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Simple Monetary-Fiscal Targeting Rules*

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ABSTRACT

We analyze the characteristics of optimal dynamics in an economy in which neither prices nor wages adjust instantaneously and lump-sum taxes are unavailable as a source of government finance. We then propose that monetary and fiscal policy should be coordinated to satisfy a pair of simple specific targeting rules, a rule for (wage) inflation and a relationship that links the growth of real wages to past price and wage developments, and output gap dynamics. We show that such simple rule-based conduct of policy can do remarkably well in replicating the dynamics of the economy under optimal policy following a given shock.

JEL Classification: E52, E61, E63.

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1. Introduction

In this paper, we propose simple 'specific targeting rules' in the spirit of Svensson (2002, 2003) to characterize 'desirable' policy in a New Keynesian economy with staggered wage and price contracts, and endogenous tax dynamics. In our complex analytical framework, which is an extension of Erceg et al. (2000) and Benigno and Woodford (2003, 2004), it is not possible to characterize optimal policy using simple analytical solutions. Our approach of looking at joint monetary-fiscal targeting rules represents an innovative way of approximating optimal policy that allows us to obtain an excellent match with the optimal dynamics of the economy following shocks.

Our work is also related to Schmitt-Grohé and Uribe (2005) and Chugh (2006) who studied jointly optimal monetary-fiscal strategies in frameworks involving costly price and wage adjustment. By doing so, they extended upon a stream of literature on monetary and fiscal policy that begins with Lucas and Stokey's (1983) flexible-price framework with complete asset markets and has gradually evolved over time to include asset market imperfections (Aiyagari et al., 2002) and price stickiness (Benigno and Woodford, 2003, Schmitt-Grohé and Uribe, 2004a and Siu, 2004).

Our analysis is distinct in the class of papers with price and wage rigidity in two important aspects. First, it follows the linear-quadratic approach of Erceg et al. (2000) and Benigno and Woodford (2004) to analyze jointly optimal monetary-fiscal strategies. This allows us to characterize optimal policy using a quadratic objective function, which is appealing from a practical point of view, and to directly compare its properties with the policy objectives derived in a similar way in the simpler economies of Erceg et al. (2000) and Benigno and Woodford (2004). Second, we depart from the conventional approach, used among others in

Erceg et al. (2000) and Schmitt-Grohé and Uribe (2004b, 2005), of looking for simple rules for policy instruments to approximate optimal policy. We propose a simple characterization of policy as an approximation to the optimal policy at a higher level of generality, as discussed in Svensson (2002, 2003) and Svensson and Woodford (2005). We concentrate on 'specific targeting rules'. These rules offer a simple joint policy target for monetary and fiscal policy makers. We show that well-specified simple specific targeting rules can guide policy very well so that when combined with the structural model, the resulting dynamics of the economy is a close approximation of the dynamics under optimal policy following a given shock. Such simple characterization of policy has thus the potential to outperform simple (ad hoc) instrument rules. In other words, conduct of policy based on conventionally considered simple instrument rules would lead to excessive welfare losses some of which can be eliminated if policy is characterized, as we propose, at a higher level of generality. We demonstrate this point by deriving 'expectations-based reaction functions' (Evans and Honkapohja, 2006) consistent with the simple targeting rules and showing that they are much more complex than the conventionally considered instrument rules. At the same time, from a practical point of view, characterization of policy via the targeting rules proposed here is no less verifiable than the conduct of policy based on commitment to instrument rules. Specific targeting rules are also equally easy to build into macroeconomic decision frameworks. As spelled out in detail in Svensson (2002, 2003), characterizing policy using targets rather than simple rules for instruments is also more consistent with the use of judgement in policy making. On the other hand, the quality of approximation provided by specific targeting rules is prone to be affected by the dynamic structure of the economy modelled. Nevertheless, and especially given the reluctance of policy makers to commit themselves to an explicit instrument rule, and the popularity of target-based conduct of monetary policy, we consider specific targeting rules an attractive and policy-relevant way of characterizing 'good' policy.

In our sticky-price, sticky-wage framework, we find that the central government's policy objective includes a wage inflation volatility term in addition to the objective of stabilizing price inflation and output gap volatility. Hence, this result from Erceg et al. (2000) and Benigno and Woodford (2004) carries over to a situation when monetary policy has fiscal implications too.[1] We also find that the relative weights in the policy objective are little changed compared with Benigno and Woodford (2004). As expected, optimal nominal wage volatility falls dramatically with wage-stickiness and this causes real wages to be much more stable compared with the sticky-price but flexible-wage model of Benigno and Woodford (2003), for instance. The presence of nominal wage stickiness also introduces endogenous persistence into the dynamics of the model. In contrast to the flexible-wage Benigno and Woodford (2003) economy, our endogenous variables converge to their (new) steady state levels only gradually, even if the disturbance that initially causes the economy to abandon its steady state is purely transitory. Non-stationarity in the dynamics of public debt, tax rate and the output gap carries over from Benigno and Woodford (2003) as the optimal solution to an economy where wage stickiness is present in addition to price stickiness. We find some support for the claim that price stickiness is the single most important factor justifying price stability as the principal goal of monetary policy (Schmitt-Grohé and Uribe, 2005). We also show that it is enough for one of the markets to be imperfectly flexible to make the policy of strict price- and wage-level targeting undesirable.

[1] Adding further frictions to the model such as rule-of-thumb behaviour, various indexation mechanisms or habit persistence would be likely to affect the nature of the policy objective, as in Amato and Laubach (2003), for instance. Our purpose here is to examine the effect of the absence of lump-sum taxation on the policy objective and optimal dynamics in isolation.

We then propose that the branches of central government authority operating in our sticky-price, sticky-wage economic framework should coordinate their efforts to gradually stabilize nominal wage growth and also make sure that a simple relationship linking the growth in the real wage rate to past price and wage inflation and the dynamics of the output gap is satisfied. We rank alternative policies in the family of policies given by these simple targeting rules. Benigno and Woodford (2006b) discuss in great detail the issues associated with ranking suboptimal rules in frameworks such as ours. The method for ranking suboptimal simple rules proposed in Benigno and Woodford (2006b), which entails separating the trend and the cyclical components of difference-stationary series (output gap) and ranking of policies implying the same trend component using a similar decomposition of welfare, cannot be applied in our case, since the policies we wish to examine do not converge to the same long-run outcome (i.e. have different trend components). Instead, we identify the best policy in our class using a simple grid search over a (constrained) range of parameters to obtain the parameter values that would calibrate our rules so that the implied impulse response functions for the variables in the policy objective come closest to those describing the behaviour of the optimal economy following the same shock. A similar criterion was shown to be a good alternative to analyses based directly on utility-based measures of welfare in Schmitt-Grohé and Uribe (2005).

The structure of the paper is as follows. Section 2 presents the microeconomic foundations of our sticky-price, sticky-wage model. In Section 3, we outline the corresponding macroeconomic model and discuss the policy makers' objective. We also examine the feasibility of some simple policy strategies. In Section 4, we derive the 'timelessly optimal' plan. In Section 5, we propose a pair of simple targeting rules to approximate optimal policy and analyze how the economy performs relative to the optimal economy under such a characterization of policy. Since the steps involved in deriving the model of the macroeconomy and the approximate Ramsey problem directly follow from Benigno and Woodford (2003, 2004), the Appendix to this paper only lists some key relationships between parameters and variables. The derivations of the structural relationships and of the policy objective are available upon request from the author.

2. The microeconomic foundations

Our model economy is inhabited by an infinite number of identical households of measure one. The representative household derives positive utility from total consumption C of differentiated goods and incurs disutility from supplying labour h, which is captured by the utility function

$$U_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} u_T; \tag{2.1}$$

$$u_t = U\left(C_t; \widehat{G}_t\right) - \int_0^1 \Lambda\left(h_t\left(j\right)\right) dj. \tag{2.2}$$

$0 < \beta < 1$ is the subjective discount rate. The household supplies industry-specific labour to j industries. As explained in Woodford (2003, Chapter 3), this is equivalent to assuming that each household is employed in one type of industry only and the existence of perfect capital markets to enable risk-sharing across industries. We assume the following specific functional forms

$$U\left(C_t; \widehat{G}_t\right) = \frac{C_t^{1-\widetilde{\sigma}^{-1}}}{1-\widetilde{\sigma}^{-1}}, \tag{2.3}$$

$$\Lambda\left(h_t\left(j\right)\right) = \frac{h_t\left(j\right)^{1+\omega_w}}{1+\omega_w}, \tag{2.4}$$

where $\widetilde{\sigma}$ and ω_w are constants. In the utility function, $\widehat{G}_t$ stands for a shock to government expenditures, which is the only source of disturbance in our model.

