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Regulation of Bank Reserves and Early Returns in a Model of Bank Runs*

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ABSTRACT

This paper uses a global game framework of bank runs to analyse how banks choose reserves and short term interest rates (early returns). The analysis shows that even though there is no need for regulation when a bank.s reserve policy is transparent, there should be regulation if there is no transparency. When the bank has private information about its reserve level, it chooses lower reserves and higher early returns than what maximises depositor welfare, which in turn increases the probability of bank runs. Therefore the regulators should .x a maximum limit for early return and a minimum level for reserves.

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The banking industry plays a key role in the well being of an economy. A sound banking system will pave way for more savings, investment and economic growth. There is much empirical evidence that banking crises affect economic growth (Caprio and Klingebiel (1996), Lindgren et.al. (1996), Mojon et al. (2008), Ennis and Keister (2003) to name a few). Despite the long history of banking crises and their detriments, they keep happening. A database describing all the banking crises suffered by countries has been put together by Laeven and Valencia (2008), according to which, there have been 124 systemic banking crises during 1970 - 2007.

Banking crises can happen because of inefficient management, bad economic conditions or fraud. In addition, a bank can fail because of a bank run. A bank run occurs when the depositors make withdrawals because they anticipate insolvency of the bank and act on self-fulfilling beliefs that other depositors would also withdraw. Theoretical and empirical research suggest that bank runs are caused by both self-fulfilling beliefs and information about the banks' efficiency.[1] Bank runs have plagued the banking industry for centuries, with some of the many casualties during the 2007/09 banking crises being Northern Rock, Bank of Antigua, Indymac Bank and Washington Mutual.

The possibility of bank runs affects the behaviour of banks, depositors and regulators. The existence of multiple equilibria in the influential Diamond and Dybvig (1983) model of bank runs made it difficult to conduct useful analysis and comparison of policies. However a significant breakthrough in this strand of literature has been made to establish a unique equilibrium using global games, enabling us to arrive at important policy decisions (Carlson and Van Damme (1993) initiated global games, of which, Morris and Shin (1998, 2002) and Goldstein and Pauzner (2005) are excellent applications). If depositors receive noisy signals about the economic fundamental that influences the uncertain return of the long term investment, a threshold fundamental exists, below which there would be a bank run and above which there will not be a bank run.

[1] Bougheas (1999), Chari and Jegannathan (1988), Cooper and Ross (1998), Demirgue-Kunt, Detragiache (1998), Gorton (1985, 1988), Jacklin and Bhattacharya (1988), Loewy (1998).

Banks are vulnerable to bank runs because they finance long term projects with short term liabilities. Reserves reduce the earning capacity of the bank, increasing the availability of funds to meet depositor demand. Even though depositors care about the bank's profitability, if they believe that the bank does not have sufficient liquidity to meet the demand for withdrawals, there will be a bank run. Therefore it is important that the right balance is struck between long term earnings and sufficient short term liquidity.

In this current paper, the global game framework is used to throw some light on two important decisions that a bank has to make: *level of reserves* it keeps and *early return* it offers early withdrawers. These two choice variables are crucial elements which determine a bank's solvency and its susceptibility to bank runs. In light of the latest banking crises in 2007/09, loss of confidence in the banking sector, deregulations that preceded the crises, banks' solvency problems, governments being forced to intervene with rescue plans, have all made it essential to analyse banks' decisions regarding their reserves, interest rates and regulation of these.

Higher early return will give better insurance to depositors in the event of a liquidity shock that forces them to withdraw early. An important result in this paper is that the optimal early return is shown to be less than the deposit itself. Higher early returns increases the probability of bank runs because it encourages those who are not hit by the liquidity shock to also withdraw early. In order to discourage them from doing so, even those who are genuinely hit by the liquidity shock have to be penalised. Another result is that the optimal level of reserves is shown to be in excess of what is required to meet the demand of those who are genuinely hit by the liquidity shock, and this excess increases with the probability of liquidity shocks.

The main aim of this paper is to look at how the choice of reserve level and early return depend on the transparency of information about a bank's solvency policy, and how this influences the need for regulation. Transparency of a bank's solvency depends on the information that is made public, its accessiblity, and the ability of depositors to make use of it. If a bank operates in a competitive environment, and there is transparency regarding its reserves and early returns, then it has to choose them to maximise the expected utility of the depositors. However, if a bank has private information about its reserve level, which in turn affects its solvency, it will exploit this information asymmetry to maximise its own profits. Regulation becomes important when there is non-transparency of vital information.

The three pillars of Basel II are capital requirement, supervisory review (which includes liquidity risk) and market discipline (which includes disclosure requirements). Dermine (2009)'s discussion about Basel II's recommendation of capital regulation shows that capital regulation should incorporate liquidity risk of the bank. More importantly it shows that since credit risk diversification and probability of loan defaults can reduce capital requirements, it can increase the probability of bank runs. There is support for regulation in the literature where it is predicted that banks would not keep sufficient reserves if it is not observable.[2] Is there a case for regulation in the model we have set up?

This paper shows that when there is no transparency of its reserves policy, the bank will choose a higher early return and lower level of reserves than what maximises the welfare of depositors, which also increases the probability of bank runs[3]. The paper recommends that even though there is no need for regulation when the level of reserves is transparent, there is indeed a need for regulation when it is not transparent. More specifically, the regulators should fix a maximum limit for early return, and a minimum level for reserves.

The analysis is not complete without discussing what happens if the bank has access to external funds when there is a shortfall in short term liquidity, though this is not the main objective. When the depositors are aware that the bank can survive even when

[2]Bhattacharya and Boot (1998), Clouse and Dow (2002), Cothren (1987), Ringborn et al. (2004).

