CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS
WORKING PAPER SERIES

CDMA08/10

Financial shocks and the US business cycle*

Charles Nolan[†]
University of St Andrews

Christoph Thoenissen[‡]
University of St Andrews

DECEMBER 2008

ABSTRACT

Employing the financial accelerator (FA) model of Bernanke, Gertler and Gilchrist (1999) enhanced to include a shock to the FA mechanism, we construct and study shocks to the efficiency of the financial sector in post-war US business cycles. We find that financial shocks are very tightly linked with the onset of recessions, more so than TFP or monetary shocks. The financial shock invariably remains contractionary for sometime after recessions have ended. The shock accounts for a large part of the variance of GDP and is strongly negatively correlated with the external finance premium. Second-moments comparisons across variants of the model with and without a (stochastic) FA mechanism suggests the stochastic FA model helps us understand the data.

JEL Classification: E30, E44, E52.

Keywords: Financial accelerator; financial shocks; macroeconomic volatility

* We would like to thank Arnab Bhattacharjee for advice on econometric matters and Max Gillman for a number of helpful discussions. We are also grateful for many useful discussions with participants at the 1st COOL Macro meeting in Cambridge. The usual disclaimer applies

[†] Address: Department of Economics, Castlecliffe, The Scores, St Andrews, Fife KY16 9AL, Scotland, UK. Tel +44 (0) 1334 462425. E-mail: Charles.Nolan@st-andrews.ac.uk, <http://www.st-andrews.ac.uk/economics/staff/pages/c.nolan.shtml>

[‡] Address: Department of Economics, University of St Andrew, St Andrews, Fife, KY16 9AL, Scotland, UK. Tel: +44 1334 462449. E-mail: christoph.thoenissen@st-andrews.ac.uk, <http://www.st-andrews.ac.uk/economics/staff/pages.c.thoenissen.shtml>

CASTLECLIFFE, SCHOOL OF ECONOMICS & FINANCE, UNIVERSITY OF ST ANDREWS, KY16 9AL

TEL: +44 (0)1334 462445 FAX: +44 (0)1334 462444 EMAIL: cdma@st-and.ac.uk

1. Introduction

This paper aims to document the role of a particular class of shocks in post-war US business cycles; specifically, shocks to the efficiency of the financial sector. The quantitative framework that we adopt is the financial accelerator model of Bernanke, Gertler and Gilchrist (1999)¹. Drawing on Townsend (1979), the key contribution of that work is to demonstrate that optimal financial contracting may amplify the responses of the macroeconomy to some shocks; financial markets may *unavoidably* increase the volatility of the economy. It is important to recognize that the financial structure of the economy is not an *independent* source of volatility in these models, but solely plays a role of leveraging other shocks. However, more recently, as we detail below, some researchers have modelled financial markets as providing an additional source of macroeconomic volatility prompted in part, no doubt, by Greenspan's oft-quoted remark about "irrational exuberance". However, the sense that corporate sector net worth and asset-price fluctuations can be important has been around for a very long time, certainly amongst policymakers^{2,3}.

1.1. Literature

The recent empirical literature is not entirely clear-cut on whether the financial accelerator (FA) model is a useful addition to DSGE models of the US economy. For example, Meier and Müller (2006) suggest that the financial frictions model improves only marginally the ability of their specification of the New Keynesian model to replicate the response of the economy to a monetary shock. They extract the empirical impulse responses to a monetary policy shock from a vector autoregression and 'fit' their model to the US data by matching impulse

---

¹The work of Carlstrom and Fuerst (1997), which developed a quantitative version of Bernanke and Gertler (1989), represents important progress in nesting financial frictions in a DSGE setting.

²Kiyotaki and Moore (1997) emphasize the role of asset prices in endogenously propagating cycles in credit extension.

³Recent events in the financial markets may also have reinforced perceptions that financial markets may not only propagate shocks, but contribute a few of their own.

responses. They argue that other features of the model, such as investment adjustment costs, are more important. Christensen and Dib (2008), on the other hand, use a maximum-likelihood procedure to estimate a new Keynesian model with and without a financial accelerator mechanism, and incorporating a wider set of shocks compared with Meier and Müller. In contrast, they find that the quantitative significance of the FA mechanism is somewhat more important in understanding monetary shocks, although it is less important for understanding output volatility.

There has been a number of applications of the FA framework on non-US data. For example, Gertler, Gilchrist and Natalucci (2007) use the FA model, nested in a small open economy framework, to interpret the Korean data following the financial crisis of the late 1990s. That contribution, whilst not a ‘test’ of the FA *per se*, does seem to attest to the usefulness of the model. Similarly, Hall (2001) suggests that some UK corporate sector behavior is consistent with the predictions of the FA model.

All of these papers analyze a version of the FA model where the financial sector is not an independent source of shocks. An alternative approach is Christiano, Motto and Rostagno (CMR, 2003, 2007) who use Bayesian techniques to estimate a model incorporating net wealth shocks, along with many other types of shocks. CMR (2007) is especially significant in that they estimate variants of their model on Euro area data as well as the US. Their variance decompositions generally suggest a significant role for net wealth shocks.⁴ Finally, De Graeve draws attention to stochastic variation in the external finance premium as an important element in the FA model’s explanation of the post-war US data.

A somewhat different strategy to that of CMR (2007) or De Graeve (2008) is adopted in this paper. Instead of estimating a model with a large number of shocks, we concentrate on three key drivers of the business cycle: total factor productivity shocks, monetary policy

---

⁴Whether or not the FA (including a shock or not) should be important for actual policy is another matter. Certainly some researchers (Gilchrist and Leahy, 2002, Faia and Monacelli, 2007 and Christiano, Motto and Rostagno, 2007) would basically argue that for all practical purposes monetary policy can generally do little better than stabilize inflation quite robustly. However, the unsettled issue is what happens when misalignments are really large. We do not address these issues in this paper.

shocks and financial friction shocks. We isolate FA (and other) shocks employing the approach of Benk, Gillman and Kejak (2005, 2008). Briefly, we use the Markov decision rules of the linearized solution of the model, along with actual data on predetermined and other endogenous variables, to back out the relevant shocks. The procedure is iterative so that the assumptions we use to derive the Markov decision rules are ultimately consistent with the shocks we recover (and any cross-correlations among the innovations to the drivers). By focusing on a limited number of familiar shocks, our aim is to emphasize any *incremental* contribution of the stochastic version of the FA model. So, we compare a baseline New Keynesian model driven by only monetary and productivity shocks; we then add a FA friction; and then we incorporate shocks to the FA friction. For reasons we discuss below, we think of this shock as a shock to the efficiency of the financial sector.

Whilst our model is somewhat simpler than CMR (2007), our identification of shocks radically different to theirs, our sample period somewhat longer and the stochastic structure of our model much simpler, we come to many similar, complimentary conclusions. The bottom line is that incorporating a stochastic FA sector in our model seems to help us interpret the US data somewhat better than a DSGE model without one.

The paper is set out as follows. Section 2 describes the key elements of the models. Section 3 discusses calibration issues and the following section discusses how we identify our stochastic driving processes. Section 5 analyzes our financial friction shock. First, we look at how that variable correlates with the NBER recession dates. We also uncover a very close link between our estimated shock and a measure of the external finance premium, giving us comfort that our estimated financial shocks are usefully interpreted as such. The role of our financial shock in fluctuations in key macroeconomic time series is assessed via variance decomposition analysis. In section 6 we compare our various models using standard second moment comparisons. Finally, section 7 offers a concluding discussion.

2. The Model

At its core, the model is a New Keynesian model with Calvo-style nominal stickiness in prices and wages and an economy-wide capital market. We incorporate monetary policy via a money-supply growth rule. Hence, to motivate the demand for money, we follow Sidrauski-Brock and include money in the utility function of the representative consumer. Along with the financial accelerator, endowed with a shock, we add habit persistence in consumption. All the features (except the FA shock) of our model are more or less standard. Our specific modelling choices (in particular sticky wages, a money growth rule and habit persistence in consumption) were motivated as follows: Modelling monetary policy as a money growth rule helps us conduct our analysis over a longer sample than if we had adopted a Taylor Rule perspective; the adoption of sticky wages helps us track the data on real wages and labour input more easily than by assuming flexible wages; and, habits in consumption helps to generate persistent responses in a number of macro aggregates following certain shocks.