Consumption of individual goods is aggregated into a total consumption index using a standard Dixit-Stiglitz (1977) aggregator

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}, \tag{2.5}$$

in which ε_p is a constant and represents the elasticity of substitution across goods in the goods market. Minimization of an expenditure function subject to (2.5) yields an expression for the optimal consumption of good i. A standard income identity then implies the demand function

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\varepsilon_p}. \tag{2.6}$$

The corresponding price index is written as

$$P_t = \left[\int_0^1 p_t(i)^{1-\varepsilon_p} di \right]^{\frac{1}{1-\varepsilon_p}}. \tag{2.7}$$

We introduce imperfect competition into the labour market in a similar way. We assume the existence of a continuum of monopolistically competitive households, supplying differentiated labour to the production sector. The total quantity of labour used in the production of good i is an aggregate of different types of labour indexed j.

$$H_t(i) = \left[\int_0^1 h_t(j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \tag{2.8}$$

in which ε_w is the elasticity of substitution in the labour market. It is assumed that there exists an employment agency that bundles together different types of labour needed in the production of a good i exactly in the same way as the firm producing that good would want it. Cost minimization by wage-taking firms subject to (2.8) yields the demand for labour of type j

$$h_t(j) = H_t \left(\frac{w_t(j)}{W_t} \right)^{-\varepsilon_w} \tag{2.9}$$

and the nominal wage index

$$W_t = \left[\int_0^1 w_t\left(j\right)^{1-\varepsilon_w} dj \right]^{\frac{1}{1-\varepsilon_w}}. \tag{2.10}$$

Note that aggregate labour supply in the economy can then be expressed as $H_t = \int_0^1 H_t\left(i\right) di$. Combining this with the production function and (2.6) yields the expression for total labour supply in the economy

$$H_t = Y_t^{\alpha} \delta_{p,t}, \tag{2.11}$$

where $\delta_{p,t}$ refers to price dispersion and is given by

$$\delta_{p,t} = \int_0^1 \left(\frac{p_t\left(i\right)}{P_t} \right)^{-\alpha\varepsilon_p} di. \tag{2.12}$$

The total disutility from supplying labour can then be expressed as

$$\int_0^1 \Lambda\left(h_t\left(j\right)\right) dj = \frac{1}{1+\omega_w} Y_t^{\alpha(1+\omega_w)} \delta_{p,t}^{1+\omega_w} \delta_{w,t}, \tag{2.13}$$

where $\delta_{w,t}$ stands for wage dispersion and is given by

$$\delta_{w,t} = \int_0^1 \left(\frac{w_t\left(j\right)}{W_t} \right)^{-\varepsilon_w(1+\omega_w)} dj. \tag{2.14}$$

2.1. The wage-setting decision

Each household maximizes the difference between the utility derived from wage income and the disutility from labour supply

$$\frac{U_c\left(C_t; \widehat{G}_t\right)}{P_t} \left(1 - \tau_{w,t}\right) w_t\left(j\right) h_t\left(w_t\left(j\right)\right) - \Lambda\left(h_t\left(w_t\left(j\right)\right)\right). \tag{2.15}$$

Moreover, it does so in a forward-looking way, evaluating an expected stream of net utility gains. As in Erceg et al. (2000), we assume a wage setting mechanism of the type put forward by Calvo (1983) with $\gamma_w \in (0,1)$ denoting the probability of not being able to adjust wages in any period. The intertemporal first-order condition can then be written as

$$E_t \sum_{T=t}^{\infty} \left\{ \gamma_w^{T-t} Q_{t,T} H_t \left(\frac{w_T(j)}{W_T} \right)^{-\varepsilon_w} \right.$$
$$\left. \times \left[(1-\tau_{w,T}) - \mu_w H_T^{\omega_w} \frac{P_T}{W_T} \frac{1}{U_c\left(C_T; \widehat{G}_T\right)} \left(\frac{w_T(j)}{W_T} \right)^{-\varepsilon_w \omega_w - 1} \right] \right\} = 0$$

in which τ_w is the rate of distortive tax on wage income. Under the baseline Calvo framework, $w_T(j) = w_t(j)$ for all T. This implies that firms will charge the price chosen in period t, if they do not receive a signal that they can adjust their prices in period T (with probability γ_w^{T-t}). $Q_{t,T}$ is a standard stochastic asset pricing kernel that can be derived from a simple household utility maximization problem subject to a flow budget constraint equating wage and dividend income together with asset returns to consumption and change in assets. μ_w is the wage markup and is given by

$$\mu_w = \frac{\varepsilon_w}{\varepsilon_w - 1}. \tag{2.16}$$

In a symmetric equilibrium, suppliers of labour who change their prices in period t set a common wage so that $w_t(j) = w_t^*$. We can now solve the first-order condition from the wage-setting problem for wage dispersion

$$\begin{aligned} \frac{w_t^*}{W_t} &= \left[\frac{E_t \sum_{T=t}^{\infty} (\gamma_w \beta)^{T-t} \mu_w Y_T^{\alpha(1+\omega_w)} \delta_{p,T}^{1+\omega_w} \left(\frac{W_T}{W_t} \right)^{\varepsilon_w(1+\omega_w)}}{E_t \sum_{T=t}^{\infty} (\gamma_w \beta)^{T-t} U_c\left(C_T; \widehat{G}_T\right) \frac{W_T}{P_T} Y_T^{\alpha} \delta_{p,T} (1-\tau_{w,t}) \left(\frac{W_T}{W_t} \right)^{\varepsilon_w - 1}} \right]^{\frac{1}{1+\varepsilon_w \omega_w}} \\ &= \left(\frac{K_{w,t}}{F_{w,t}} \right)^{\frac{1}{1+\varepsilon_w \omega_w}}. \end{aligned} \tag{2.17}$$

The assumption of baseline Calvo-pricing implies the following law of motion for the wage index

$$W_t = \left[(1-\gamma_w) w_t^{*1-\varepsilon_w} + \gamma_w W_{t-1}^{1-\varepsilon_w} \right]^{\frac{1}{1-\varepsilon_w}}. \tag{2.18}$$

Combining (2.17) and (2.18) gives us an implicit definition of the nominal wage growth

$$\left[\frac{1-\gamma_w \Pi_{w,t}^{\varepsilon_w - 1}}{1-\gamma_w}\right]^{\frac{1+\varepsilon_w \omega_w}{\varepsilon_w - 1}} = \frac{F_{w,t}}{K_{w,t}}, \tag{2.19}$$

where $\Pi_{w,t} = \frac{W_t}{W_{t-1}}$. It is now also easy to show that

$$\delta_{w,t} = \gamma_w \Pi_{w,t}^{\varepsilon_w (1+\omega_w)} \delta_{w,t-1} + (1-\gamma_w) \left[\frac{1-\gamma_w \Pi_{w,t}^{\varepsilon_w - 1}}{1-\gamma_w}\right]^{-\frac{\varepsilon_w (1+\omega_w)}{1-\varepsilon_w}}. \tag{2.20}$$

2.2. The price-setting decision

Firms maximize a future stream of profits with wages being the only cost item in their balance sheets. The nominal profit of firm i is written as

$$\digamma_t (i) = p_t (i) \, y_t (i) - W_t H_t (i) \, . \tag{2.21}$$

Here, we also assume a baseline Calvo price-setting mechanism with γ_p being the probability of having to leave prices unchanged in a given period. The representative firm is then choosing the optimal price, taking the wage index as given, and the intertemporal first-order condition is written as

$$E_t \sum_{T=t}^{\infty} \gamma_p^{T-t} Q_{t,T} Y_T \left(\frac{p_t (i)}{P_T}\right)^{-\varepsilon_p} \left[1 - \mu_p \frac{W_T}{P_T} \alpha Y_T^{\alpha - 1} \left(\frac{p_t (i)}{P_T}\right)^{-\varepsilon_p (\alpha - 1) - 1}\right] = 0 \tag{2.22}$$