[3]The objective of this paper is not to eleminate bank runs, but to look at the welfare of depositors in a set up where bank run is a possibility. There are papers which show that some probability of bank runs is actually optimal for the depositors (Alonso (1996), Peck and Shell (2003)).

it runs out of reserves, obviously the probability of bank runs become lower. The more access to such safety nets the bank has, the more confidence the depositors will have, and the less probability there is of bank runs. This result is also supported by Rochet and Vives (2004). The model goes on to show that the bank would be able to hold lower reserves, increasing long term earnings. This does not mean the banks can down-play the responsibility they have in holding sufficient reserves and making suitable investments. In order to secure access to such funds, the bank has to demonstrate that it is solvent in the long term.

The rest of the paper is organised as follows. In Section I, the basic model is set up. Section II is devoted to the analysis when the bank's reserve policy is transparent. It discusses the reserve level and early return chosen by the bank and the resulting probability of a bank run. The model is modified in Section III so that the bank has private information about its reserve policy and the results are compared with the outcome in the previous section. Section IV compares the outcome when the bank has access to external funds when it runs out of reserves. Section V concludes and the Appendix contains the proofs of the Propositions.

I. THE BASIC MODEL

The basic model follows the set up in Goldstein and Pauzner (2005). The model has three periods (t_0, t_1, t_2). There is a continuum $[0, 1]$ of agents who are the depositors, and a representative bank which operates in a competitive market. All the agents have endowments of one unit at the beginning of t_0. Expected returns from investing in a bank are high enough such that all the agents will invest their endowments of one unit in the bank at date t_0. Consumption happens only in periods t_1 and t_2, when the depositors withdraw their deposits.

Agents can be of two types, patient or impatient, which is their private information. The depositors learn their types only at the beginning of t_1, when proportion λ of the depositors receive a liquidity shock which would require them to definitely withdraw in t_1

itself. The depositors who don't receive the shock can postpone withdrawal till t_2. Those who receive the liquidity shock in t_1 are the impatient agents and those who don't are the patient agents. The impatient agents can derive utility only by consuming in t_1 while the patient agents can derive utility from consuming in either t_1 or t_2. The agents' utility function is increasing, twice continuously differentiable and strictly concave. There is no aggregate uncertainty (i.e. λ is fixed and known), but the bank cannot distinguish the type of each agent individually.

Once the agents deposit their endowments in t_0, the bank chooses proportion ρ to keep as reserves to meet the demand by withdrawers in t_1. The balance $(1-\rho)$ is invested in a long term risky project. Those agents who withdraw early in t_1 will receive an early return of r. Each unit that is invested in the long term project in t_0 realises a return of θ in t_2. This long term return, θ, is uncertain and unknown in t_0. Once θ is realised in t_1, each agent observe a private signal and decide whether to withdraw in t_1 or wait till t_2. This decision is based on their beliefs about θ, which affects the bank's financial success and the number of agents who would withdraw in t_1. More discussion about θ and its signals is found in the sub-section that follows.

The analysis is based on the existence of a unique equilibrium, θ^*, which is derived in Section II. If a patient agent i observes a signal $\theta_i > \theta^*$, he will not withdraw because he would then believe that the return on the project is going to be high enough and sufficient number of other depositors will also decide the same. If he observes $\theta_i < \theta^*$, he will withdraw because he believes that sufficiently high enough number of the other agents would have received signals which prompt them to withdraw, which means he would get nothing if he waited till t_2.

If the level of reserves, ρ, is sufficient to meet the early demand for withdrawals in t_1, the bank survives till t_2. If the demand in t_1 exceeds ρ, the bank has to liquidate the long term project at a very small value and therefore will have to close down in t_1 (unless the bank can borrow from another source, which we discuss separately in Section IV). In such a situation where the bank crashes, all those who want to withdraw early will receive r with equal probability, while those who waited without withdrawing early will receive nothing.

It is clear that reserves have to be at least λr to meet the demand of the impatient agents who will definitely withdraw in t_1. However, reserves can be more than this basic requirement in order to cater to the patient agents who might decide to withdraw early. Level of reserves is defined as $\rho = \lambda r + (1 - \lambda)\,\pi$, where π indicates the extent of excess reserves over and above what is needed to serve the genuinely impatient agents. On the whole, the bank's solvency depends on ρ, r and θ. The bank has to choose π and r in t_0. (note that ρ, is directly related to π.)

Economic Fundamental θ and Noisy Signals

The economic fundamental θ determines the solvency and profitability of the bank in the longer term, which depends on everything that will affect the return of the investment such as the economic and political environment. The θ is uncertain at t_0 and is drawn from a uniform distribution on $\left[\underline{\theta}, \overline{\theta}\right]$, where $\underline{\theta}= 0$ and $\overline{\theta} > 0$ and large. The θ is realised only in t_1, when each agent i observes a private signal that is noisy, $\theta_i = \theta + \epsilon_i$. The noise ϵ_i is uniformly and independently distributed among the depositors with support $[-e, +e]$. Higher the θ_i, higher is agent i's own expectation of the return on the investment. In addition to that, when an agent receives a high signal he expects the other agents to have also received higher signals and therefore to be less likely to withdraw. If the depositors believe that the bank is going to collapse, they will prefer to withdraw early and cut their losses. The inclusion of noise in the model is only to fascilitate a unique equilibrium in the global game set up, and therefore assumed to be very small, $e \to 0$.