2.1. Representative agent: demand and supply decisions

There are a large number of agents in the economy who evaluate their utility in accordance with the following utility function:

$$E_t \left\{ U(C_t, \frac{M_t}{P_t}, N_t) \right\} \equiv E_t \left\{ \frac{(C_t - h\bar{C}_{t-1})^{1-\rho}}{1-\rho} + \frac{\zeta}{1-b} (m_t)^{1-b} - \Psi \frac{N_t^{1+\eta}}{1+\eta} \right\}. \quad (2.1)$$

E_t denotes the expectations operator at time t , β is the discount factor, C is consumption, $\bar{C}$ is aggregate consumption, M is the nominal money stock, P is the price-level, m is the stock of real money balances, and N is labour supply. h , ζ and Ψ are all parameters greater than zero. ρ is the coefficient of relative risk aversion, b reflects money demand elasticity and η captures labour supply elasticity. Consumption is defined over a basket of goods

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad (2.2)$$

where the price level is

$$P_t = \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \quad (2.3)$$

The demand for each good is given by

$$c_t^d(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t^d, \quad (2.4)$$

where Y_t^d denotes aggregate demand. Agents face a time constraint each period (normalized to unity) such that leisure, L_t , is given by

$$L_t = 1 - N_t. \quad (2.5)$$

Agents also face the following flow budget constraint:

$$C_t + E_t \left\{ Q_{t,t+1} d_{t+1} \frac{P_{t+1}}{P_t} \right\} + m_t = d_t + m_{t-1} \frac{P_{t-1}}{P_t} + w_t N_t + \Pi_t + \tau_t. \quad (2.6)$$

Here d_{t+1} denotes the real value at date $t+1$ of the asset portfolio held at the end of period t . $Q_{t,T}$ is the stochastic discount factor between period t and T , and

$$\frac{1}{R_t} = E_t \{ Q_{t,t+1} \} \quad (2.7)$$

denotes the nominal interest rate on a riskless one-period bond. w_t denotes the real wage in period t , and Π_t is the real value of income from the corporate sector remitted to the individual (e.g., think of rental income from the capital stock along with a proportionate share in any final profits and transfers of entrepreneurial equity that accrue when entrepreneurs exit or die)⁵. Finally, τ_t is the lump-sum transfer from the government or central bank. In addition to the standard boundary conditions, necessary conditions for an optimum include:

$$U_{C_t}(\cdot) = \mu_t; \quad (2.8)$$

---

⁵Our set up follows Meier and Müller, as well as Christiano, Motto and Rostagno (2003) and has the advantage that aggregate consumption is determined exclusively by the intertemporal optimisation of households, without having to account separately for entrepreneurial consumption, as in Bernanke et al (1999).

$$\mu_t = R_t \beta E_t \mu_{t+1} \frac{P_t}{P_{t+1}}; \quad (2.9)$$

$$\frac{U_{M_t}(\cdot)}{U_{C_t}(\cdot)} = \frac{R_t - 1}{R_t}. \quad (2.10)$$

μ is the Lagrange multiplier associated with both the consumer's and the firm's optimization problem.

2.2. Entrepreneurs

The entrepreneurial sector follows closely the exposition of BGG. Other helpful recent expositions of this part of the model can be found in Christensen and Dib (2008), Meier and Müller (2006) and Gertler, Gilchrist and Natalucci (2007). The entrepreneurial sector is the source of the financial accelerator mechanism. Here, entrepreneurs combine hired labour and purchased capital in a constant returns to scale technology to produce intermediate goods. There are a large number of risk neutral entrepreneurs who each have a finite planning horizon. The probability that an individual entrepreneur will survive until the next period is denoted γ_t . When an entrepreneur 'dies', his net wealth is distributed amongst the households. This assumption is vital, as it ensures that entrepreneurs never accumulate enough net wealth to finance new capital expenditure entirely out of net wealth, ensuring that the entrepreneur has to go to the capital market to borrow funds prior to purchasing capital. Even though entrepreneurs die, the size of the entrepreneurial sector is constant, with new arrivals replacing departed entrepreneurs. It is usually assumed in this class of model that entrepreneurs are endowed with N_t^e units of labour, supplied inelastically as a managerial input of production. The wage from this activity acts as 'seed money' for newly arrived entrepreneurs.

The aggregate production function for any period t can be written as:

$$Y_t = Z_t K_{t+1}^\alpha H_t^{1-\alpha} \quad (2.11)$$

where, as in BGG, Y_t is aggregate output of intermediate goods, K_t is the aggregate amount of capital purchased by entrepreneurs in period $t - 1$, Z_t an exogenous technology parameter

capturing total factor productivity and H_t is the amount of labour input. Labour input is an aggregate of labour supplied by the household union, N_t , and labour supplied by the entrepreneurs, N_t^e , where:

$$H_t = N_t^\Omega (N_t^e)^{1-\Omega}. \quad (2.12)$$

The aggregate capital stock evolves according to

$$K_{t+1} = (1 - \delta)K_t + \phi\left(\frac{I_t}{K_t}\right) K_t, \quad (2.13)$$

where I_t denotes aggregate investment expenditure, and δ the depreciation rate of the capital stock. Aggregate investment expenditure yields a gross output of capital goods of $\phi\left(\frac{I_t}{K_t}\right) K_t$. Different from the standard new Keynesian model, we follow BGG and assume that adjustment costs are external to the intermediate goods producing firm. In equilibrium, our adjustment cost function implies that the price of a unit of capital in terms of the numeraire good, Q , is given by

$$Q_t = \left[\phi' \left(\frac{I_t}{K_t} \right) \right]^{-1}. \quad (2.14)$$

The shape of $\phi\left(\frac{I_t}{K_t}\right)$ is such that in the steady state $Q = 1$. Entrepreneurs sell their output to retailers. Recall that the markup of retail goods over intermediate goods is X_t so that the relative price of intermediate goods is $1/X_t$. Given the production function, (2.11), the rental rate of capital in $t + 1$ is

$$\frac{1}{X_{t+1}} \frac{\alpha Y_{t+1}}{K_{t+1}}. \quad (2.15)$$

Given the capital accumulation equation, and the fact that adjustment costs are external to the firm, the expected gross return to holding a unit of capital from t to $t + 1$ is

$$E_t\{R_{t+1}^k\} = E_t \left\{ \frac{\frac{1}{X_{t+1}} \frac{\alpha Y_t}{K_{t+1}} + Q_{t+1}(1 - \delta)}{Q_t} \right\}. \quad (2.16)$$

Finally, the optimal demand for household and entrepreneurial labour are given by:

$$w_t = \frac{1}{X_t} \partial Y_t / \partial N_t; \quad (2.17)$$

$$w_t^e = \frac{1}{X_t} \partial Y_t / \partial N_t^e. \quad (2.18)$$

2.2.1. Financial frictions

Entrepreneurs have insufficient funds to meet their investment needs. Hence, there is a demand for loanable funds, supplied by private agents via financial intermediaries. The financial intermediaries know that a fixed proportion of firms that it lends to will go under. Furthermore, the returns to a particular investment is known with certainty only to the entrepreneur, the financial intermediary can only verify the return at some cost. It turns out (see Townsend, 1979 and BGG, 1999 for details) the optimal contract charges a premium on funds borrowed which is proportional to entrepreneurs' net wealth. The higher is net wealth and the more funds the entrepreneur sinks into a project, the more closely aligned are the incentives of entrepreneur and investor. This implies that the expected gross return to holding a unit of capital is linked to the risk free rate through a risk premium as in

$$\frac{Q_t K_{t+1}}{NW_{t+1}} = \varphi \left(\frac{E_t R_{t+1}^k}{R_{t+1}} \right). \quad (2.19)$$