We can define $\omega_p = \alpha - 1$ and as before, the markup is given by

$$\mu_p = \frac{\varepsilon_p}{\varepsilon_p - 1}. \tag{2.23}$$

For $p_t (i) = p_t^*$, we obtain a closed-form solution

$$\frac{p_t^*}{P_T} = \left(\frac{K_{p,t}}{F_{p,t}}\right)^{\frac{1}{1+\varepsilon_p \omega_p}} \tag{2.24}$$

with

$$K_{p,t} = E_t \sum_{T=t}^{\infty} \left(\gamma_p \beta\right)^{T-t} U_c\left(C_T; \widehat{G}_T\right) \mu_p \frac{W_T}{P_T} \alpha Y_T^{\alpha} \left(\frac{P_T}{P_t}\right)^{\varepsilon_p(1+\omega_p)}, \tag{2.25}$$

$$F_{p,t} = E_t \sum_{T=t}^{\infty} \left(\gamma_p \beta\right)^{T-t} U_c\left(C_T; \widehat{G}_T\right) Y_T \left(\frac{P_T}{P_t}\right)^{\varepsilon_p - 1}. \tag{2.26}$$

The price index evolves according to

$$P_t = \left[\left(1-\gamma_p\right) p_t^{*1-\varepsilon_p} + \gamma_p P_{t-1}^{1-\varepsilon_p}\right]^{\frac{1}{1-\varepsilon_p}}, \tag{2.27}$$

which together with (2.24) implies the following implicit definition of price inflation $\Pi_{p,t} = \frac{P_t}{P_{t-1}}$

$$\left[\frac{1-\gamma_p \Pi_{p,t}^{\varepsilon_p - 1}}{1-\gamma_p}\right]^{\frac{1+\varepsilon_p \omega_p}{\varepsilon_p - 1}} = \frac{F_{p,t}}{K_{p,t}}. \tag{2.28}$$

The law of motion for price dispersion as defined in (2.12) is

$$\delta_{p,t} = \gamma_p \Pi_{p,t}^{\varepsilon_p(1+\omega_p)} \delta_{p,t-1} + \left(1-\gamma_p\right)\left[\frac{1-\gamma_p \Pi_{p,t}^{\varepsilon_p - 1}}{1-\gamma_p}\right]^{-\frac{\varepsilon_p(1+\omega_p)}{1-\varepsilon_p}}. \tag{2.29}$$

2.3. Central government

Monetary and fiscal authorities, the two branches of the central government, coordinate their actions to ensure that social welfare given by (2.1) is maximized. The government raises revenues via distortive taxes on wage income to finance exogenous government spending G. It issues one-period nominal bonds to bridge the gap between taxation and spending. The government therefore faces the flow budget constraint

$$B_t = \left(1 + i_{t-1}\right) B_{t-1} - P_t \Delta_t, \tag{2.30}$$

where B denotes the volume of one-period nominal bonds issued by the fiscal authority, Δ_t is the t-period primary budget surplus and i_t is the nominal interest rate. Aggregate tax receipts are given by

$$T_t = \tau_{w,t} w_{R,t} Y_t^{\alpha} \delta_{p,t}. \tag{2.31}$$

This constraint can be rewritten as

$$\frac{b_t}{(1+i_t)} = \frac{b_{t-1}}{\Pi_{p,t}} - \Delta_t. \tag{2.32}$$

The nonlinear Ramsey problem is then to maximize (2.1) subject to the first-order conditions from the optimization problems of the consumers and firms, the fiscal solvency requirement, the dynamics of the price and wage dispersion, and the relevant initial and terminal conditions, alongside appropriate (initial) commitments that ensure time-consistency of the solution in the absence of full commitment.[2] This problem would yield a system of nonlinear first-order conditions. One can then solve for optimal dynamics using the nonlinear model or linearize the nonlinear system to obtain an approximate solution. An equivalent way of solving for the approximate optimal plan is to formulate a linear-quadratic approximate policy problem (see Benigno and Woodford, 2003, 2006b for a thorough treatment of a general class of problems which includes the one considered here). In frameworks where stabilization policy has significant first-order welfare effects, the construction of a correct second-order-accurate welfare objective requires a second-order approximation to the structural equations. In the next section, we present the structural elements of the approximate problem.

3. The macroeconomic model

We find that the absence of lump-sum taxation as a source of government finance does not change the nature of policy makers' preferences as expressed by a

[2] Such formulation of the policy problem is now standard and is not presented here to economize on space.

quadratic loss function. It follows from our setup that the monetary and fiscal branches of central government should conduct monetary and fiscal policy to minimize the quadratic loss function

$$L_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \frac{1}{2} q_y y_T^2 + \frac{1}{2} q_{\pi p} \pi_{p,T}^2 + \frac{1}{2} q_{\pi w} \pi_{w,T}^2 \right\}. \tag{3.1}$$

As in Benigno and Woodford (2003, 2004), y_t is the welfare-relevant output gap defined as the difference between the actual and the target deviation in output $\widehat{Y}_t^*$. The target deviation follows from the approximation to individual utility, hence it is determined by preferences and is independent of policy (see the Appendix). $\pi_{p,t}$ stands for price inflation and $\pi_{w,t}$ stands for nominal wage growth or wage inflation. If appropriate initial commitments are satisfied, this objective represents a second-order-accurate welfare-ranking criterion, while the rest of the model is specified with only first-order accuracy. The functional form is thus the same as in Erceg et al. (2000) and Benigno and Woodford (2004). The objective contains a wage-inflation-stabilization term, which is absent in flexible-wage frameworks such as Benigno and Woodford (2003). As one would expect though, the coefficients as well as the target deviation in output will, in general, be different given the different structural setup caused by the unavailability of lump-sum taxation.

For common parameter values, the coefficients in the policy objective will be positive, implying that the function is convex and the optimal solution that we obtain represents a minimum from the perspective of welfare losses.

The setup presented in the previous section also implies the following structural relationships for the economy. The supply side is characterized by

$$\pi_{p,t} = \kappa_p y_t + \zeta_p \left(\widehat{w}_{R,t} - \widehat{w}_{R,t}^* \right) + \beta E_t \pi_{p,t+1}; \tag{3.2}$$

$$\pi_{w,t} = \kappa_w y_t + \zeta_w \left[\chi_w \left(\widehat{\tau}_{w,t} - \widehat{\tau}_{w,t}^* \right) - \left(\widehat{w}_{R,t} - \widehat{w}_{R,t}^* \right) \right] + \beta E_t \pi_{w,t+1}. \tag{3.3}$$

Equation (3.2) is the price aggregate supply relationship. $\widehat{w}^*_{R,t}$ is the deviation in the real wage rate from its steady state value that brings about the target deviation in output without affecting the price level. Equation (3.3) is the wage aggregate supply relationship. $\widehat{\tau}^*_{w,t}$ is the deviation in the tax rate that aligns the natural rate of output with its target level, as in Benigno and Woodford (2003). It is determined based on the difference in the degree to which the natural and the welfare-efficient levels of output are affected by the rise in government spending.[3]

The dynamics of real wages is given by

$$\widehat{w}_{R,t} - \widehat{w}_{R,t-1} = \pi_{w,t} - \pi_{p,t}. \tag{3.4}$$

Chugh (2006) highlights the central importance of this identity in determining the optimal dynamics of the economy and optimal inflation volatility when wages are sticky.[4] The flow government budget constraint, together with a standard Euler equation and a transversality condition, implies the following fiscal sustainability condition

$$\begin{aligned}\widehat{b}_{t-1} - \pi_{p,t} - \sigma^{-1} y_t &= -\varphi_t + (1-\beta)\left[f_y y_t + f_\tau \left(\widehat{\tau}_{w,t} - \widehat{\tau}^*_{w,t}\right) + f_w \left(\widehat{w}_{R,t} - \widehat{w}^*_{R,t}\right)\right] \\ &\quad + \beta E_t \left[\widehat{b}_t - \pi_{p,t+1} - \sigma^{-1} y_{t+1}\right] \end{aligned} \tag{3.5}$$

φ_t is again the 'fiscal stress' term introduced in Benigno and Woodford (2003). The coefficients f_y, f_τ, and f_w are defined in the Appendix. Finally, the demand

[3]Briefly, the natural rate of output is the level of output that would prevail in an economy with no nominal rigidities. See Woodford (2003) for a thorough treatment.

[4]Under staggered wage adjustment, nominal wage growth is costly in welfare terms, as it generates an inefficient allocation of resources due to wage dispersion and hence its desired volatility will be low. Assuming that real wage dynamics is determined elsewhere in the system and is fairly stable, inflation varies only to deliver the desired path of real wages. This role of inflation, according to Chugh (2006), dominates its role of a shock absorber. Hence, he obtains low volatility of inflation to be optimal even when only wage adjustment is costly. This contrasts with Schmitt-Grohé and Uribe (2005) who find prices to be optimally volatile when wages only are inflexible.

side of the economy is given by the 'IS' relationship

$$y_t = E_t y_{t+1} - \sigma \left(\widehat{i}_t - E_t \pi_{p,t+1} - \widehat{r}_t^* \right), \tag{3.6}$$

in which $\widehat{r}_t^*$ is the 'efficient' deviation in the interest rate that is consistent with the target deviation in output. This equation follows from a standard Euler equation.