According to the literature on global games, one of the conditions that is needed to get this unique equilibrium is that θ should have an upper dominant region and a lower dominant region. Explaining this further, it should be feasible to get an extremely good signal where the return is so high that no patient agent would want to withdraw early regardless of the behaviour of other agents. These signals are said to belong to the upper dominant region, given by $\left[\widetilde{\theta}, \overline{\theta}\right)$. There also exists a lower dominant region where agents receive an extremely bad signal and the returns are so low that they would definitely withdraw even if no other patient agent withdraws. The lower dominant region is $\left(\underline{\theta}, \underset{\sim}{\theta}\right]$.

A very small probability of an occurrence of these dominant regions is sufficient (never the less it is needed) to drive a unique equilibrium θ^*, so that the patient agent i would not withdraw in t_1 if and only if $\theta_i > \theta^*$, and would withdraw if $\theta_i < \theta^*$. If an agent receives a signal close to a dominant region, there is a probability that there would be some who have received signals within that dominant region and therefore have a dominant strategy. This will ensure that this player also to follow that strategy. This process can be iterated so that we eventually arrive at the unique threshold point where the agent will be indifferent between withdrawing or not withdrawing. The formal derivation of threshold θ^* in this model set up is in the next section.

II. TRANSPARENT RESERVE POLICY

The analysis in this section is under the assumption that the bank, depositors and regulators have the same information about the bank's solvency because the depositors can monitor the level of reserves, ρ, which is transparent.

Threshold θ^*: Probability of bank runs

The following discussion shows how the threshold value θ^* is derived. It is assumed that the dominant regions are extreme enough that they will not have an influence over θ^*. Once θ is realised, player i observes a signal $\theta_i = \theta + \varepsilon_i$. We consider threshold strategies and set out the conditions for θ^* to be a symmetric equilibrium. The strategy in t_1 for an impatient agent (who is hit by the liquidity shock) is to withdraw irrespective of his signal. The strategy for a patient player is to withdraw if $\theta_i < \theta_i^*$ and not withdraw if $\theta_i > \theta_i^*$. Symmetric threshold strategy would mean $\theta_i^* = \theta^*$, for every player i.

For now, let patient agents withdraw if they receive a signal less than $\widehat{\theta}$. Agent i's (who has received a signal θ_i) posterior distribution of the θ that is realised, which we call $y\,(= \theta/\theta_i)$, is uniform on $[\theta_i - e, \theta_i + e]$.

$$
\begin{aligned}
f(y) &= \frac{1}{2e} \; if \;\; y \in [\theta_i - e, \theta_i + e] \, . \qquad (1) \\
&= 0 \;\; if \;\; y \notin [\theta_i - e, \theta_i + e] \, .
\end{aligned}
$$

This is true for all except those points very close to the ends in the dominant regions. For each point $y \in [\theta_i - e, \theta_i + e]$, he will believe that all the other agents would have received independent and uniformly distributed signals $[y - e, y + e]$ and hence the proportion of patient agents whom he believes would withdraw (i.e. those who received a signal less than $\widehat{\theta}$) is a distribution $\widetilde{\omega}(y) \in [0, 1]$ given by:

$$
\widetilde{\omega} = \left\{ \begin{array}{ccc} 0 & \text{if} & y > \widehat{\theta} + e \\ 1 & \text{if} & y < \widehat{\theta} - e \\ \frac{\widehat{\theta} - y + e}{2e} & \text{if} & \widehat{\theta} - e \leq y \leq \widehat{\theta} + e \end{array} \right\} \qquad (2)
$$

At each point, y, his belief about the proportion who withdraw would be $\lambda + \widetilde{\omega}(y) * (1 - \lambda)$.

When the depositors receive private signals about θ in t_1, they will withdraw if their signal was less than $\widehat{\theta}$. But if any depositor observes a value more than $\widehat{\theta}$ he will withdraw only if he was impatient. Proportion λ of the depositors withdraw early irrespective of θ because of the liquidity shock. Proportion $\widetilde{\omega}$ out of the $(1 - \lambda)$ patient depositors would withdraw because of the bad signal they received.

Recall that the bank keeps a reserve $\rho \, (= \lambda r + \pi \, (1 - \lambda))$. The amount λr is to cater for the demand by the impatient agents who will definitely withdraw. In addition to that, $\pi \, (1 - \lambda)$ is kept as reserves for the patient agents who might want to withdraw early in t_1. If demand for withdrawals exceed the liquidity available, the bank would crash. In such an eventuality, those who wait till t_2 will end up getting nothing, while those who do run on the bank will receive r with probability $\frac{\rho}{\lambda r + (1 - \lambda) \widetilde{\omega}(y) r}$.

If reserves are sufficient to meet the demand, early withdrawers will receive r. The $(1 - \lambda) \, (1 - \widetilde{\omega}(y))$ agents who waited till t_2 will share the long term return given by,

$(1-\lambda)(\pi - \widetilde{\omega}(y) r) + (1-\rho) y$.[4] Expected returns to agent i who receives signal θ_i are given in Table 1.

	$\lambda r + (1-\lambda)\widetilde{\omega}(y) r \leq \rho$	$\lambda r + (1-\lambda)\widetilde{\omega}(y) r > \rho$
Withdraw early	r	r with probability $\frac{\rho}{\lambda r + (1-\lambda)\widetilde{\omega}(y) r}$
Not withdraw early	$\frac{(1-\lambda)(\pi - \widetilde{\omega}(y) r) + (1-\rho) y}{(1-\lambda)(1-\widetilde{\omega}(y))}$	0