The greater is entrepreneurs' net wealth, NW_{t+1} , relative to the aggregate capital stock, the smaller will be the external finance premium. Entrepreneurial net wealth evolves as follows:

$$\begin{aligned} NW_{t+1} &= v_t [\gamma V_t] + w_t^e; \\ NW_{t+1} &= v_t \gamma \left[R_t^k Q_{t-1} K_t - R_t \Upsilon_{t-1} - \frac{\mu \int_0^{\bar{\omega}_t} \omega dF(\omega) R_t^k Q_{t-1} K_t}{(Q_{t-1} K_t - NW_t)} \Upsilon_{t-1} \right] + w_t^e, \end{aligned} \quad (2.20)$$

where $\Upsilon_{t-1} \equiv (Q_{t-1} K_t - NW_t)$, γ is the survival probability of the entrepreneur and v_t is a random disturbance term. Aggregate entrepreneurial net wealth is equal to the equity held by entrepreneurs at $t-1$ who are still in business at t , plus the entrepreneurial wage. Christiano, Motto and Rostagno (2007) interpret the shift factor v_t as a reduced form way to capture what Alan Greenspan has called 'irrational exuberance', or simply asset price bubbles. It raises entrepreneurial net wealth independently of movements in fundamentals. In CMR, the

shock directly affects the survival probability of entrepreneurs.⁶ Our preferred interpretation of v_t follows Gilchrist and Leahy (2002), who interpret their shock to entrepreneurial net wealth as a shock to the efficiency of contractual relations between borrower and lenders. That seems an attractive interpretation since the friction is present in the first place because of a costly state verification problem. In the steady state, $v = 1$, but away from steady state we assume that it follows an AR(1) process

$$\ln v_t = \rho^v \ln v_{t-1} + \varepsilon_t^v. \quad (2.21)$$

2.2.2. Retailers

Retailers purchase intermediate goods from entrepreneurs and transform these into differentiated goods using a linear technology. These differentiated goods are used for both consumption and investment. Prices are sticky in a time-dependent manner. The retailer will reprice as in Calvo (1983). That is, if the retailer reprices in period t it faces the probability $(\alpha^p)^k$ of having to charge the same price in period $t + k$. The criterion facing a retail firm presented with the opportunity to reprice is given by

$$\max \sum_{k=0}^{\infty} (\alpha^p \beta)^k E_t \left\{ \frac{\mu_{t+k}}{\mu_t} \left[\frac{p_t(i)}{P_{t+k}} \left(\frac{p_t(i)}{P_{t+k}} \right)^{-\theta} Y_{t+k}^d - X_{t+k} \left(\frac{p_t(i)}{P_{t+k}} \right)^{-\theta} Y_{t+k}^d \right] \right\}, \quad (2.22)$$

where the terms in marginal utility ensure that the price set is what would have been chosen by any individual in the economy had they been in charge of price-setting. The optimal price is given by

$$p'_t(i) = \frac{\theta \sum_{k=0}^{\infty} (\alpha^p \beta)^k E_t \{ \mu_{t+k} X_{t+k} P_{t+k}^{\theta} Y_{t+k}^d \}}{(\theta - 1) \sum_{k=0}^{\infty} (\alpha^p \beta)^k E_t \{ \mu_{t+k} P_{t+k}^{\theta-1} Y_{t+k}^d \}}. \quad (2.23)$$

Any retailer given the chance to reprice will choose this value. As a result the price-level evolves in the following way:

$$P_t = [(1 - \alpha^p) p_t'^{1-\theta} + \alpha^p P_{t-1}^{1-\theta}]^{\frac{1}{1-\theta}}. \quad (2.24)$$

---

⁶Letting γ_t be the random variable implies restricting the variance of γ_t to ensure that it always remains in the zero-one range. Our approach does not restrict the variance of γ_t .

2.3. Wage setting

We follow the work of Erceg *et al.* (2000) by assuming that labour is supplied by ‘household unions’ acting non-competitively. Household unions combine individual households’ labour supply according to:

$$N_t = \left[\int_0^1 N_t(i)^{\frac{\theta_w-1}{\theta_w}} di \right]^{\frac{\theta_w}{\theta_w-1}}. \quad (2.25)$$

If we denote by W the price index for labour inputs and by $W(i)$ the nominal wage of worker i , then total labour demand for household i ’s labour is:

$$N_t(i) = \left[\frac{W_t(i)}{W_t} \right]^{-\theta_w} N_t. \quad (2.26)$$

The household union takes into account the labour demand curve when setting wages. Given the monopolistically competitive structure of the labour market, if household unions have the chance to set wages every period, they will set it as a mark-up over the marginal rate of substitution of leisure for consumption. In addition to this monopolistic distortion, we also allow for the partial adjustment of wages using the same Calvo-type contract model as for price setters. This yields the following maximization problem:

$$\max \sum_{k=0}^{\infty} (\alpha^w \beta)^k E_t \left\{ \frac{\mu_{t+k}}{\mu_t} \left[\frac{W_t(i)}{P_{t+k}} \left[\frac{W_t(i)}{W_{t+k}} \right]^{-\theta_w} N_{t+k} - mrs_{t+k} \left[\frac{W_t(i)}{W_{t+k}} \right]^{-\theta_w} N_{t+k} \right] \right\} \quad (2.27)$$

where mrs is the marginal rate of substitution.

2.4. Monetary policy

We assume that the monetary authority exogenously sets the growth rate of money, $g_{M,t}$, such that supply of real money balance evolves according to

$$m_t = (1 + g_{M,t}) m_{t-1} \frac{P_{t-1}}{P_t}. \quad (2.28)$$

The real money growth rate, $g_{M,t}$, is assumed to follow a stochastic AR(1) process. The seigniorage from this activity is redistributed in a lump sum fashion to the consumer yielding real money transfers of

$$\tau_t = g_{M,t} m_{t-1} \frac{P_{t-1}}{P_t}. \quad (2.29)$$

2.5. Market clearing conditions

The aggregate market clearing condition states that output is the sum of consumption, investment, government expenditure plus the aggregate cost of monitoring associated with bankruptcies,

$$Y_t = C_t + I_t + G_t + \mu \int_0^{\bar{\omega}_t} \omega dF(\omega) R_t^k Q_{t-1} K_t. \quad (2.30)$$

3. Calibration

As we describe below, the parameters of the model are central to our shock extraction process and so we have sought to keep close to what we think is a standard choice for the values of the key deep parameters. For example, $\rho = \eta = 1.5$, α^w and θ_w are the same as in Erceg et al (2000), the values of the habit persistence parameter, h and the Calvo price parameter, α^p are the same as in CMR (2007). We describe the parameters and their assumed values in Table 3.1

Parameters pertaining to the financial accelerator are taken from BGG, specifically the values for μ and σ . We have chosen γ to match the average spread of the yield on AAA rated corporate bonds over the 3-month Treasury bill rate over our sample period (1960:Q1 to 2006:Q4).

4. Construction of Shocks

To construct the shocks driving the model, we follow the procedure of Benk, Gillman and Kejak (2005, 2007). Specifically, we assume that each of the drivers follows a stochastic

Table 3.1: Parameters of the models

Parameter	Description	Value
<i>Parameters in utility function</i>		
β	Discount factor	0.99
ρ	Consumption	1.5
b	Money	1.5
η	Labour	1.5
h	Habit persistence	0.6
<i>Parameters in production of goods</i>		
α	Capital share	0.25
Ω	Share of entrepreneurial labour	0.1
δ	Depreciation rate	0.025
$-\phi''(x/k)\delta/\phi'(x/k)$	Curvature of adjustment cost fn.	1
<i>Parameters in retail sector</i>		
X	Steady state markup (prices)	1.1
α^p	Calvo parameter prices	0.5
α^w	Calvo parameter wages	0.75
θ_w	Elasticity of labour demand	4
<i>Parameters in financial accelerator</i>		
μ	proportion of output lost to monitoring	0.12
σ	volatility of firm-specific shock	0.28
γ	Survival probability of entrepreneurs	0.978
$\frac{R^k}{R}$	External finance premium (bps)	211
$\frac{QK}{NW}$	Capital stock to net worth ratio	1.956
$F(\omega)$	Quarterly business failure rate	0.007
ω	Cutt-off rate for default	0.487
χ	Elasticity	0.037

AR(1) process. We linearize the model about its nonstochastic steady state and recover the Markov decision rules⁷. The decision rules are written in state-space form as

$$Y(t) = \Pi S(t); \quad (4.1)$$

$$S(t) = MS(t-1) + Ge(t). \quad (4.2)$$

The model's endogenous variables, including 'jump' variables, are stacked in the Y vector and the predetermined and exogenous variables are contained in the S vector, ordered in such a way that the predetermined state variables, (matrix $\kappa_t \equiv [k_t, w_{t-1}, q_{t-1}, r_{t-1}, nw_t, i_{t-1}, c_{t-1}, m_{t-1}]^8$), appear first, followed by the exogenous driving processes, $S(t) = [\kappa_t z_t u_t v_t]'$. Recall that to solve the model we need to take a guess as to the value of the autocorrelation coefficient of each of the driving processes and on any cross correlation to their innovations. One can re-write (4.1) and (4.2) in the following way:

$$Y(t) = A\kappa(t) + B[z(t), u(t), v(t)]', \quad (4.3)$$

where $A = \Pi(Y, \kappa)$ and $B = \Pi(Y, (z, u, v))$.