3.1. Some simple policy considerations

Erceg et al. (2000) have argued that the policy maker is able to achieve maximum welfare—the Pareto optimum, as they referred to a value of zero of L_t—if prices in either the product or labour market are perfectly flexible. In that case, a policy of zero inflation in the 'non-flexible' market will be consistent with maximum welfare. Benigno and Woodford (2004), however, overturned their result showing that the presence of cost-push shocks in economies where stabilization policy has significant first-order welfare effects will prevent such policies being feasible.

In our model, there is a new element. Since the dynamics of the tax rate is endogenous, the tax rate as a policy variable can be used to counteract cost-push pressures in the economy. On the other hand, the optimal economy also has to satisfy the constraint on government financing. Therefore, in the context of our model, we can formulate the following proposition.

Proposition 3.1. *a) Perfect stabilization of nominal wages with the aim of attaining the Pareto optimum is a feasible policy under flexible prices but sticky wages if and only if*

$$\widehat{b}_{t-1} + \varphi_t = \widehat{w}_{R,t-1} - \widehat{w}_{R,t}^* \text{ for all } t. \tag{3.7}$$

b) Similarly, it is feasible to engineer a policy of zero price inflation to attain the Pareto optimum under flexible wages but sticky prices if and only if

$$\widehat{b}_{t-1} + \varphi_t = 0 \text{ for all } t. \tag{3.8}$$

Proof a) Since prices are fully flexible, the price aggregate supply relationship is vertical and price inflation volatility carries zero weight in (3.1). Price inflation π_p can then serve the purpose of maintaining

$$\widehat{w}_{R,t} = \widehat{w}^*_{R,t} \text{ for all } t, \tag{3.9}$$

given that $\pi_{w,t} = 0$ at all times, so that with $\widehat{\tau}_{w,t} = \widehat{\tau}^*_{w,t}$ for all t, zero output gaps are realized for all t and the Pareto optimum is attained. However, price inflation is also the only means available to satisfy the fiscal solvency constraint. With no output gap and no 'tax gap' either, (3.5) is satisfied if and only if

$$\widehat{b}_{t-1} + \varphi_t = \pi_{p,t} \text{ for all } t. \tag{3.10}$$

Equations (3.4), (3.9) and (3.10) lead to (3.7). With this, all constraints are satisfied and the zero-inflation, zero-output-gap Pareto optimum is feasible. Now suppose

$$\widehat{b}_{t-1} + \varphi_t \neq \widehat{w}_{R,t-1} - \widehat{w}^*_{R,t} \text{ for any } t. \tag{3.11}$$

For the Pareto optimum to be feasible, fiscal solvency and hence (3.10) has to be satisfied. We can thus replace the left-hand side in (3.11) with $\pi_{p,t}$. By (3.9), which also must hold in the Pareto optimum, $\widehat{w}_{R,t}$ replaces $\widehat{w}^*_{R,t}$ on the right-hand side. Combining this with (3.4) leads to the contradiction $\pi_{p,t} \neq \pi_{p,t}$. Thus, if (3.7) is not satisfied in any t, the Pareto optimum will not be attainable.

b) In the flexible-wage case, with a vertical wage aggregate supply relationship, π_w carries zero weight in the policy objective. It can vary freely to ensure (3.9) is satisfied, given that $\pi_{p,t} = 0$ at all times, so that with $\widehat{\tau}_{w,t} = \widehat{\tau}^*_{w,t}$ for all t, zero output gaps are realized for all t. The Pareto optimum is feasible if and only if fiscal solvency is maintained. With zero output gap, zero 'tax gap' and zero price inflation, (3.5) is satisfied if and only if (3.8) holds. This is a formal presentation of the argument in Benigno and Woodford (2003). ■

Neither of the conditions (3.7) and (3.8) is likely to be satisfied under general circumstances. We can therefore conclude that the presence of distortive taxes, even though affecting the structure of the economy and of the argument, it does not alter the principal conclusions from Benigno and Woodford (2004). Strict price- and/or wage-level targeting remains an undesirable policy strategy in the presence of nominal rigidity. The need to satisfy the fiscal solvency condition precludes the attainment of the Pareto optimum when either of the product or labour markets is flexible.

4. Optimal dynamics

It is well known that optimal policy in forward-looking rational expectation frameworks is in general time inconsistent and so Woodford (2003) has proposed formulating optimal policy from a timeless perspective; one should model optimal policy as the policy that would have been intended for the current period had such a policy been formulated far in the past. Such a perspective rules out the possibility of policy makers 'exploiting' certain initial conditions. In order to characterize this timelessly optimal policy, we need to restrict the policy choices for period t so that the policy maker uses the same procedure to formulate policy as in later periods. The optimal policy is then chosen so that it satisfies certain initial (self-consistent) commitments, in addition to the structural equations of the economy, and is characterized by a time-invariant policy rule. Our policy Lagrangian thus includes commitments regarding the t-period values of endogenous variables the knowledge of which in period $t-1$ would have influenced the determination of the equilibrium. These commitments reflect the long-run optimal solution to the dynamics of the relevant endogenous variables. The policy Lagrangian is written as

$$\begin{aligned}
J_t \;=\; & E_t \sum_{T=t}^{\infty} \beta^{T-t} \Bigg\{ \left[\frac{1}{2} q_y y_T^2 + \frac{1}{2} q_{\pi p} \pi_{p,T}^2 + \frac{1}{2} q_{\pi w} \pi_{w,T}^2 \right] \\
& + \phi_{1,T} \left[\pi_{p,T} - \kappa_p y_T - \zeta_p \left(\widehat{w}_{R,T} - \widehat{w}_{R,T}^* \right) - \beta E_t \pi_{p,T+1} \right] \\
& + \phi_{2,T} \left[\pi_{w,T} - \kappa_w y_T - \zeta_w \chi_w \left(\widehat{\tau}_{w,T} - \widehat{\tau}_{w,T}^* \right) + \zeta_w \left(\widehat{w}_{R,T} - \widehat{w}_{R,T}^* \right) - \beta E_t \pi_{w,T+1} \right] \\
& + \phi_{3,T} \left[\widehat{w}_{R,T} - \widehat{w}_{R,T-1} - \pi_{w,T} + \pi_{p,T} \right] \\
& + \phi_{4,T} \Big[\widehat{b}_{T-1} - \pi_{p,T} - \sigma^{-1} y_T + \varphi_T - (1-\beta) f_y y_T - (1-\beta) f_\tau \left(\widehat{\tau}_{w,T} - \widehat{\tau}_{w,T}^* \right) \\
& - (1-\beta) f_w \left(\widehat{w}_{R,T} - \widehat{w}_{R,T}^* \right) - \beta \widehat{b}_T + \beta E_t \pi_{p,T+1} + \beta E_t \sigma^{-1} y_{T+1} \Big] \Bigg\} \\
& + \left(\phi_{4,t-1} - \phi_{1,t-1} \right) \pi_{p,t} - \phi_{2,t-1} \pi_{w,t} + \sigma^{-1} \phi_{4,t-1} y_t
\end{aligned} \tag{4.1}$$

The first-order conditions (with $T = t$) form the following dynamic system

$$q_y y_t = \kappa_p \phi_{1,t} + \left[(1-\beta) f_y + \sigma^{-1} - \kappa_w \frac{(1-\beta) f_\tau}{\zeta_w \chi_w} \right] \phi_{4,t} - \sigma^{-1} \phi_{4,t-1}, \tag{4.2}$$

$$q_{\pi p} \pi_{p,t} = \left(\phi_{4,t} - \phi_{4,t-1} \right) - \left(\phi_{1,t} - \phi_{1,t-1} \right) - \phi_{3,t}, \tag{4.3}$$

$$q_{\pi w} \pi_{w,t} = \phi_{3,t} + \frac{(1-\beta) f_\tau}{\zeta_w \chi_w} \left(\phi_{4,t} - \phi_{4,t-1} \right), \tag{4.4}$$

$$\phi_{3,t} = \beta E_t \phi_{3,t+1} + \zeta_p \phi_{1,t} + (1-\beta) \left[f_w + \frac{f_\tau}{\chi_w} \right] \phi_{4,t}, \tag{4.5}$$

$$\phi_{4,t} = E_t \phi_{4,t+1}. \tag{4.6}$$

The optimal dynamics of the key variables can be obtained by solving a system comprising (4.2) to (4.6) and the constraints in (4.1). Unfortunately, the complexity of the system does not allow us to obtain closed-form solutions for the dynamics of endogenous variables, which is the main disadvantage of this

Parameter	Value
β	0.99
σ^{-1}	0.157
$\varepsilon_p = \varepsilon_w$	10
α	1.25
$\gamma_p = \gamma_w$	0.65
ω	0.473
$\overline{\tau}_w$	0.3
c	0.8

Table 4.1: Parameter values

more complex setup. Nor can we specify optimal policy at a deeper level by deriving tractable policy targets and implied rules for monetary and fiscal policy. We are therefore constrained to present the optimal dynamics in the form of numerical results. We do this below. In the next section, we shall examine simple targeting rules that do well in generating dynamics that closely matches the optimal dynamics.