Table 1: Expected returns under transparent policy

Therefore the difference in the expected utility of agent i who received signal θ_i between withdrawing and not withdrawing is given by $g(\theta_i(r, \pi, \lambda))$.[5]

$$g(\theta_i(r, \pi, \lambda)) = \frac{1}{2e} \left(\begin{array}{c} \int_{\theta_i - e}^{\widehat{\theta} + e - \frac{2e\pi}{r}} \frac{\rho}{\lambda r + (1-\lambda)\widetilde{\omega}(y) r} u(r)\, dy + \int_{\widehat{\theta} + e - \frac{2e\pi}{r}}^{\theta_i + e} u(r)\, dy \\ - \int_{\widehat{\theta} + e - \frac{2e\pi}{r}}^{\theta_i + e} u\left(\frac{(1-\lambda)(\pi - \widetilde{\omega}(y) r) + (1-\rho) y}{(1-\lambda)(1-\widetilde{\omega}(y))} \right) dy \end{array} \right). \tag{3}$$

The threshold point $\theta_i = \theta^*$ is such that $g(\theta^*(r, \pi, \lambda)) = 0$ where player i is indifferent between withdrawing and not. Since all agents are identical, all patient players would withdraw if they received a signal below θ^*, and will not withdraw if they received a signal above θ^*. Proposition 1 states that such an equilibrium exists, which is proved in Appendix A1.

Proposition 1. *There exists a unique equilibrium θ^* such that agents who are not hit by a liquidity shock would not withdraw in t_1 if they received a signal above θ^*, but would withdraw in t_1 if they received a signal below θ^*.*

[4] Return from the long term investment at each point y is $(1-\rho) y$. In addition, there will be the unused reserves, $\rho - (\lambda r + (1-\lambda) * \widetilde{\omega}(y) * r)$.

[5] There is sufficient reserves to meet early demand when $\lambda r + (1-\lambda)\widetilde{\omega} r < \rho \Longrightarrow \lambda r + (1-\lambda)\left(\frac{\widehat{\theta} - y + e}{2e}\right) r < \lambda r + (1-\lambda)\pi r \Longrightarrow y > \widehat{\theta} + e - \frac{2e\pi}{r}$.

Probability of Bank Runs

It is useful to discuss how λ, r and π affect the probability of bank runs, which is given in Proposition 2, the proof of which is found in Appendix A2.

Proposition 2. *The probability of bank runs increases with early return, r and proportion of impatient depositors, λ; while it decreases with excess reserves, π.*

Having higher early return provides an incentive for agents to withdraw early. So, if we want to minimise the probability of bank runs (i.e. minimise θ^*), those who withdraw early, including those who are genuinely hit by the liquidity shock should be penalised. Furthermore, depositors realise that more the early return, more the probability that the reserves might not be sufficient to meet early demand. Therefore θ^* increases with r, which increases the probability of bank runs.

Higher the probability of liquidity shock, higher reserves need to be kept, reducing the long-term return. Furthermore, the depositors will want higher insurance in the form of higher r. All these factors encourage the patient agents also to withdraw early. This is the intuition for why threshold θ^* increases with the proportion of impatient investors, λ.

When excess reserves π increases, the agents will have more confidence that the bank will survive till t_2 because of excess reserves, and will be willing to wait. Therefore θ^* will decrease with π, which means the probability of bank runs decreases.

Effects of Banking Competition and Transparent Reserve Policy

In this sub-section we analyse the effects of banking competition on the choice variables, reserves and early returns. When the level of reserves and early returns of the bank are transparent in a competitive environment, the bank will choose them to maximise the expected utility of the agent subject to zero profit, taking into account the possibility that the bank can collapse if there are too much withdrawals.

Recall that at the time of choosing ρ and r in t_0, the θ is unknown to the bank and expects it to be any point between $\left[\underline{\theta}, \overline{\theta}\right]$. The λ proportion of agents who are impatient would always withdraw early. If $\underline{\theta} < \theta < \theta^* - e$ there would be a total run on the bank because everyone would receive a signal that is below the threshold value. If $\theta^* + e < \theta < \overline{\theta}$ no patient agent will withdraw early because all the agents would receive signals that are above θ^*. But when $\theta^* - e < \theta < \theta^* + e$ there would be a partial run. However, the probability of partial run is very small because noise is assumed to be very small with $e \to 0$. Therefore the bank can simplify its expectations that, if $\theta < \theta^*$ everybody will withdraw early, whose demand have to be met with the available reserves ρ. On the other hand, if $\theta > \theta^*$, only those who are impatient will withdraw early. In this no bank run case, the player has λ probability of being hit by the liquidity shock and receive r, while he has $(1 - \lambda)$ probability of not being hit by the liquidity shock and receive $R(\theta)\left(= \frac{\rho - \lambda r + (1-\rho)\theta}{1-\lambda}\right)$. Therefore the expected utility of the agents, EU, at the beginning of t_0 can be written as follows:

$$EU = \left(\frac{1}{\overline{\theta} - \underline{\theta}}\right)\left(\int_{\underline{\theta}}^{\theta^*} \frac{\rho}{r} u(r)\, d\theta + \int_{\theta^*}^{\overline{\theta}} \left[\lambda u(r) + (1-\lambda) u(R(\theta))\right] d\theta\right). \quad (4)$$

The optimal reserve level and early return will be decided by maximising EU given by equation (4). First, we will look at some interesting results about the optimal early return which are summarised in Proposition 3. To maximise EU, early return should be less than the deposit itself (i.e. $r^* < 1$). Institutional restraints compelling banks to give early returns $r \geq 1$ is a way of ensuring better insurance when there is a probability of being hit by liquidity shocks. However, this has the repercussion of increasing the probability of bank runs because it will be an incentive for those who are not hit by the liquidity shock to also withdraw early. To reduce this eventuality, even those who are genuinely impatient have to pay a price by being given a lower early return.