Given Π and therefore A and B , as well as data on Y and κ , it is straightforward to obtain an estimated series for $[z(t), u(t), v(t)]'$ via the following transformation:

$$[z(t), \widehat{u(t)}, v(t)]' = (B'B)^{-1}B'[Y(t) - A\kappa(t)] \quad (4.4)$$

As we are interested in estimating three shocks, we need data on κ and at least three variables contained in Y , the choice of which we discuss presently.⁹

---

⁷We use the King and Watson solution algorithm.

⁸Here, k_t is the capital stock, w_t the hourly real wage, q_t is Tobin's q , r_t denotes the real interest rate, nw_t our measure of entrepreneurial net wealth, i_t is the federal funds rate, c_t is consumption and m_t are real money balances. Data on entrepreneurial net wealth, which in the model is deflated by the consumer price index and is in per capita units, is taken from the Flow of Funds Accounts Table B 102. We use the 'nonfarm, nonfinancial corporate business net worth (market value)' series which we seasonally adjust and deflate by the consumer price index and by the size of the US population. This data series comes closest to the model's definition of entrepreneurial net wealth. We list further data sources and definitions in the appendix.

⁹We use quarterly, seasonally adjusted (where relevant, per capita) data which has been linearly detrended.

Next, we take the estimated series for $z(t)$, $u(t)$ and $v(t)$ and estimate the following equations as a system of seemingly unrelated regressions:

$$z_t = \rho^z z_{t-1} + \varepsilon_t^z; \quad (4.5)$$

$$u_t = \rho^u u_{t-1} + \varepsilon_t^u; \quad (4.6)$$

$$v_t = \rho^v v_{t-1} + \varepsilon_t^v. \quad (4.7)$$

We thus obtain estimates of the first-order auto-correlation coefficients of z , u and v . Because the matrix Π is a function of the triple $\{\rho^z, \rho^u, \rho^v\}$, we now proceed in an iterative fashion. To summarize: We start with an initial guess for $\{\rho^z, \rho^u, \rho^v\}$, using that guess to calculate the matrix Π and hence a new estimate of $\{\rho^z, \rho^u, \rho^v\}$. We calculate successive versions of Π and the process ends when the triple converges. Once this procedure has converged we use the values for ρ^z , ρ^u and ρ^v and ε^z , ε^u and ε^v in our solution algorithm and obtain impulse responses as well as filtered second moments of the model economy.

5. Estimated shocks: 1960:Q1 to 2006:Q4

As noted in Benk *et al* (2005), not all combinations of variables in $Y(t)$ yield the same time series for the shocks. However, as two of the shocks that we wish to identify are quite straight forward to construct using conventional methods, we focus on combinations of variables that when included in $Y(t)$ produce estimated processes for TFP and the money growth rate shock that are highly correlated with their conventionally constructed counterpart.¹⁰

The easiest shock to derive conventionally is the money growth rule shock, requiring only data on per capita M1. In our preferred combination of $Y(t)$ variables, we use logged and linearly detrended data on inflation, investment, real per capita M1, the real hourly wage rate and the quarterly real interest rate.¹¹ That combination combined with our choice of

---

¹⁰That is, we can easily construct a candidate TFP sequence via the (detrended) Solow residual, using per capita data on GDP, capital and labour input. The money shock is even more straightforward to recover. We use these conventionally constructed shocks as described in the text.

¹¹Please see the appendix for details of the data construction.

Figure 5.1: Money growth rule shocks. DSGE derived versus traditionally estimated shocks.

structural parameters in A and B yields a series for the money growth shock that has a correlation coefficient of 0.94 with the traditionally estimated shock. The corresponding correlation between our and the traditionally estimated TFP shock is 0.76. Figures (5.1) and (5.2) plot both the traditionally derived as well our money and TFP shocks.

Our financial friction shock is shown in Figure (5.3); of course, it has no extant conventional counterpart.

As noted, we may use different combinations of endogenous variables to construct our shocks. Using three variables at a time, we have analysed 120 such combinations. Figures (5.4) and (5.5) show eight combinations other than our preferred one that satisfy our auxiliary conditions of generating series for TFP and money growth shocks that are highly correlated with their conventional counterparts. The average correlation between our preferred shock and the other reported combinations is 0.94.

Figure 5.2: Total factor productivity. DSGE derived versus traditionally estimated shocks.

Figure 5.3: DSGE derived financial friction (FA) shock

Figure 5.4: DSGE derived FA shocks using different combinations of endogenous variables.

Figure 5.5: More DSGE derived FA shocks using different combinations of endogenous variables.

Figure 5.6: Our preferred measure of the FA and the average of eight differently derived FA measures.

Figure (5.6) plots our benchmark measure of v_t against the mean of the measures reported in Figures (5.4) and (5.5). Again, there is a very strong correlation (0.985) between these financial friction shocks and the one derived using our preferred combination of endogenous variables.

Finally, Figures (5.1), (5.2) and (5.3) plot the AR(1) driving processes, where $\rho_z = 0.9353$, $\rho_u = 0.5757$ and $\rho_v = 0.9782$. Hence, we estimate that TFP is more persistent than the growth rate of M1, but somewhat less persistent than v_t . Turning to the variance-covariance matrix of the ε_t^z , ε_t^u and ε_t^v , we find that:

$$VCM^{DSGE} = 1.0e^{-4} \times \begin{bmatrix} 0.5762 & 0.3542 & -0.2252 \\ 0.3542 & 0.9085 & -0.4927 \\ -0.2252 & -0.5761 & 0.7694 \end{bmatrix}$$

The innovations or ‘shocks’ to TFP are positively correlated with the shocks to the money

growth term. Danthine and Kurmann (2004) interpret this as indicative of an historical accommodation of supply-side shocks by the Fed. Innovations to our v_t process are negatively correlated with both innovations to TFP and to money growth.

We can compare the characteristics of our shocks to ‘traditionally’ derived shocks. We find that just as for our shocks, TFP is more persistent than the growth rate of per capita M1, $\rho_z = 0.9035$ compared with $\rho_u = 0.6723$. Compared to our shocks, the traditionally estimated Solow residual is somewhat less volatile, but is also positively correlated with money growth shock innovations. And similarly, money growth innovations are somewhat more volatile than those of TFP:

$$VCM^{Trad} = 1.0e^{-4} \times \begin{bmatrix} 0.6083 & 0.0618 \\ 0.0618 & 0.7214 \end{bmatrix}$$

5.1. Financial friction shocks and the external finance premium

How ‘reasonable’ is our estimated financial friction shock? Figure (5.7) shows the (HP filtered) spread between AAA rated corporate bonds and the three-month Treasury Bill rate. That data gives us an approximate measure of the external finance premium.¹² We also include the NBER reference periods between peaks and troughs of the business cycle. Figure (5.8) shows that in most cases, troughs in the business cycle correspond to peaks in the external finance premium.

Figure (5.8) shows the external finance premium along with our HP filtered series for the FA shock. We emphasize, the estimated FA shock is not constructed using data on the external finance premium. Nevertheless, we find a strong negative correlation between our FA shock and the external finance premium of -0.64 . Just as the financial accelerator model predicts, a shock that reduces net wealth raises the external finance premium. This

---

¹²The spread between corporate bonds and Treasury bills comes closest to our model’s definition of the external finance premium: $\frac{R_{t+1}^k}{R_{t+1}}$. Using the spread between BAA rated corporate bonds and the 3-month Treasury bill yields very similar results. An alternative measure, that does not correspond directly with the model’s definition of the risk premium is the BAA-AAA spread.

Figure 5.7: External finance premium, defined as the spread of AAA rated corporate bonds over the three-month Treasury bill rate. H-P filtered.

Figure 5.8: External finance premium and the DSGE derived FA shock.

correlation is much stronger than the correlation between our TFP measure and the spread (0.15) or our monetary policy shock and the spread (0.38).

