$ and $\Gamma(\bar{\omega}) - \mu G(\bar{\omega})$ which under the assumption that $z \equiv (\ln(\bar{\omega}) + 0.5\sigma^2)/\sigma$ BGG find that

$$\Gamma(\bar{\omega}) = \Phi(z - \sigma) + \bar{\omega}[1 - \Phi(z)]$$

$$\Gamma(\bar{\omega}) - \mu G(\bar{\omega}) = (1 - \mu)\Phi(z - \sigma) + \bar{\omega}[1 - \Phi(z)]$$

where Φ is the standard normal c.d.f.

$$\begin{aligned}
\Gamma'(\bar{\omega}) &= \Phi'(z - \sigma)\frac{1}{\bar{\omega}\sigma} + [1 - \Phi(z)] - \bar{\omega}\Phi'(z)\frac{1}{\bar{\omega}\sigma} \\
&= [1 - \Phi(z)]
\end{aligned}$$

$$\begin{aligned}
\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}) &= \Phi'(z - \sigma)\frac{1 - \mu}{\bar{\omega}\sigma} + [1 - \Phi(z)] - \bar{\omega}\Phi'(z)\frac{1}{\bar{\omega}\sigma} \\
&= [1 - \Phi(z)] - \Phi'(z - \sigma)\frac{\mu}{\bar{\omega}\sigma}
\end{aligned}$$

$$\lambda(\bar{\omega}) = \frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})} \tag{9}$$

$$s(\bar{\omega}) = \frac{\lambda(\bar{\omega})}{\Psi(\bar{\omega})} \tag{10}$$

$$k(\bar{\omega}) = \frac{\Psi(\bar{\omega})}{1 - \Gamma(\bar{\omega})} \tag{11}$$

Now in theory, one is able to solve the following equation, given $s(\bar{\omega})$, $k(\bar{\omega})$ and $\Gamma(\bar{\omega})$ for $\bar{\omega}$.

$$\beta(k(\bar{\omega}))^{-1} = \gamma(1 - \Gamma(\bar{\omega}))s(\bar{\omega}) + \frac{(1 - \alpha)\Omega}{\alpha}(s(\bar{\omega}) - (1 - \delta)\beta) \tag{12}$$

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The next step is to determine the elasticity of the risk premium with respect to k.

$$\left(E\hat{r}^k_{t+1} - \hat{r}_{t+1}\right) = -\frac{\varphi\left(\frac{R^k}{R}\right)}{\varphi'\left(\frac{R^k}{R}\right)}\frac{R}{R^k}\left(n_{t+1} - \hat{q}_t - \hat{k}_{t+1}\right)$$

specifically, we have to get a value for $-\frac{\varphi\left(\frac{R^k}{R}\right)}{\varphi'\left(\frac{R^k}{R}\right)}\frac{R}{R^k}$. Consider the following relationship:

$\frac{Q_tK_{t+1}}{N_{t+1}} = \varphi\left(\frac{ER^k_{t+1}}{R_{t+1}}\right)$ which in the steady state can be expressed as: $k = \varphi(s)$. Let g be a function that relates $\bar{\omega}$ to s and h be a function that relates $\bar{\omega}$ to k, i.e. funtions (6) and (7), respecitvely.

$$\begin{aligned} g &\equiv& s\left(\bar{\omega}\right) = \frac{\lambda(\bar{\omega})}{\Psi(\bar{\omega})} \\ h &\equiv& k(\bar{\omega}) = \frac{\Psi(\bar{\omega})}{1 - \Gamma(\bar{\omega})} \end{aligned}$$

then

$$\begin{aligned} k &=& \varphi\left(s\right) = h(g^{-1}(s)) \\ \varphi'\left(s\right) &=& \frac{h'}{g'} \end{aligned}$$

$$\begin{aligned} g' &=& \frac{\lambda'(\bar{\omega})\Psi(\bar{\omega}) - \lambda(\bar{\omega})\Psi'(\bar{\omega})}{[\Psi(\bar{\omega})]^2} \\ h' &=& \frac{\Psi'(\bar{\omega})(1 - \Gamma(\bar{\omega})) + \Psi(\bar{\omega})\Gamma'(\bar{\omega})}{[1 - \Gamma(\bar{\omega})]^2} \end{aligned}$$

$$\frac{\varphi\left(\frac{R^k}{R}\right)}{\varphi'\left(\frac{R^k}{R}\right)}\frac{R}{R^k} = \frac{\varphi\left(s\right)}{\varphi'\left(s\right)}\frac{1}{s} = \frac{k}{s}\frac{g'}{h'}$$

we already know what some of these terms are, but others we have yet to derive:

$$\begin{aligned} \lambda(\bar{\omega}) &=& \frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})} \\ \lambda'(\bar{\omega}) &=& \frac{\Gamma''(\bar{\omega})\left[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})\right] - \left[\Gamma''(\bar{\omega}) - \mu G''(\bar{\omega})\right]\Gamma'(\bar{\omega})}{[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})]^2} = \frac{\mu\left[G''(\bar{\omega})\Gamma'(\bar{\omega}) - \Gamma''(\bar{\omega})G'(\bar{\omega})\right]}{[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})]^2} \end{aligned}$$

$$\begin{aligned} \Psi(\omega) &=& 1 - \Gamma(\bar{\omega}) + \lambda(\omega)\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] \\ \Psi'(\omega) &=& -\Gamma'(\bar{\omega}) + \lambda'(\omega)\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] + \Gamma'(\bar{\omega})\lambda(\omega) - \lambda(\omega)\mu G'(\bar{\omega}) \end{aligned}$$

$$\Gamma'(\bar{\omega}) \quad = \quad [1 - \Phi(z)]$$

$$\Gamma''(\bar{\omega}) \quad = \quad -\Phi'(z)\frac{1}{\bar{\omega}\sigma}$$

$$\mu G'(\bar{\omega}) \quad = \quad \mu\Phi'(z - \sigma)\frac{1}{\bar{\omega}\sigma}$$

$$\mu G''(\bar{\omega}) \quad = \quad \mu\Phi''(z - \sigma)\frac{1}{(\bar{\omega}\sigma)^2}$$

$\Phi(z)$ is the standard normal c.d.f at z and $\Phi'(z)$ is the standard normal pdf at z.

$$G'(\bar{\omega}) = \Phi'(z - \sigma)\frac{1}{\bar{\omega}\sigma} = \frac{1}{\bar{\omega}\sigma}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(z(\bar{\omega}))^2}$$

where $z(\bar{\omega}) \equiv (\ln(\bar{\omega}) + 0.5\sigma^2)/\sigma$, we can differentiate this expression with respect to $\bar{\omega}$

$$G''(\bar{\omega}) \quad = \quad -\frac{1}{\bar{\omega}^2\sigma}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(z(\bar{\omega}))^2} - z(\bar{\omega})z'(\bar{\omega})\frac{1}{\bar{\omega}\sigma}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(z(\bar{\omega}))^2}$$

$$G''(\bar{\omega}) \quad = \quad -\frac{1}{\bar{\omega}\sigma}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(z(\bar{\omega}))^2}\left[\frac{1}{\bar{\omega}} + z(\bar{\omega})z'(\bar{\omega})\right]$$

$$G''(\bar{\omega}) \quad = \quad -G'(\bar{\omega})\left[\frac{1}{\bar{\omega}} + z(\bar{\omega})\frac{1}{\bar{\omega}\sigma}\right]$$

... and this is all one needs to work out the elasticity $\frac{k}{s}\frac{g'}{h'}$.

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