As the benchmark, we look at the first-best scenario where the bank knows exactly who is hit by the liquidity shock so that the bank will be able to offer a contract which promises early return in t_1 only to the depositors who are hit by the liquidity shock. In this case, there will be no bank run because the patient agents will not receive anything early. So the bank does not have to use the early return as an instrument to dissuade patient agents from withdrawing early, because they cannot withdraw early anyway. In the first-best scenario the bank will keep reserves $\rho = \lambda r^*_{FB}$ which is just sufficient to meet the demand from the impatient agents. The first-best early return, denoted by r^*_{FB}, is greater than one. Optimal early return being less than under the first-best scenario (i.e. $r^* < r^*_{FB}$), is a similar result as shown in the Goldstein and Pauzner model. The proof of Proposition 3 is presented in Appendix A3.

Proposition 3. *(1) The optimal early return is less than the deposit itself,* $0 < r^* < 1$.

(2) The first-best early return, r^*_{FB}, is higher than the optimal early return r^*.

Next, we look at some results about the optimal level of reserves, which are summarised in Proposition 4, proof of which is found in Appendix A4. Recall that the amount $\pi(1-\lambda)$ is put aside to cater to the demand of patient agents who might withdraw early because of self fulfilling beliefs. It can be shown that $\pi^* > 0$, so that the optimal level of reserves $\rho^* > \lambda r$. This means the bank is keeping as reserves more than what is needed to meet the demand of those who are genuinely hit by the liquidity shock. Keeping excess reserves reduces the amount that can be invested in profitable projects.

Proposition 3 goes on to say that excess reserves, captured by π, increases with the proportion of impatient agents λ. The inefficiency of keeping excess reserves is caused by the existence of impatient agents. Higher the λ, more the reserves that should be put aside for the patient agents. This not only reduces the opportunity cost of withdrawing early to the patient agents but also increases their risk if they do not withdraw early. Therefore higher the λ, higher the probability that patient agents will panic and withdraw early, and hence more reserves should be put aside to meet the likely early demand of patient depositors.

Proposition 4. *(1) Optimal level of reserves is over and above what is needed to meet the demand of impatient agents, when $\lambda > 0$, $\pi^* > 0$.*

(2) This inefficiency of excessive reserves, denoted by π^*, increases with λ, $\frac{d\pi^*}{d\lambda} > 0$.

This analysis is based on the fact that the level of reserves could be observed by the depositors. In this scenario there is no need for regulation on the level of reserves because the banks in a competitive environment would choose reserves and early returns to maximise expected utility of the agents.

III. NON-TRANSPARENT RESERVE POLICY

As pointed out in the Introduction, there is support for some forms of banking regulation among academics and authorities. We observe that in many countries reserves of banks are indeed regulated, and have been encouraged to do so. This section argues that there is indeed reason to regulate banks on reserves and early returns by introducing another dimension to our analysis - what happens if the *bank reserves is private information* to the bank and cannot be observed by the depositors. Information on reserves may not be freely available, costly to obtain and the depositors may not be sophisticated enough to make use of the information. The basic model remains the same as described in Section I. Now that the bank has private information about an important component of its solvency, it gives rise to moral hazard.

In the basic model, all the remaining returns in t_2 were distributed to the agents and the objective was to maximise the expected utility of the agents. However, now that the bank has private information, only a proportion δ of the profits is distributed equally among the agents who did not withdraw early, *maximising the bank's profit.* For any θ, the profit Δ for the bank is given as follows:[6]

$$\Delta = \frac{(1-\rho)\,\theta + (1-\lambda)\,(\pi - \omega r)}{(1+\delta)}. \tag{5}$$

The difference in the expected utility of agent i who received signal θ_i between withdrawing

[6]

$$\Delta = \rho - \lambda r - (1-\lambda)\,\omega r + (1-\rho)\,\theta - \frac{(1-\lambda)\,(1-\omega)}{(1-\lambda)\,(1-\omega)}\delta\Delta.$$

and not withdrawing in t_1, when reserves cannot be observed is given by $g^n\left(\widehat{\theta}(\pi,\lambda)\right)$ in equation (6). Note that only the last term is different from equation (3).

$$g^n\left(\widehat{\theta}(\pi,\lambda)\right) = \frac{1}{2e} * \left(\begin{array}{c} \int_{\theta_i-e}^{\widehat{\theta}+e-\frac{2e\pi}{r}} \frac{\rho}{\lambda r+(1-\lambda)\widetilde{\omega}(y)r} u(r)\,dy + \int_{\widehat{\theta}+e-\frac{2e\pi}{r}}^{\theta_i+e} u(r)\,dy \\ -\int_{\widehat{\theta}+e-\frac{2e\pi}{r}}^{\theta_i+e} \frac{1}{2e} * u\left(\frac{\delta\Delta}{(1-\lambda)(1-\widetilde{\omega}(y))}\right) dy \end{array} \right). \tag{6}$$

When reserves cannot be monitored by the depositors, the threshold value is such that $g^n(\theta^{n*}(\pi,\lambda)) = 0$. This can be proved in the same way as Proposition 1. The profit to the bank is $\Delta = 0$ if $\theta < \theta^*$ because the bank fails due to a bank run, and $\Delta = \frac{(1-\lambda)\pi+(1-\rho)\theta}{1+\delta}$ if $\theta > \theta^*$. Therefore the ex ante expected profit, $E\Delta$, for the bank is given by,

$$E\Delta = \int_{\theta^*}^{\overline{\theta}} \frac{1}{\overline{\theta}-\underline{\theta}} \left(\frac{(1-\lambda)\pi + (1-\rho)\theta}{1+\delta}\right) d\theta. \tag{7}$$

Since the level of reserves is private information to the bank, it can exploit this information asymmetry and choose reserves, ρ^{n*}, to *maximise its own profits*, $E\Delta$.