We can also compare our model-derived FA shock to the γ_t shock analyzed in CMR (2007). Even though their model, the sample period of their data, as well as their estimation technique, differs from ours, their shock to the survival probability of entrepreneurs, which is comparable to our FA shock, has similar characteristics. They estimate an AR(1) coefficient of 0.9373 (mode of the posterior distribution) and a variance of $1.0e^{-4} \times 0.3969$.

5.2. Financial friction shocks and business cycle reference dates

In this section, we compare our DSGE generated shocks (H-P filtered) with the NBER business cycle reference dates. In particular, we track recessions which start at the peak of a business cycle and end at the trough. Our sample encompasses the following recessions: 1969:4 - 1970:4, 1973:4 - 1975:1, 1980:1 - 1980:3, 1981:3 - 1982:4, 1990:3 - 1991:1 and 2001:1

Figure 5.9: H-P filtered financial friction shock and NBER business cycles.

- 2001:4. Figure (5.9) overlays the NBER business cycle reference dates with our financial friction shock. The conformity of our derived financial shock with these recessions is quite striking. For the first two recessions, a peak in the business cycle corresponds to a local peak of our DSGE derived time series for v_t . In every subsequent recession, our financial friction either lags the peak of the business cycle by one or two quarters (1980:1 - 1980:3, 1981:3 - 1982:4, 1990:3 - 1991:1) or leads the peak, by one quarter, as in the 2001:1 - 2001:4 recession. In general, bar the 1980:1 - 1980:3 recession, our shock to financial efficiency continues to decline past the trough of the recession.

It is interesting again to note the strong similarities between our FA shock and the γ_t shock shown in Figure 3b in CMR (2007). Both measures of financial frictions show an easing of borrowing conditions from the early 1990s until 2001, where upon financial frictions imply a reduction in entrepreneurial net wealth or a worsening of lending conditions for firms.

Figure 5.10: H-P filtered money growth rate and NBER business cycles.

The link between the peaks and troughs of the business cycle and the realization of the money growth and TFP shocks, Figures (5.10) and (5.11), is rather less obvious than for our FA shock.

5.3. Variance decomposition

In this section, we measure the contribution of each of our three shock processes, TFP, the money growth shock and the FA shock, to the fluctuations in key macroeconomic time series. Because our shock processes are correlated, we follow Ingram, Kocherlakota and Savin (1994) and perform variance decompositions by imposing a recursive ordering scheme that orthogonalizes the correlated shocks derived from our DSGE model. The appendix describes how we calculated the data reported in Table 5.1

Ingram et al (1994) have shown that the relative contribution of a particular shock to

Figure 5.11: H-P filtered TFP shock and NBER business cycles.

the decomposition of the variance of a given variable depends on the ordering of the shocks in our recursive ordering scheme. Because of this, and because we have no strong prior as to the order of precedence of the shocks, we compute the variance decomposition for all six possible orderings. Table 5.1 reports the maximum, median and minimum percentage variation in each variable that is explained by each shock.

Focusing only on the median values of the relative variances, Table 5.1 suggests that our FA shock is a key driver for output, investment, the external finance premium, the federal funds rate and hours worked. In each case, the median of the share in the variance attributed to the FA shock is larger than that for the other shocks. For the external finance premium and investment the FA shock contributes by far the most to the variance. The median contribution of the FA shock for investment is about 45%, this median reflects a range from 9.8% to 85.8% depending on the ordering of the shocks. For the external finance premium, the medium contribution of the FA shock is 35% in a range between 15% and 70%, depending on the ordering of shocks. For output and hours worked, the FA shock contributes about as much to the variance as does total factor productivity. The median contribution of the TFP shock for output is 44%, while that of FA shock is 45%. Only for inflation and consumption is the shock relatively unimportant.

Gilchrist and Leahy (2002) argue that the monetary authorities should not respond directly to FA shocks, but that the best they can do is to vigorously stabilize inflation. Interestingly, when we decompose the variance of the federal funds rate we find that FA shocks play a key role. Indeed, the median contribution of FA shocks is larger than that of either TFP or money growth shock.

With the exception of inflation, the money growth rate shock contributes the least to the variance of the macroeconomic data analyzed. The key driver of the variance of inflation appears to be total factor productivity, and not the money growth rule shock. Total factor productivity contributes most to the variance of inflation and consumption.

The relatively large range between the minimum and the maximum contribution of each

Table 5.1: Variance decompositions

	Statistic	z_t	u_t	v_t
Output	[min, max]	[13.35, 63.79]	[11.43, 28.28]	[11.77, 61.93]
	median	43.95	24.41	44.63
	$median^2$	59.90	25.23	60.52
	$median^3$	20.67	18.74	20.25
Consumption	[min, max]	[20.93, 69.61]	[14.59, 42.03]	[10.49, 59.08]
	median	56.60	19.93	28.82
	$median^2$	64.84	22.30	37.03
	$median^3$	34.64	19.63	15.54
Investment	[min, max]	[2.48, 59.76]	[2.78, 45.11]	[9.80, 85.78]
	median	28.16	21.02	44.83
	$median^2$	52.33	37.74	68.99
	$median^3$	6.95	7.23	23.63
External finance premium	[min, max]	[12.52, 62.53]	[10.32, 44.63]	[14.85, 70.38]
	median	29.79	19.85	34.86
	$median^2$	51.37	33.61	56.45
	$median^3$	15.94	13.72	21.03
Inflation	[min, max]	[18.97, 69.67]	[15.32, 50.24]	[10.65, 54.69]
	median	58.51	23.00	22.87
	$median^2$	66.35	26.29	30.70
	$median^3$	36.48	17.52	12.85
Federal funds rate	[min, max]	[9.44, 69.10]	[9.02, 51.27]	[15.91, 68.57]
	median	27.50	18.43	30.47
	$median^2$	50.85	36.62	53.82
	$median^3$	15.93	11.96	18.89
Hours worked	[min, max]	[13.44, 65.58]	[7.53, 41.06]	[6.53, 73.20]
	median	35.72	20.55	36.06
	$median^2$	58.90	34.43	59.21
	$median^3$	16.33	10.42	16.72

shock confirms that our variance decompositions are sensitive to the ordering of shocks. As noted in Ingram et al (1994), the last, in our case third, shock in each ordering contributes the least to the variance of the variable in question. As a sensitivity check on our results, we recalculate the median dropping those decompositions where the shock concerned occurs last. We report these variance decompositions in rows labelled ‘median²’. With the exception of inflation, the order of importance of the three shocks in terms of their contribution to the variance of our macroeconomic variables remains unchanged. For example, the FA shock still contributes most to the variance of the external finance premium and investment. For inflation, the FA now actually plays a larger role than the money growth shock.

Recalculating the median while dropping those variance decompositions where the shock in question occurs first, tends to reduce the median contribution of a shock to the variance of a specific variable. We report this sensitivity exercise in rows labelled ‘median³’. Again, most orderings remain unchanged. The FA shock still has the highest median contribution for the variance of investment, the external finance premium and hours worked. A notable exception is output, where all three shocks now have similar median contributions to the variance. For consumption, the FA shock now has the lowest median contribution.

In summary, our variance decompositions suggest that shocks to financial efficiency contribute significantly to the variance of key macroeconomic time series. The relative contributions of FA shocks are comparable to those of total factor productivity and exceed those of shocks to the growth rate of the money supply.

6. Second moments

Table 6.1. compares with the quarterly, detrended and filtered US data, the data generated by three models that are identical except that: Model 3 has no FA mechanism; Model 2 adds the FA to Model 3; Model 1 adds shocks to the FA mechanism of Model 2.

Our baseline model driven by TFP, money growth and financial friction shocks comes close to matching the standard deviation of GDP and its components, hours worked, the

Table 6.1: Second Moments from Data and Models

	US Data	Model 1	Model 2	Model 3
<i>stdev</i>		all shocks	no nw shocks	no FA
<i>y</i>	0.015	0.013	0.018	0.018
<i>c</i>	0.012	0.015	0.013	0.013
<i>i</i>	0.048	0.046	0.046	0.046
<i>n</i>	0.017	0.014	0.017	0.017
<i>w</i>	0.009	0.008	0.009	0.009
<i>r</i>	0.004	0.0001	0.0001	0.0001
π	0.005	0.005	0.006	0.006
<i>m</i>	0.030	0.020	0.019	0.018
<i>efp</i>	0.002	0.002	0.0008	0
<i>nw</i>	0.024	0.103	0.048	na
<i>corr</i> ($\cdot, y$)				
<i>c</i>	0.861	0.801	0.974	0.973
<i>i</i>	0.791	0.367	0.952	0.951
<i>n</i>	0.868	0.638	0.746	0.752
<i>w</i>	0.223	0.519	0.552	0.554
<i>r</i>	0.355	-0.151	-0.227	-0.224
π	0.380	0.515	0.474	0.478
<i>m</i>	0.316	0.746	0.970	0.978
<i>efp</i>	-0.599	-0.216	-0.910	0
<i>nw</i>	0.276	0.195	0.814	na

real wage, inflation and, importantly, the external finance premium (efp). The model comes reasonably close to matching the volatility of the real per capita money supply (M1), but over-predicts the volatility of entrepreneurial net wealth by a factor of four. Our model also fails to account for most of the volatility of the nominal interest rate.