4.1. Numerical analysis: Calibration and baseline results

We use the King and Watson (1998) algorithm to solve the dynamic system of first-order conditions and structural constraints. The parameter values used in the calibration exercise are given in Table 4.1.

The implied steady-state value of the primary budget balance as well as the values of the coefficients in the policy objective are displayed in Table 4.2. We see that the relative weight of output gap stabilization remains very small. The relative weights in the objective function roughly correspond to the relative weights obtained in the monetary policy model of Benigno and Woodford (2004).[5] The system, as calibrated, satisfies the Blanchard and Kahn (1980) stability

[5] Given our calibration, the corresponding coefficient values in the Benigno and Woodford (2004) model would be $q_y = 0.9680, q_{\pi p} = 287.9683$ and $q_{\pi w} = 183.2466$.

Variable	Value
$\frac{\overline{\overline{\Delta}}}{\overline{Y}}$	0.0160
q_y	2.3496
$q_{\pi p}$	666.77
$q_{\pi w}$	424.29

Table 4.2: Steady-state values and coefficients

conditions and has a unique, stable solution, which we present below.

Based on the discussion above on the optimality of simple price- and wage-level targeting strategies, it is easy to postulate that the optimal dynamics under sticky prices and sticky wages will not involve the pair $\left\{\widehat{w}^*_{R,T}, \widehat{\tau}^*_{w,T}\right\}_{T=t}^{\infty}$ and we shall observe price and wage inflation and non-zero output gaps. The optimal dynamics in response to a single, serially uncorrelated positive innovation to government spending of the magnitude of one percent of steady-state output are plotted in Figure 4.1.[6] We find that the absence of the possibility of instantaneous real-wage adjustment introduces endogenous persistence into the optimal dynamics of the economy. We also see that the solution includes non-stationary elements, as in the flexible-wage case of Benigno and Woodford (2003). While price and wage inflation as well as the interest rate are stationary, the output gap, real wages, the tax rate and outstanding liabilities all converge to a new equilibrium. This equilibrium is characterized by a higher tax rate and a lower output level. Hence, the non-stationarity property of the optimal dynamic solution to public debt, the tax rate and output survives in the sticky-price, sticky-wage framework as well. The near unit root property of the solution for debt is also consistent with both Schmitt-Grohé and Uribe (2005) and Chugh (2006).

[6] The value of 1 on the vertical axes denotes 1 percent deviation from the pre-shock steady state. The inflation and wage rates are quarterly.

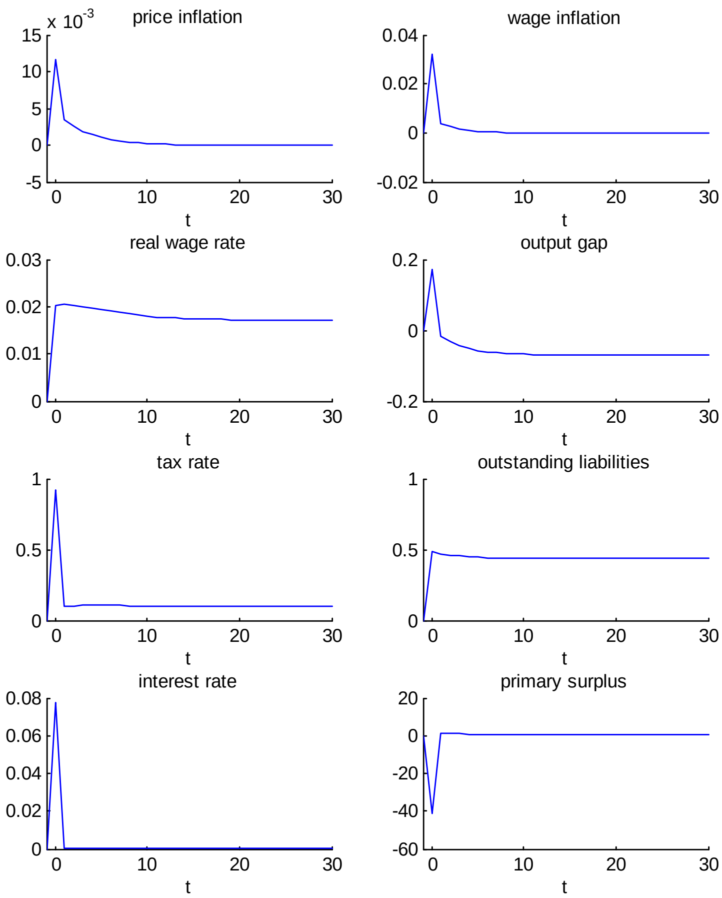

Figure 4.1: Optimal dynamics under imperfect price and wage flexibility

4.2. Sensitivity analysis

In this section, we examine how the above reported results change when one varies the length of contracts in both product and labour markets. Benigno and Woodford (2003) have performed similar diagrammatic sensitivity analysis in their sticky-price, flexible-wage framework. They concluded that optimal (price) inflation volatility falls dramatically for even a small degree of price stickiness. The optimal long-run tax policy is shown to be fairly robust to the degree of price stickiness, except for the limiting case of full flexibility when taxes are optimally stabilized at their pre-shock steady state level in the long run. They, however, report substantial variation both in the size and the sign of the short-term response in the tax rate.

Our results for the sticky-price, sticky-wage economy are shown in Figures 4.2 and 4.3. There are several things to note about these results. First, the optimal response in the real wage rate is much smaller relative to the response in aggregate production compared with the flexible-wage economy at even small degrees of wage stickiness. For any length of wage contracts, we get a subdued, hump-shaped response in the real wage rate, which brings the dynamics of the model in line with the empirical literature.[7] Second, both price and wage inflation vary with the degree of wage stickiness but wage inflation is remarkably insensitive to changes in the contract length in the product market. Our analysis suggests optimal price inflation volatility increases considerably as one shortens the duration of price contracts while keeping wage contract duration at the baseline length. These results give some support to the conclusions of Schmitt-Grohé and Uribe

[7] Stock and Watson (1999), for instance, observe that real wages in the United States have displayed 'essentially no contemporaneous comovement with the business cycle'. A similar point has been made by Christiano et al. (1997, 1999). Chadha et al. (2002) also find statistically insignificant correlation between output and real wages, though the fact that this need not hold for the entire history of UK business cycle fluctuations is shown in Chadha and Nolan (2000).

(2005) that price stickiness is the single most important distortion in the economy justifying price stability as the central goal of monetary policy. Third, the optimal long-run tax policy is as robust to both price and wage rigidity as it was in the case of the sticky-price economy of Benigno and Woodford (2003). However, the short-term response in the tax rate is sensitive to the degree of wage stickiness but not the length of price contracts. The tax policy here is affected by the changes in the slope of the wage aggregate supply relationship and the changes in the costliness of wage inflation resulting from changes in the duration of wage contracts. When wages are highly rigid, the wage aggregate supply schedule is flat and the impact of a given tax rise on wages is only small. At the same time, nominal wage growth is very costly in welfare terms. Our results indicate that it is optimal to raise taxes sharply in the short term in response to the shock, implying that the first effect dominates. As we approach the other extreme, the aggregate supply schedule is now steep but a given rate of wage inflation is less costly. Our analysis indicates that tax policy principally remains unchanged compared with the case of highly rigid wages, implying that now the second effect dominates. The intermediate values of wage stickiness then imply more tax smoothing over time, characterized by a subdued initial response in taxes followed by slower adjustment to the new long-run equilibrium level. Finally, we find that the degree of wage or price stickiness has remarkably little effect on the optimal dynamics of the output gap both in the short run and the long run.[8] These observations carry important information that we shall utilize when specifying simple targeting rules in the next section.

[8]Several of the observations here are in line with Erceg et al. (2000). Their sensitivity analysis implies that the variance of price inflation in the optimal economy is more sensitive to wage contract duration than wage inflation volatility to price contract duration. They also find that the optimal variance of the output gap is pretty stable across different combinations of price and wage stickiness.