Proposition 5. *When the level of reserves cannot be observed, the bank will choose optimal reserves, $\rho^{n*} = \lambda r$, which is just sufficient to meet the early demand of the impatient depositors: $\rho^{n*} > \rho^*$.*

Proof. The proof is very simple. Rearranging equation (7),

$$E\Delta = \int_{\theta^{n*}}^{\overline{\theta}} \frac{1}{\overline{\theta}-\underline{\theta}} \left(\frac{\theta - \lambda r\theta - \pi(1-\lambda)(\theta-1)}{1+\delta}\right) d\theta. \tag{8}$$

Because π cannot be observed by the agents, θ^{n*} will not be affected by π.

$$\frac{dE\Delta}{d\pi} = -\int_{\theta n^*}^{\overline{\theta}} \frac{(1-\lambda)(\theta-1)}{\left(\overline{\theta}-\underline{\theta}\right)(1+\delta)} d\theta < 0. \tag{9}$$

Therefore with private information about reserves, the bank will choose $\pi^{n*} = 0$ to maximise profit - i.e. $\rho^{n*} = \lambda r$. □

When the agents cannot observe the reserves that the bank keeps, it cannot increase the confidence of agents through reserves. Therefore it will keep reserves as low as possible. However, it will have to keep reserves to meet the demand of those hit by the liquidity shock who would definitely withdraw. Therefore, the bank will keep as reserves, just sufficient to meet the demand of those who are hit by the liquidity shock- i.e. $\rho^{n*} = \lambda r$. This is opposed to what happened when reserves could be observed, where the bank would put aside $\rho^* > \lambda r$, which is over and above what is needed to meet the early demand of impatient depositors.

Next, we look at the choice of early return. The bank is set in a competitive environment and since early return is observable, r has to be chosen to maximise expected utility of depositors, given $\pi = 0$. The expected utility of depositors when reserve policy is not observable is given by EU^n.

$$EU^n = \frac{\int_{\underline{\theta}}^{\theta^{n*}} \rho u\left(r\right) d\theta + \int_{\theta^{n*}}^{\overline{\theta}} \left[\lambda u\left(r\right) + \left(1-\lambda\right) u\left(\frac{\delta\Delta}{(1-\lambda)}\right)\right] d\theta}{\overline{\theta} - \underline{\theta}}. \tag{10}$$

The early return that is chosen when the reserve level is not transparent is denoted by r^{n*}.

Proposition 6. *Early return when reserves cannot be observed, is higher than the early return when the level of reserves is transparent. i.e.* $r^{n*} > r^*$.

Proof. The first-order condition for r^{n*} is used so that,

$$\left.\frac{dEU^n}{dr}\right|_{\pi=0,r=1} = \frac{1}{\overline{\theta}-\underline{\theta}} \left(\begin{array}{c} \int_{\underline{\theta}}^{\theta^{n*}} 2\lambda d\theta - \left(1-\lambda\right) \frac{d\theta^{n*}}{dr} u\left(\frac{\delta\theta^{n*}}{(1+\delta)}\right) \\ +\lambda\alpha \int_{\theta^{n*}}^{\overline{\theta}} \left(1 - \left(\frac{\delta\theta}{(1+\delta)}\right)^{\alpha}\right) d\theta \end{array} \right). \tag{11}$$

The second term is the late return at one point, θ^{n*} whereas the first term is over the whole range $(\underline{\theta}, \theta^{n*})$. When θ^{n*} is large, $\frac{d\theta^{n*}}{dr}$ will be small (threshold θ^{n*} cannot increase more than upper dominant region which makes $\frac{d\theta^{n*}}{dr}$ negligible). $\theta^{n*}/_{\pi=0}$ is close to the upper dominant region. Therefore the last term is negligible. The first term is significant.

Therefore $\frac{dEU^n}{dr}\Big|_{\pi=0,r=1} > 0$, which means $r^{n*} > 1$.

We have shown in Proposition 3 that when there is transparency $r^* < 1$. This means that when the bank has private information about its reserves, it chooses a higher early return - i.e. $r^{n*} > r^*$. □

The bank chooses an early return that is high in order to attract customers because it operates in a competitive environment. It is also noteworthy to recall that the probability of bank runs increases with early returns. Therefore, when reserves cannot be observed, the banks would choose an early return that increases the probability of bank runs, compared to when the reserve level is transparent. Therefore when the reserve level cannot be observed, the regulators have another role to play, which is to control the early return that the banks offer and fix an upper bound to it.

In a non-transparent environment, the depositors will be aware of this tight reserve level. Bank runs can be more easily triggered if patient depositors have reason to believe that other patient depositirs might withdraw. Because this makes the banks prone to bank runs, regulators have a role to play.

Regulators can stipulate reserve requirements that takes into consideration the welfare of the agents and make the agents aware that they are thus regulating. In countries where agents are able to comprehend such information, the regulators can make it a statutory requirement to make the level of reserves transparent.

IV. BANK HAS ACCESS TO EXTERNAL FUNDS

To complete this analysis, this section looks at what happens if the bank has access to external funds to meet the early demand from the depositors when it runs out of reserves. It is assumed that such lenders have perfect information about θ (Noisier the information about θ the worse the outcome - i.e. more reluctant the regulators will be to help the bank). It is reasonable to assume that the lender will have better information than the depositors. Those who lend funds or take-over the bank will obviously research into the bank's financial solvency thoroughly.