The model correctly predicts the sign of the correlation with GDP for all variables except for the nominal interest rate. Importantly, the model captures the fact that the external finance premium is counter cyclical and that entrepreneurial net wealth is pro-cyclical in the data.

Table 6.1 also reports second moments generated by the model in the absence of financial friction shocks and in the absence of the financial accelerator mechanism, Model 2 and Model 3, respectively. For both of these models, we derive TFP and money growth rule shocks in the manner described above.¹³ Our FA shock increases the volatility of investment for a given calibration. In order to make a comparison across models easier, we have changed the capital adjustment cost parameter vis-à-vis our baseline calibration in Models 2 and 3, so that investment is as volatile in these models as in our baseline model.¹⁴ An important difference between Models 1 and 2 on the one hand and Model 3 on the other, is that GDP is more volatile in models without FA shocks. We relate this finding to the fact that our FA shock is negatively correlated with both TFP and money growth shocks. Impulse responses presented in the appendix (see also those in Gilchrist and Leahy (2002)) show that FA shocks can cause the components of GDP (consumption and investment in our case) to move in opposite directions. As a result, our model with FA shocks displays a lower correlation between GDP and either consumption or investment than do the alternative models without FA shocks. A key difference between our models is that only the model with FA shocks can generate a realistic amount of volatility in the external finance premium.

---

¹³In both cases, the highest correlation between model-generated and traditionally estimated TFP and money growth shocks is obtained by using data on the endogenous variables: inflation, real money supply and real wages. The correlation between model-derived and traditional shocks is somewhat lower than in the baseline case 65% for TFP and 88% for the money growth shocks.

¹⁴The adjustment cost parameter is set at -1.8 in Model 2 and at -2.35 in Model 3.

Model 2 generates a series for the external finance premium which is only $1/3$ as volatile as in the data.

Comparing across models without FA shocks reveals only minimal differences attributable to the presence of a financial accelerator. A similar conclusion is reached by Meier and Müller (2006) who find that financial accelerator plays only a minor role in the transmission of monetary policy shocks.

The main contribution of FA shocks in terms of matching the data's second moments over our sample period lies in the model's ability to match the second moments of the external finance premium. FA shocks, being negatively correlated with TFP and money growth shocks, also help reduce the excessively large correlation between GDP on the one hand and consumption, investment and real money balances on the other. However, given the importance of FA shocks in terms of the variance decomposition of US data and in their correlation with major post-war recessions, it is perhaps somewhat surprising that we do not find a stronger role for FA shocks in explaining the second moments of US data over our sample period. Our analysis comparing the dynamics of the FA shock with NBER reference dates for major post-war recessions suggests that potentially FA shocks are more important during large downturns than during business cycle fluctuations of smaller magnitude.

7. Conclusion

Our analysis identifies an important source of cyclical variation for the US economy. We identify and gauge the importance of shocks emanating from the financial accelerator mechanism put forward by BGG (1999). Gilchrist and Leahy (2002) interpret this source of variation as a shock to the efficiency of contractual relations between borrower and lenders. Our analysis suggests that the role of these financial shocks seems to be important in understanding the post-war US data. Our results suggest that such shocks have a very strong link to the business cycle. Our approach is not the only way to extract these shocks but our findings seem to be robust given the results of CMR (2007) and De Graeve (2008).

References

- [1] Benk, Szilárd, Max Gillman and Michal Kejak (2005). Credit Shocks in the Financial Deregulatory Era: Not the Usual Suspects, *Review of Economic Dynamics*, vol. 8(3), July pages 668-687.
- [2] Benk, Szilárd, Max Gillman and Michal Kejak (2008). Money Velocity in an Endogenous Growth Business Cycle with Credit Shocks, *Journal of Money Credit and Banking* Vol. 40, No. 6, September pages 1281-1293.
- [3] Bernanke, Ben S., Mark Gertler, and Simon Gilchrist (1999). *The Financial Accelerator in a Quantitative Business Cycle Framework*, in Handbook of Macroeconomics, Volume 1C, Handbooks in Economics, vol. 15. Amsterdam: Elsevier, pp. 1341-93.
- [4] Bhattacharjee, Arnab and Christoph Thoenissen (2007). Money and monetary policy in dynamic stochastic general equilibrium models, *The Manchester School*, 2007, vol. 75, Issue S1, pp 88 - 122.
- [5] Blankenau, William, M. Ayhan Kose and Kei-Mu Yi (2001). Can world real interest rates explain business cycles in a small open economy? *Journal of Economic Dynamics and Control* Volume 25, Issues 6-7, June Pages 867-889
- [6] Calvo, Guillermo A. (1983). Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics* Volume 12, Issue 3, September 1983, Pages 383-398
- [7] Christensen, Ian and Ali Dib (2008). The financial accelerator in an estimated New Keynesian model, *Review of Economic Dynamics* Volume 11, Issue 1, January Pages 155-178.
- [8] Christiano, Lawrence J., Roberto Motto and Massimo Rostagno (2004). The Great Depression and the Friedman-Schwartz Hypothesis March ECB Working Paper No. 326.

- [9] Christiano, Lawrence J., Roberto Motto and Massimo Rostagno (2007). Financial Factors in Business Cycles, Working Paper, mimeo.
- [10] De Graeve, Ferre (2008). The external finance premium and the macroeconomy: US post-WWII evidence, *Journal of Economic Dynamics and Control*, 32 (11) 3415-3440.
- [11] Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin (2000). Optimal monetary policy with staggered wage and price contracts, *Journal of Monetary Economics*, vol. 46(2), pages 381-413.
- [12] Ester, Faia and Tommaso Monacelli (2007). Optimal Monetary Policy Rules, Asset Prices and Credit Frictions, *Journal of Economic Dynamics and Control*, vol. 31(10), October pages 3228-3254.
- [13] Gertler, Mark, Simon Gilchrist and Fabio M. Natalucci (2007). External Constraints on Monetary Policy and the Financial Accelerator, *Journal of Money, Credit and Banking*, vol. 39(2-3), pages 295-330.
- [14] Gilchrist, Simon and John V. Leahy (2002). Monetary policy and asset prices, *Journal of Monetary Economics*, Volume 49, Issue 1, January pages 75-97.
- [15] Hall, Simon (2001) Financial Accelerator Effects in UK Business Cycles, Bank of England Working Paper December, No. 150.
- [16] Ingram, Beth Fisher, Narayana R. Kocherlakota and N. E. Savin (1994). Explaining business cycles: A multiple-shock approach, *Journal of Monetary Economics*, Volume 34, Issue 3, December 1994, Pages 415-428.
- [17] Meier, Andre and Gemot J. Müller (2006). Fleshing Out the Monetary Transmission Mechanism: Output Composition and the Role of Financial Frictions, *Journal of Money, Credit, and Banking* 38(8), 2099-2134.

- [18] Townsend, Robert M. (1979). Optimal contracts and competitive markets with costly state verification, *Journal of Economic Theory* Volume 21, Issue 2, October pages 265-293.

A. Data Sources

- k_t is a quarterly series for the US capital stock constructed using annual capital stock data and quarterly data on investment expenditure. Source: BEA
- w_{t-1} is the first lag of the real wage defined as real hourly compensation (non farm business sector) PRS85006153.
- q_{t-1} is the lag of Tobin's q defined as $q_t = 1/\phi(k_t - x_t)$ where x_t is real per capita investment, constructed using BEA Table 5.3.3. Real Private Fixed Investment by Type, Quantity Indexes as well as population size.
- r_{t-1} is the lag of the real interest rate, defined as $i_t - E_t\pi_{t+1}$ where i_t is the quarterly federal funds rate and $E_t\pi_{t+1}$ is constructed using a centered moving average of past, future and current inflation rates.
- nw_t is the real per capita stock of entrepreneurial wealth constructed using the nonfarm nonfinancial corporate business net worth (market value) series taken from the flow of funds account, B.102, seasonally adjusting this series, and dividing by population and the consumer price index.
- m_{t-1} is the lag of the growth rate of real per capita M1.
- $\hat{c}_t$: per capita real consumption. Source: BEA. Data used: Personal consumption expenditures (NIPA 2.3.5), Implicit price deflator for Personal consumption expenditures (NIPA 1.1.9), US population (NIPA 7.1).