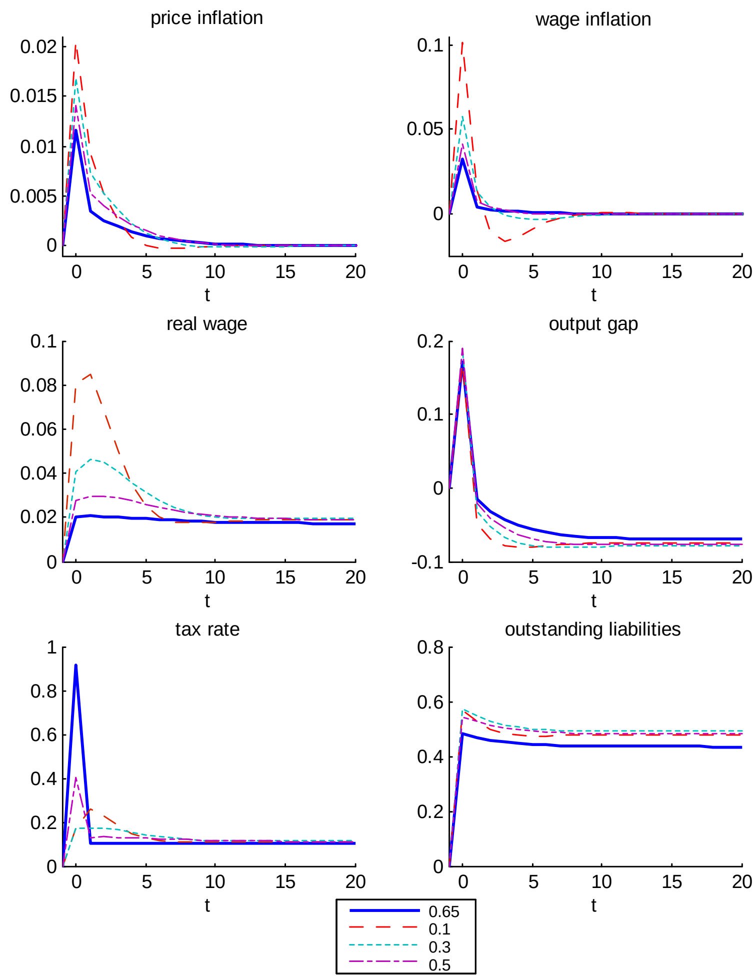

Figure 4.2: Wage contract length and optimal dynamics

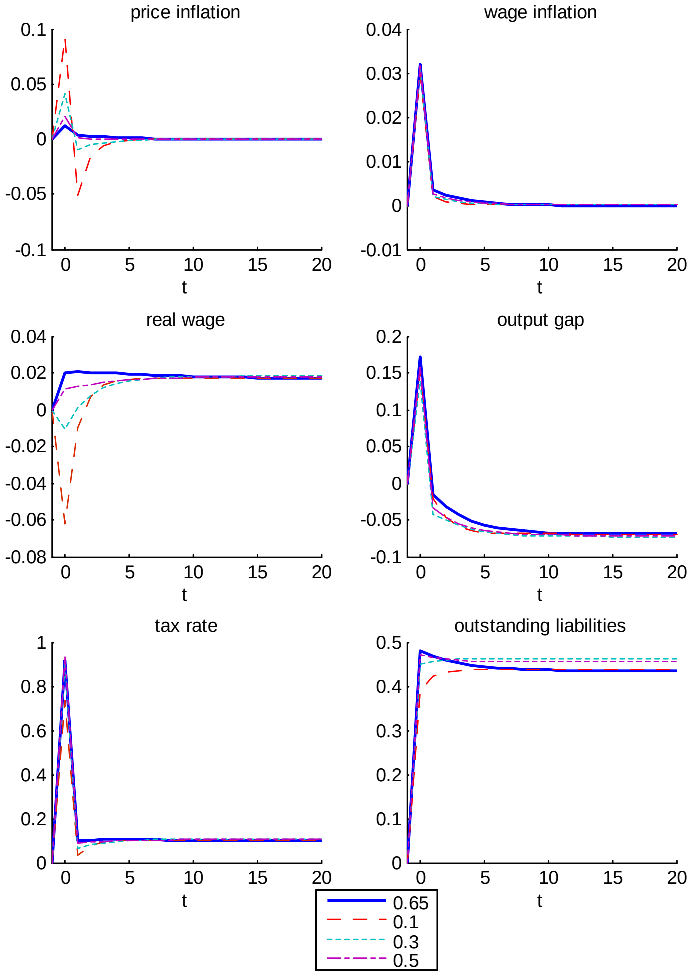

Figure 4.3: Price contract length and optimal dynamics

5. Simple specific targeting rules

The previous section has highlighted one of the considerable weaknesses of optimal policy analysis in Ramsey-type welfare-maximizing frameworks. Even in a linear-quadratic setup that would normally yield tractable results which enable us to describe optimal paths and policy in terms of analytically simple solutions, it ultimately becomes impossible to characterize optimal policy at all levels of generality—in the sense of Svensson and Woodford (2005)—once the modelling environment becomes more complicated. One way to analyze issues in policy design is then to search for simple (linear) policy rules that can to some extent replicate the dynamics of the optimal economy and produce limited welfare losses compared with the optimal plan. In the words of Lucas (1986), such rules,

> 'though certainly less efficient than a monetary [and fiscal] policy that reacted to real shocks in just the right way, would have welfare consequences differing trivially from the optimum policy and, unlike the latter, would be easy to spell out and monitor.'

Such rules can either be ad hoc or one can employ a formal optimization procedure to determine the parameters of a 'quasi-optimal' rule (Currie and Levine, 1987). Erceg et al. (2000) have found simple hybrid rules that would do well compared with the fairly complicated optimized simple rule. Schmitt-Grohé and Uribe (2004b, 2005) also search for suitable rules for the interest rate and the tax rate that would approximate optimal policy.

In this section, we take a different approach. Rather than looking for 'good' instrument rules directly, we look for suitable characterization of policy at a higher level of generality. We identify a pair of joint policy targets which, in conjunction with the structural equations, generate dynamics similar to the dynamics of the optimal economy. Subsequently, we specify 'expectations based reaction functions' for monetary and fiscal policy following Evans and Honkapohja (2006). The complexity of these reaction functions highlights the advantage of specifying policy at a higher level of generality, namely that it allows us to approximate a system with complex dynamic behaviour much more closely than simple instrument rules would normally do. There are thus potential gains in terms of welfare associated with the use of targeting rules as opposed to simple instrument rules.

The family of specific targeting rules we found useful to examine for our income-tax economy describes the policy makers' aim in terms of a rule for real wage growth as follows

$$\widehat{w}_{R,t} - \widehat{w}_{R,t-1} = -\Lambda_\pi \left(\lambda_{\pi w}\pi_{w,t-1} + \lambda_{\pi p}\pi_{p,t-1}\right) - \Lambda_y \left(y_t - y_{t-1}\right) \tag{5.1}$$

and a wage inflation targeting rule

$$E_t \pi_{w,t+1} = \rho \pi_{w,t}, \tag{5.2}$$

with $0 < \rho < 1$. $\Lambda_\pi, \lambda_{\pi w}, \lambda_{\pi p}, \Lambda_y$ and ρ are policy parameters, where $\lambda_{\pi w}$ and $\lambda_{\pi p}$ add up to one. We motivate this choice of targeting rules by the following considerations. Benigno and Woodford (2003) show that optimal policy in a sticky-price monetary-fiscal framework can be described by a pair of targeting rules (rather than a single rule, as in monetary policy models): a future (price) inflation target and a relationship that links current inflation to past inflation and the dynamics of the output gap. Benigno and Woodford (2004) solve for a rather more complex optimal relationship in their sticky-price, sticky-wage setup and conclude that a good approximation to optimal policy will likely entail a dynamic relationship featuring price and wage inflation as well as the output gap. Our targeting rule (5.1) reflects some of the features of their more complex rule.[9] We have also seen from the sensitivity analysis that optimal tax policy

[9] Adding further dynamic elements into (48) did not improve the results much more and comes at a cost of significant reductions in the simplicity of policy specification.

is sensitive to wage contract duration but not to the degree of price stickiness. The same holds for nominal wage growth. It is therefore natural to think of wage inflation as a good candidate for an intermediate target in our income-tax economy.[10] Unlike in flexible-wage models, there is some persistence in the behaviour of our endogenous variables—including wage inflation—in the optimal economy, therefore, one should expect the targeting rule to be defined as a gradual adjustment process converging to a long-run target value. Hence the choice of (5.2). It follows that the choice of this second rule will likely depend on the nature of the tax system.[11]