The model remains the same as in Section II, which is the transparent scenario. Let a patient agent receives a signal θ_i and $y = \theta/\theta_i$. The reserves are sufficient to meet the early demand if $y \geq \widehat{\theta} + e - \frac{2e\pi}{r}$, so that the returns are the same as before. However if the level of reserves is not sufficient, now the bank will be able to borrow from a lender for a return, $l\,(> 1)$, so long as the lender is satisfied that the bank can be expected to have sufficient returns in t_2 to repay the loan. This condition is given by the inequality (12) which needs to be satisfied for the bank to be able to secure external funds.

$$(\lambda r + (1-\lambda)\,\widetilde{\omega} r - \rho)\, l < (1-\rho)\, y. \tag{12}$$

Therefore, when $\widehat{\theta} + e - \frac{2e\pi}{r} > y > \frac{\left(\left(\widehat{\theta}+e\right)r - 2e\pi\right)(1-\lambda)l}{2e(1-\rho)+(1-\lambda)l}$, the bank will be able to survive by borrowing from a lender. Within this range the depositors who withdraw early will receive r while those who wait will receive $\frac{(1-\rho)y-(1-\lambda)(\widetilde{\omega}(y)r-\pi)l}{(1-\lambda)(1-\widetilde{\omega}(y))}$. Table 2 presents the expected returns.

	Withdraw in t_1	Not withdraw in t_1
$\theta_i + e \geq y > \widehat{\theta} + e - \frac{2e\pi}{r}$	r	$\frac{(1-\lambda)(\pi-\widetilde{\omega}(y)r)+(1-\rho)y}{(1-\lambda)(1-\widetilde{\omega}(y))}$
$\widehat{\theta} + e - \frac{2e\pi}{r} \geq y \geq \frac{\left(\left(\widehat{\theta}+e\right)r-2e\pi\right)(1-\lambda)l}{2e(1-\rho)+(1-\lambda)l}$	r	$\frac{(1-\rho)y-(1-\lambda)(\widetilde{\omega}(y)r-\pi)l}{(1-\lambda)(1-\widetilde{\omega}(y))}$
$\frac{\left(\left(\widehat{\theta}+e\right)r-2e\pi\right)(1-\lambda)l}{2e(1-\rho)+(1-\lambda)l} > y \geq \theta_i - e$	r with probability $\frac{\rho}{\lambda r+(1-\lambda)\widetilde{\omega}(y)r}$	0

Table 2: Expected returns with access to external funds

So the difference in the expected utility of agent i who receives signal θ_i between withdrawing and not withdrawing when the bank can borrow is given by $g^l\left(\theta_i\left(r,\pi,\lambda\right)\right)$.

$$g^l\left(\theta_i\left(r,\pi,\lambda\right)\right) = \frac{1}{2e} * \left(\begin{array}{c} \int_{\theta_i-e}^{\frac{\left(\left(\widehat{\theta}+e\right)r-2e\pi\right)(1-\lambda)l}{2e(1-\rho)+(1-\lambda)l}} \frac{\rho}{\lambda r+(1-\lambda)\widetilde{\omega}(y)r} u\left(r\right) dy \\ + \int_{\frac{\left(\left(\widehat{\theta}+e\right)r-2e\pi\right)(1-\lambda)l}{2e(1-\rho)+(1-\lambda)l}}^{\theta_i+e} u\left(r\right) dy \\ - \int_{\frac{\left(\left(\widehat{\theta}+e\right)r-2e\pi\right)(1-\lambda)l}{2e(1-\rho)+(1-\lambda)l}}^{\widehat{\theta}+e-\frac{2e\pi}{r}} u\left(\frac{(1-\rho)y-(1-\lambda)(\widetilde{\omega}(y)r-\pi)l}{(1-\lambda)(1-\widetilde{\omega})}\right) dy \\ - \int_{\widehat{\theta}+e-\frac{2e\pi}{r}}^{\theta_i+e} u\left(\frac{(1-\lambda)(\pi-\widetilde{\omega}(y)r)+(1-\rho)y}{(1-\lambda)(1-\widetilde{\omega}(y))}\right) dy \end{array} \right). \tag{13}$$

The equilibrium point $\theta_i = \theta^{l*}$ is such that $g^l\left(\theta^{l*}(r, \pi, \lambda)\right) = 0$ when depositor i is indifferent between withdrawing and not withdrawing. Proposition 7 says what happens to the probability of bank-runs and reserves when the bank has access to external funds compared to when it did not. The proof is in Appendix A5.

Proposition 7. *If the bank has access to external funds, compared to when it did not, the probability of bank run is lower and the inefficiency of excess reserves,* π*, is lower.*

When the depositors are aware that there is a possibility of receiving a return even when the reserve level is not sufficient to meet the early demand, the probability of bank runs become lower. This is because the depositors are less likely to panic that they will lose their deposits if the bank collapses. This is true, even though the bank would now hold lower reserves, by reducing the excess reserves. For one thing, this increases the expected return in t_2, while giving confidence to the depositors that even if the bank runs out of reserves, they don't necessarily lose out. The more access to such safety nets the bank has, the more confidence the depositors will have, and the less probability there is of bank runs. This does not mean the banks can be irresponsibility in their solvency decisions. In order to receive financial assistance in times of need, the bank has to prove its ability to repay the loan as indicated by inequality (12).

V. CONCLUSION

A model of bank runs in a global game framework was used to study how a bank chooses reserves and early return. The analysis was done under two scenarios, namely, when the level of reserves could be observed by the depositors, and when it could not be. Under the first scenario, it is found that compared to the first-best case (when the bank and identify and only meets the demand of impatient agents), the optimal early return is lower. The level of reserves is more than what is needed to cater to the impatient agents, and this inefficiency increases with the proportion of impatient agents. Policy makers should take note that if banks are operating in a competitive environment where the agents can observe reserves, the bank would voluntarily choose reserves and early return to maximise the expected utility of depositors. Therefore there is no need for regulation of these. However it should be ensured that reserves are made known and there is a competitive environment. However in the second scenario where the bank has private information about the level of reserves, the model recommends there should be regulation of reserves and early returns. It was found that when the level of reserves cannot be observed, the bank chooses a level of reserves that is lower and early return that is higher. This not only lowers the welfare of the depositors, but also creates financial instability by increasing the probability of bank runs. Therefore when the level of reserves cannot be observed, the regulators do have a role to play by fixing a minimum reserve requirement and a maximum limit on early return which maximise the welfare objective.