- $\hat{x}_t$: per capita real investment. Source: BEA. Data used: Real private non-residential fixed investment (NIPA 5.3.3), US population (NIPA 7.1)
- $\hat{y}_t$: real per capital GDP. Source: BEA. Data used: Selected Per Capita Product and Income Series in Current and Chained Dollars (NIPA 7.1)

B. The linearized model

$$\hat{\mu}_t = \hat{\mu}_{t+1} + i_t - E\pi_{t+1} \quad (\text{B.1})$$

$$\omega_t = \beta E\omega_{t+1} - \kappa_w \hat{\mu}_t + \phi \kappa_w \hat{n}_t - \kappa_w \hat{w}_t \quad (\text{B.2})$$

$$\frac{\beta}{\beta - 1} i_t = b \hat{m}_t + \hat{\mu}_t \quad (\text{B.3})$$

$$\hat{w}_t = \widehat{mc}_t + ((1 - s_k)(1 - s_e) - 1) \hat{n}_t + s_k \hat{k}_t + \hat{z}_t \quad (\text{B.4})$$

$$\hat{\rho}_t = \widehat{mc}_t + (1 - s_k)(1 - s_e) \hat{n}_t - (1 - s_k) \hat{k}_t + \hat{z}_t \quad (\text{B.5})$$

$$\hat{k}_{t+1} = \delta \hat{x}_t + \delta \hat{k}_t \quad (\text{B.6})$$

$$\hat{r}_t^k = \frac{1 - \delta}{r^k} \hat{q}_t - \hat{q}_{t-1} + \left(1 - \frac{1 - \delta}{r^k}\right) \hat{\rho}_t \quad (\text{B.7})$$

$$\hat{q}_t = \phi''(x/k) \delta / \phi'(x/k) \left[\hat{x}_t - \hat{k}_t \right] \quad (\text{B.8})$$

$$\hat{y}_t = \hat{z}_t + s_k \hat{k}_t + (1 - s_k)(1 - s_e) \hat{n}_t \quad (\text{B.9})$$

$$\begin{aligned}
\hat{y}_t &= \frac{c}{y}\hat{c}_t + \frac{x}{y}\hat{x}_t + \frac{DK}{Y}\Phi_t \\
\Phi_t &\equiv \ln \left[\frac{\mu \int_0^{\bar{\omega}} \omega R_t^k Q_{t-1} K_t f(\omega) d\omega}{\mu \int_0^{\bar{\omega}} \omega R^k f(\omega) d\omega K} \right] \\
D &\equiv \mu \int_0^{\bar{\omega}} \omega R^k f(\omega) d\omega
\end{aligned} \tag{B.10}$$

$$\hat{m}_t = \hat{m}_{t-1} - \pi_t + u_t \tag{B.11}$$

$$\pi_t = \beta E \pi_{t+1} + \kappa_p \widehat{m} c_t \tag{B.12}$$

$$\hat{\mu}_t = -\rho \frac{1}{1-h} \hat{c}_t + \rho \frac{h}{1-h} \hat{c}_{t-1} \tag{B.13}$$

$$\hat{w}_t = \hat{w}_{t-1} + \omega_t - \pi_t \tag{B.14}$$

$$rr_t = i_t - E \pi_{t+1} \tag{B.15}$$

$$\begin{aligned}
\frac{nw}{k} \widehat{nw}_{t+1} &= \frac{\gamma}{\beta} \frac{nw}{k} \widehat{nw}_t + \gamma r^k \hat{r}_t^k + \frac{\gamma}{\beta} \left(\frac{nw}{k} - 1 \right) \hat{r}_{t-1}^k + \gamma (r^k - 1/\beta) \left[\hat{q}_{t-1} + \hat{k}_t \right] \\
&+ \frac{(1-s_k)s_e}{s_k} (r^k - 1/\beta) [\hat{y}_t + \widehat{m} c_t] + \hat{v}_t \gamma \frac{V}{K}
\end{aligned} \tag{B.16}$$

$$E \left(\hat{r}_{t+1}^k - rr_t \right) = \chi \left(\hat{q}_t + \hat{k}_{t+1} - \widehat{nw}_{t+1} \right) \tag{B.17}$$

B.1. Variance decomposition calculations

Our shock processes are correlated so we follow Ingram, Kocherlakota and Savin (1994), Blankenau et al (2001) and Benk et al (2008) and perform variance decompositions by imposing a recursive ordering scheme that orthogonalizes the correlated shocks derived from our DSGE model. This appendix describes how we calculated the data reported in Table 5.1

Let $\tilde{y}_t$, $\tilde{z}_t$, $\tilde{u}_t$, and $\tilde{v}_t$ be the logged, Hodrick-Prescott (1600) filtered time series of GDP, total factor productivity, the money growth rule shock and the FA shock, respectively. To illustrate our variance decomposition approach let $[\tilde{z}_t, \tilde{u}_t, \tilde{v}_t]$ for $t = 1, T$ denote the vector of time series of our three shocks. The specific ordering $[\tilde{z}_t, \tilde{u}_t, \tilde{v}_t]$ implies that movement in $\tilde{z}_t$ are responsible for any co-movement between $\tilde{z}_t$ and $\tilde{u}_t$ and between $\tilde{z}_t$ and $\tilde{v}_t$. Movements in $\tilde{u}_t$ are responsible for any co-movement between $\tilde{u}_t$ and $\tilde{v}_t$. Only the part of $\tilde{v}_t$ uncorrelated with either $\tilde{z}_t$ or $\tilde{u}_t$ is ‘assigned’ to $\tilde{v}_t$. The variance decomposition of output, $\tilde{y}_t$, into the three shocks generated by our model is obtained by running the following regression:

$$\tilde{y}_t = \underbrace{\sum_{m=0}^M \beta_{z,m} \tilde{z}_{t-m}}_{y_t^z} + \underbrace{\sum_{m=0}^M \beta_{u,m} u_{t-m}^e}_{y_t^u} + \underbrace{\sum_{m=0}^M \beta_{v,m} v_{t-m}^e}_{y_t^v} + \varepsilon_t \quad (\text{B.18})$$

where we define u_{t-m}^e as the residuals in a regression of $\tilde{u}_{t-m}$ on $[\tilde{z}_t, \dots, \tilde{z}_{t-M}]$ and v_{t-m}^e as the residuals in a regression of $\tilde{v}_t$ on $[\tilde{z}_t, \dots, \tilde{z}_{t-M}; \tilde{u}_t, \dots, \tilde{u}_{t-M}]$. As M becomes large, the $\text{var}(\varepsilon_t)$ tends towards zero. We set the lag length M so that $\text{var}(\varepsilon_t)$ is less than 0.5% of $\text{var}(\tilde{y}_t)$. In the variance decompositions reported in table 3, we set $M = 45$. The fraction of the variance of $\tilde{y}_t$ explained by each shock is $\frac{\text{var}(y_t^z)}{\text{var}(\tilde{y}_t)}$, $\frac{\text{var}(y_t^u)}{\text{var}(\tilde{y}_t)}$ and $\frac{\text{var}(y_t^v)}{\text{var}(\tilde{y}_t)}$.

As noted in the text, the relative contribution of a particular shock to the decomposition of the variance of a given variable depends on the ordering of the shocks. Hence, we compute the variance decomposition for all six possible orderings.

B.2. Impulse responses

In this section, we use impulse response analysis to examine the model's response to FA shocks. Figure (B.1), analyzes the response of the model economy to the FA shock. A positive shock raises entrepreneurial net wealth. The rise in net wealth lowers the external finance premium, which in turn stimulates investment. Compared to the response of output, the rise in investment is large. Initially, output rises due an increase in hours worked. This rise in output is, however, not sufficient to meet the demand for investment goods. To meet this demand consumption has to fall and it continues to fall in the initial periods following a shock. The finding that consumption is negatively correlated with FA shocks is related to the fact that this is a closed economy. In an open economy model, the surge in investment would result in a large current account deficit and would not necessarily require a fall in consumption. Inflation rises following a FA shock

Figure B.1: A positive shock to entrepreneurial net wealth.