As far as rules for instruments are concerned, it is straightforward to derive the analogues of the 'expectations based reaction function' of Evans and Honkapohja (2006) associated with our targeting rules. For monetary policy, we have

$$\begin{aligned}\widehat{i}_t &= \widehat{r}^*_t + E_t\pi_{p,t+1} + \sigma^{-1}E_t y_{t+1} - \frac{\sigma^{-1}}{\Lambda_y}\pi_{p,t} + \frac{\sigma^{-1}}{\Lambda_y}\pi_{w,t} \\ &\quad + \frac{\sigma^{-1}}{\Lambda_y}\Lambda_\pi\lambda_{\pi p}\pi_{p,t-1} + \frac{\sigma^{-1}}{\Lambda_y}\Lambda_\pi\lambda_{\pi w}\pi_{w,t-1} - \sigma^{-1}y_{t-1} \qquad (5.3)\end{aligned}$$

The reaction function for the tax rate can be written as

$$\begin{aligned}\widehat{\tau}_{w,t} &= \widehat{\tau}^*_{w,t} + \frac{1-\beta}{\chi_w\zeta_p}\pi_{p,t} + \frac{1-\beta\rho}{\chi_w\zeta_w}\pi_{w,t} - \frac{\omega+\sigma^{-1}}{\chi_w}y_t \\ &\quad - \frac{\beta}{\chi_w\zeta_p}\left(E_t\pi_{p,t+1} - \pi_{p,t}\right), \qquad (5.4)\end{aligned}$$

[10] In setups with endogenous tax dynamics, the policy regarding the tax rate plays a role in stabilizing prices through its impact on firms' marginal cost. This relationship between the tax rate and wage inflation is also clear from the wage aggregate supply relationship (35).

[11] In our preliminary attempts, we also experimented with a price inflation target similar to the wage inflation target used, but it proved to be an inferior policy objective in our economy relying on income tax as the source of government tax revenue. It is evident from Figure 4.3 that a policy strategy featuring a simple inflation target similar to (49) would make it very difficult to mimic the optimal economy under a shorter duration of price contracts. Such specification might, however, be appropriate under a different tax regime. See the concluding remarks for further discussion.

using (5.2). Their complexity is considerable when matched against the instrument rules normally considered in the literature.

5.1. The ranking criterion

There are obviously numerous ways to calibrate our rules with each of the calibrations implying different welfare effects. Some calibrations will do better than others and the problem is to identify the best policy in the class of policies described by the pair of simple targeting rules. Since one of the variables in the policy objective, namely the output gap, is non-stationary, it is not straightforward to assess the goodness of fit provided by our simple targeting rules relative to the optimal policy in terms of welfare, as expressed by (3.1). An additional complication is that the policies we wish to rank do not imply convergence to the same long-run outcomes. What we do in this paper is that we look at how well the dynamics of our economy following the shock matches the dynamics of the optimal economy following the same shock. In other words, we evaluate alternative rules of the form (5.1) and (5.2) by comparing the impulse response functions these rules generate vis-à-vis the impulse responses of the optimal economy. We shall concentrate on the impulse responses of the variables in the policy objective. More formally, let

$$\mathbf{IR}_{t,T} = \begin{bmatrix} \boldsymbol{\pi}_{p,t,T} \\ \boldsymbol{\pi}_{w,t,T} \\ \mathbf{y}_{t,T} \end{bmatrix}$$

where $\mathbf{x}_{t,T}$ for $x = \pi_p, \pi_w, y$ is a column vector of realizations of variable x following the baseline government spending shock between time t—when the shock occurs—and time T, which is set arbitrarily.[12] In our analysis, we set $T = 30$, which

[12] As an extension of the analysis presented here, one could evaluate responses to a variety of shocks by extending the vector $\mathbf{IR}_{t,T}$ and possibly assign weights according to the importance of each of the type of shocks in explaining business cycle variation. Having multiple types of shocks would not affect the general discussion, as the additional disturbance terms would only enter the analysis via the 'star' variables in an additive fashion.

provides a long enough time for our economy to converge to its new steady state following the spending shock. Let us also define the vector of impulse responses

$$\mathbf{IR}_{t,T}^{opt} = \begin{bmatrix} \boldsymbol{\pi}_{p,t,T}^{opt} \\ \boldsymbol{\pi}_{w,t,T}^{opt} \\ \mathbf{y}_{t,T}^{opt} \end{bmatrix}$$

that would describe the dynamics of the optimal economy following the same shock. Let us further define

$$\boldsymbol{\Psi}_{t,T} = \mathbf{IR}_{t,T} - \mathbf{IR}_{t,T}^{opt}$$

and

$$\boldsymbol{\Psi}_{t,T}^{q} = \begin{bmatrix} q_{\pi p}\left(\boldsymbol{\pi}_{p,t,T} - \boldsymbol{\pi}_{p,t,T}^{opt}\right) \\ q_{\pi w}\left(\boldsymbol{\pi}_{w,t,T} - \boldsymbol{\pi}_{w,t,T}^{opt}\right) \\ q_{y}\left(\mathbf{y}_{t,T} - \mathbf{y}_{t,T}^{opt}\right) \end{bmatrix}.$$

Then we define the best policy in the family of policies characterized by (5.1) and (5.2) as the policy that minimizes the criterion

$$\Gamma = \boldsymbol{\Psi}_{t,T}^{q\prime}\boldsymbol{\Psi}_{t,T}. \tag{5.5}$$

Γ is thus a weighted sum of squares of the differences in responses in price inflation, wage inflation and output gap in period t following the government spending shock under the simple policy specification (5.1) and (5.2) and under the 'timelessly optimal' policy. The weights are given by the importance assigned to stabilization of each of the target variables in the policy objective (3.1).

5.2. Numerical results

In this section, we present the results of our search for the best calibration of (5.1) and (5.2) in terms of (5.5). Given that we conduct our search in a four dimensional space, the number of potential combinations of parameter values is large at any level of discretization. We have therefore constrained our search.

Search	Parameter	Interval	Step
1	$\Lambda_\pi\lambda_{\pi w}$	$[0, 10]$	1
1	$\Lambda_\pi\lambda_{\pi p}$	$[0, 10]$	1
1	Λ_y	$[0.5, 10.5]$	1
1	ρ	$[0, 0.3]$	0.1
2	$\Lambda_\pi\lambda_{\pi w}$	$[5, 10]$	0.5
2	$\Lambda_\pi\lambda_{\pi p}$	$[0, 5]$	0.5
2	Λ_y	$[0.5, 10.5]$	1
2	ρ	$[0.05, 0.2]$	0.05
3	$\Lambda_\pi\lambda_{\pi w}$	$[3, 10]$	1
3	$\Lambda_\pi\lambda_{\pi p}$	$[0, 7]$	1
3	Λ_y	$[0.5, 7]$	0.5
3	ρ	$[0.05, 0.3]$	0.05

Table 5.1: Intervals for parameter value search

We looked for optimal parameter combinations on certain intervals of parameter values. We have conducted the searches reported in Table 5.1, which gives the interval size and the size of one step within the given interval.[13] The intervals were chosen after some preliminary random calibrations and also keeping in mind that the resulting rule should have a practical appeal. All other structural parameters in the model were calibrated as before. The parameter combinations we report all satisfy the Blanchard-Kahn (1980) conditions for uniqueness and stability of the results.

All of the reported three searches have selected the following calibration as the one that provides the best fit with the optimal solution: $\Lambda_\pi = 10.0, \lambda_{\pi p} = 1.0, \lambda_{\pi w} = 0, \Lambda_y = 3.5$ and $\rho = 0.20$.[14] The reported parameter values imply a long-run response coefficient of 1.40 at price inflation and a long-run coefficient

[13] The precision of our grid-search is limited by the available computer power.

[14] In this specific case, the coefficient at lagged wage inflation in the targeting rule is zero. Results obtained through a similar exercise for different lengths of wage contract duration not reported here have, however, included a positive $\lambda_{\pi w}$. Hence the general formulation of (48).

of 0.05 for wage inflation in the reaction function for monetary policy. The long-run response to the output gap is zero. Figure 5.1 below provides a sense of the closeness of the dynamics generated by our targeting rule-based policy and the optimal policy.[15] Such a close match is unlikely to be provided by a pair of simple instrument rules.