ABOUT THE CDMA

The **Centre for Dynamic Macroeconomic Analysis** was established by a direct grant from the University of St Andrews in 2003. The Centre funds PhD students and facilitates a programme of research centred on macroeconomic theory and policy. The Centre has research interests in areas such as: characterising the key stylised facts of the business cycle; constructing theoretical models that can match these business cycles; using theoretical models to understand the normative and positive aspects of the macroeconomic policymakers' stabilisation problem, in both open and closed economies; understanding the conduct of monetary/macroeconomic policy in the UK and other countries; analyzing the impact of globalization and policy reform on the macroeconomy; and analyzing the impact of financial factors on the long-run growth of the UK economy, from both an historical and a theoretical perspective. The Centre also has interests in developing numerical techniques for analyzing dynamic stochastic general equilibrium models. Its affiliated members are Faculty members at St Andrews and elsewhere with interests in the broad area of dynamic macroeconomics. Its international Advisory Board comprises a group of leading macroeconomists and, ex officio, the University's Principal.

Affiliated Members of the School

Dr Fabio Aricò.
Dr Arnab Bhattacharjee.
Dr Tatiana Damjanovic.
Dr Vladislav Damjanovic.
Prof George Evans.
Dr Gonzalo Forgeue-Puccio.
Dr Laurence Lasselle.
Dr Peter Macmillan.
Prof Rod McCrorie.
Prof Kaushik Mitra.
Prof Charles Nolan (Director).
Dr Geetha Selvaratnam.
Dr Ozge Senay.
Dr Gary Shea.
Prof Alan Sutherland.
Dr Kannika Thampanishvong.
Dr Christoph Thoenissen.
Dr Alex Trew.

Senior Research Fellow

Prof Andrew Hughes Hallett, Professor of Economics,
Vanderbilt University.

Research Affiliates

Prof Keith Blackburn, Manchester University.
Prof David Cobham, Heriot-Watt University.
Dr Luisa Corrado, Università degli Studi di Roma.
Prof Huw Dixon, Cardiff University.
Dr Anthony Garratt, Birkbeck College London.
Dr Sugata Ghosh, Brunel University.
Dr Aditya Goenka, Essex University.
Prof Campbell Leith, Glasgow University.
Prof Paul Levine, University of Surrey.
Dr Richard Mash, New College, Oxford.
Prof Patrick Minford, Cardiff Business School.
Dr Gulcin Ozkan, York University.
Prof Joe Pearlman, London Metropolitan University.

Prof Neil Rankin, Warwick University.
Prof Lucio Sarno, Warwick University.
Prof Eric Schaling, South African Reserve Bank and
Tilburg University.
Prof Peter N. Smith, York University.
Dr Frank Smets, European Central Bank.
Prof Robert Solis, Newcastle University.
Prof Peter Tinsley, Birkbeck College, London.
Dr Mark Weder, University of Adelaide.

Research Associates

Mr Nikola Bokan.
Mr Farid Boumediene.
Mr Johannes Geissler.
Mr Michal Horvath.
Ms Elisa Newby.
Mr Ansgar Rannenberg.
Mr Qi Sun.

Advisory Board

Prof Sumru Altug, Koç University.
Prof V V Chari, Minnesota University.
Prof John Driffill, Birkbeck College London.
Dr Sean Holly, Director of the Department of Applied
Economics, Cambridge University.
Prof Seppo Honkapohja, Bank of Finland and
Cambridge University.
Dr Brian Lang, Principal of St Andrews University.
Prof Anton Muscatelli, Heriot-Watt University.
Prof Charles Nolan, St Andrews University.
Prof Peter Sinclair, Birmingham University and Bank of
England.
Prof Stephen J Turnovsky, Washington University.
Dr Martin Weale, CBE, Director of the National
Institute of Economic and Social Research.
Prof Michael Wickens, York University.
Prof Simon Wren-Lewis, Oxford University.

**RECENT WORKING PAPERS FROM THE
CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS**

Number	Title	Author(s)
CDMA07/10	Optimal Sovereign Debt Write-downs	Sayantan Ghosal (Warwick) and Kannika Thampanishvong (St Andrews)
CDMA07/11	Bargaining, Moral Hazard and Sovereign Debt Crisis	Syantan Ghosal (Warwick) and Kannika Thampanishvong (St Andrews)
CDMA07/12	Efficiency, Depth and Growth: Quantitative Implications of Finance and Growth Theory	Alex Trew (St Andrews)
CDMA07/13	Macroeconomic Conditions and Business Exit: Determinants of Failures and Acquisitions of UK Firms	Arnab Bhattacharjee (St Andrews), Chris Higson (London Business School), Sean Holly (Cambridge), Paul Kattuman (Cambridge).
CDMA07/14	Regulation of Reserves and Interest Rates in a Model of Bank Runs	Geethanjali Selvaretnam (St Andrews).
CDMA07/15	Interest Rate Rules and Welfare in Open Economies	Ozge Senay (St Andrews).
CDMA07/16	Arbitrage and Simple Financial Market Efficiency during the South Sea Bubble: A Comparative Study of the Royal African and South Sea Companies Subscription Share Issues	Gary S. Shea (St Andrews).
CDMA07/17	Anticipated Fiscal Policy and Adaptive Learning	George Evans (Oregon and St Andrews), Seppo Honkapohja (Cambridge) and Kaushik Mitra (St Andrews)
CDMA07/18	The Millennium Development Goals and Sovereign Debt Write-downs	Sayantan Ghosal (Warwick), Kannika Thampanishvong (St Andrews)
CDMA07/19	Robust Learning Stability with Operational Monetary Policy Rules	George Evans (Oregon and St Andrews), Seppo Honkapohja (Cambridge)
CDMA07/20	Can macroeconomic variables explain long term stock market movements? A comparison of the US and Japan	Andreas Humpe (St Andrews) and Peter Macmillan (St Andrews)

CDMA07/21	Unconditionally Optimal Monetary Policy	Tatiana Damjanovic (St Andrews), Vladislav Damjanovic (St Andrews) and Charles Nolan (St Andrews)
CDMA07/22	Estimating DSGE Models under Partial Information	Paul Levine (Surrey), Joseph Pearlman (London Metropolitan) and George Perendia (London Metropolitan)
CDMA08/01	Simple Monetary-Fiscal Targeting Rules	Michal Horvath (St Andrews)
CDMA08/02	Expectations, Learning and Monetary Policy: An Overview of Recent Research	George Evans (Oregon and St Andrews), Seppo Honkapohja (Bank of Finland and Cambridge)
CDMA08/03	Exchange rate dynamics, asset market structure and the role of the trade elasticity	Christoph Thoenissen (St Andrews)
CDMA08/04	Linear-Quadratic Approximation to Unconditionally Optimal Policy: The Distorted Steady-State	Tatiana Damjanovic (St Andrews), Vladislav Damjanovic (St Andrews) and Charles Nolan (St Andrews)
CDMA08/05	Does Government Spending Optimally Crowd in Private Consumption?	Michal Horvath (St Andrews)
CDMA08/06	Long-Term Growth and Short-Term Volatility: The Labour Market Nexus	Barbara Annicchiarico (Rome), Luisa Corrado (Cambridge and Rome) and Alessandra Pelloni (Rome)
CDMA08/07	Seigniorage-maximizing inflation	Tatiana Damjanovic (St Andrews) and Charles Nolan (St Andrews)
CDMA08/08	Productivity, Preferences and UIP deviations in an Open Economy Business Cycle Model	Arnab Bhattacharjee (St Andrews), Jagjit S. Chadha (Canterbury) and Qi Sun (St Andrews)
CDMA08/09	Infrastructure Finance and Industrial Takeoff in the United Kingdom	Alex Trew (St Andrews)

For information or copies of working papers in this series, or to subscribe to email notification, contact:

Johannes Geissler
Castlecliffe, School of Economics and Finance
University of St Andrews
Fife, UK, KY16 9AL
Email: jg374@at-andrews.ac.uk; Phone: +44 (0)1334 462445; Fax: +44 (0)1334 462444.