6. Concluding remarks

In this paper, we have extended the Benigno and Woodford (2003) economy to include staggered wage adjustment. We have shown that preferences of the policy maker can be characterized by a quadratic welfare objective involving wage inflation variability in addition to variability in price inflation and the output gap. We have shown that perfect stabilization of prices and/or wages is not a desirable way of conducting policy, if at least one of the markets is subject to nominal rigidity. We have learned that inclusion of nominal wage rigidity causes the economy's dynamics in response to shocks to become persistent even if shocks themselves are non-persistent. The optimal dynamics of real wages is much less cyclical compared with the sticky-price, flexible-wage framework. The result that public debt, tax rate and the output gap are all non-stationary carries over to an economy with imperfect wage flexibility.

We have proposed a pair of simple specific targeting rules, one simple wage inflation rule and one real-wage-growth rule, that perform well in replicating the dynamics of the optimal economy. We have selected 'desirable' policies in the family of policies defined by these rules using a formal assessment of the match between the dynamics of the economy induced by these rules and the optimal

[15] Should one consider multiple types of shocks, the 'desirable' calibration of policy targets would likely be affected and the close match with the optimal dynamics would only hold for the shocks carrying higher weight. Nevertheless, the general concept would remain unaffected.

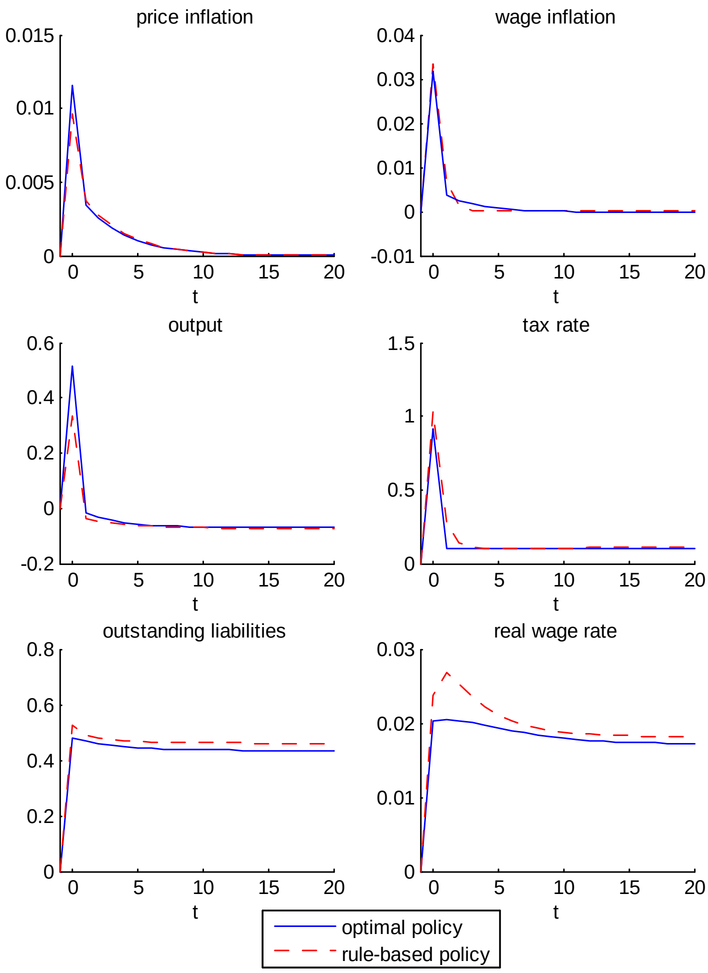

Figure 5.1: Dynamics of the economy under optimal and rule-based policy

dynamics of the economy.

In Horvath (2007), we show that these rules perform similarly well under different calibrations of price and wage contract duration. For reasons explained in Correia et al. (2003) and Benigno and Woodford (2006a), the present framework is not likely to provide useful results for economies with more complicated tax structures, involving different types of tax instruments that could be varied independently of each other. The question of the links between the tax system and the appropriate formulation of the targeting rules remains an interesting one to explore further. In Horvath (2007), we have replicated the analysis presented in this paper for an economy in which tax is on sales revenues rather than wage income and found that optimal dynamics was best approximated by a pair of specific targeting rules in which the wage inflation rule is replaced by a price inflation rule. Hence, when devising appropriate targeting-rule-based frameworks, one also needs to examine carefully the link with the tax system in addition to the dynamics of the supply side of the economy.

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A. Appendix

We define the values of the coefficients used in the text:

$$\zeta_k = \frac{(1-\gamma_k)(1-\beta\gamma_k)}{\gamma_k(1+\omega_k\varepsilon_k)} \qquad \text{for } k = p, w$$

$$\kappa_p = \zeta_p(\alpha - 1)$$

$$\kappa_w = \zeta_w\left(\alpha\omega_w + \sigma^{-1}\right)$$

$$\sigma^{-1} = \widetilde{\sigma}^{-1}c^{-1}$$

$$\alpha(1+\omega_w) = (1+\omega_p)(1+\omega_w)$$

$$\chi_w = \frac{\overline{\tau}_w}{1-\overline{\tau}_w}$$

$$d_\tau = \frac{\overline{\Delta}}{\overline{\overline{T}}}$$

$$d = \frac{\overline{\Delta}}{\overline{\overline{Y}}}$$

$$\Theta_{Y,w} = 1 - \frac{(1-\overline{\tau}_w)}{\mu_p\mu_w}$$

$$\Omega = d_\tau^{-1}\left(\alpha\omega_w + \sigma^{-1}\right) + d_\tau^{-1}(1+\chi_w)(\alpha - 1) - d_\tau^{-1}\alpha\chi_w + \sigma^{-1}\chi_w.$$

$$\begin{aligned} q_y &= \omega + \sigma^{-1} - \Theta_{Y,w}(1+\omega) - \frac{\Theta_{Y,w}}{\Omega}\chi_w d_\tau^{-1}\left(1 - 2\sigma^{-1}\right) \\ &\quad - \frac{\Theta_{Y,w}}{\Omega}\chi_w\left[\sigma^{-2} - \sigma^{-1}\left(1 - c^{-1}\right)\right] \\ &\quad + \frac{\Theta_{Y,w}}{\Omega}d_\tau^{-1}\left[(1+\omega)^2 - \left(1-\sigma^{-1}\right)^2 + \sigma^{-1}\left(1 - c^{-1}\right)\right], \end{aligned}$$

$$q_{YG} = \sigma^{-1}\left[1 + \frac{\Theta_{Y,w}}{\Omega}\chi_w\left(d^{-1} - \sigma^{-1} - c^{-1}\right) + \frac{\Theta_{Y,w}}{\Omega}d_\tau^{-1}\left(1 + \chi_w - c^{-1} - \sigma^{-1}\right)\right],$$

$$q_{\pi w} = \frac{(1-\Theta_{Y,w})}{\zeta_w\alpha}\varepsilon_w + \frac{\Theta_{Y,w}}{\Omega}d_\tau^{-1}\frac{\varepsilon_w(1+\omega_w)}{\zeta_w},$$

$$q_{\pi p} = \frac{(1-\Theta_{Y,w})}{\zeta_p}\varepsilon_p + \frac{\Theta_{Y,w}}{\Omega} d_\tau^{-1} \frac{\varepsilon_p (1+\omega)}{\zeta_p}.$$

$$\widehat{Y}_T^* = \frac{q_{YG}}{q_y}\widehat{G}_T.$$

$$\widehat{w}_{R,t}^* = -(\alpha - 1)\frac{q_{YG}}{q_y}\widehat{G}_t$$

$$\widehat{\tau}_{w,t}^* = \frac{(\omega + \sigma^{-1})}{\chi_w}\left[\frac{\sigma^{-1}}{(\omega+\sigma^{-1})} - \frac{q_{YG}}{q_y}\right]\widehat{G}_t.$$

$$f_y = \left(d_\tau^{-1}\alpha - \sigma^{-1}\right)$$

$$f_\tau = f_w = d_\tau^{-1}$$

$$\begin{aligned} f_t \;=\; & \sigma^{-1}\left(1 - \frac{q_{YG}}{q_y}\right)\widehat{G}_t + (1-\beta)E_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{ d_\tau^{-1}(\alpha-1)\frac{q_{YG}}{q_y} \right. \\ & -\left(d_\tau^{-1}\alpha - \sigma^{-1}\right)\frac{q_{YG}}{q_y} - \sigma^{-1} + d^{-1} \\ & \left. -\frac{d_\tau^{-1}}{\chi_w}\left[\sigma^{-1} - \left(\alpha\omega_w + \sigma^{-1}\right)\frac{q_{YG}}{q_y} - (\alpha-1)\frac{q_{YG}}{q_y}\right]\right\}\widehat{G}_T. \end{aligned}$$

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