Appendix

A1. Proof of Proposition 1 - existence of threshold θ^*

Recall equation (3) which gives the difference in the expected utility of agent i who received signal θ_i between withdrawing and not withdrawing:

$$\begin{aligned} g\left(\theta_i\left(r,\pi,\lambda\right)\right) &= \int_{\theta_i-e}^{\widehat{\theta}+e-\frac{2e\pi}{r}} \frac{1}{2e} * \frac{\rho}{\lambda r+\left(1-\lambda\right)\widetilde{\omega}\left(y\right)r} u\left(r\right)dy \\ &+ \int_{\widehat{\theta}+e-\frac{2e\pi}{r}}^{\theta_i+e} \frac{1}{2e} u\left(r\right)dy + \\ &- \int_{\widehat{\theta}+e-\frac{2e\pi}{r}}^{\theta_i+e} \frac{1}{2e} * u\left(\frac{\rho-\lambda r-\left(1-\lambda\right)*\widetilde{\omega}\left(y\right)*r+\left(1-\rho\right)y}{\left(1-\lambda\right)\left(1-\widetilde{\omega}\left(y\right)\right)}\right)dy. \end{aligned}$$

$g : R \rightarrow R$

Figure 1 is a representation of what is being proved below.

Because of the lower dominant region, $\underset{\theta_i \rightarrow \underline{\theta}}{Lim} g\left(\theta_i\right) > 0$ so that if an agent receives a signal $\underline{\theta}$, he will withdraw. On the other hand, because of the upper dominant region, $\underset{\theta_i \rightarrow \overline{\theta}}{Lim}\, g\left(\theta_i\right) < 0$ so that an agent who receives a signal $\overline{\theta}$ will not withdraw. This is because the expected return from not withdrawing is very high.

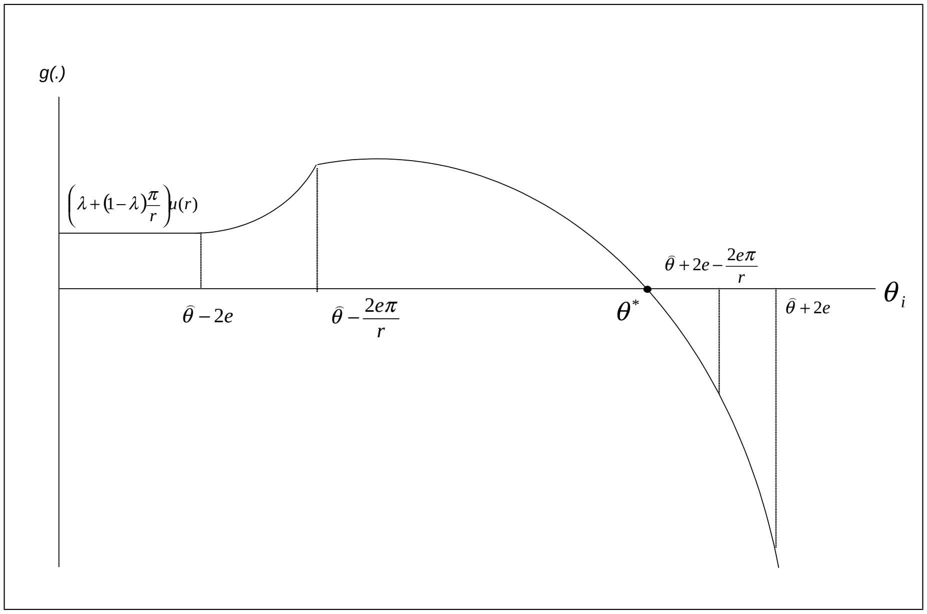

FIGURE 1: Threshold θ^*

Therefore, as long as the function is continuous a value exists for θ_i such that $g\left(\theta^*, \widehat{\theta}\right) = 0$. Because of symmetry, all agents are indifferent between withdrawing early and not doing so when they receive a signal θ^*.

The expected utility of withdrawing early is continuous. When $\theta_i \leq \widehat{\theta} - 2e$, $\widetilde{\omega}(y) = 1 \; \forall \; y$. This means we have a constant expected utility, $\left(\lambda + (1-\lambda)\frac{\pi}{r}\right) u(r)$. When $\theta_i \geq \widehat{\theta} + 2e$, $\widetilde{\omega}(y) = 0 \; \forall \; y$ giving a constant expected utility of $u(r)$. In between the expected utility continuously increases.

The expected utility of not withdrawing early is zero for $\theta_i \leq \widehat{\theta} - \frac{2e\pi}{r}$. After that, it increases continuously. Therefore the function $g(.)$ is continuous in θ_i.

Now we show that the point where $g\left(\theta^*, \widehat{\theta}\right) = 0$ is unique.

For the range $\theta_i \leq \widehat{\theta} - 2e$, $\widetilde{\omega}(y) = 1 \; \forall \; y$. The expected utility of not running is zero. This means we have a constant $g(.)|_{\left(\theta_i \leq \widehat{\theta} - 2e\right)} = \frac{\rho}{\lambda r + (1-\lambda) r} u(r) > 0$.

For the range $\widehat{\theta} - 2e \leq \theta_i \leq \widehat{\theta} - \frac{2e\pi}{r}$, $\widetilde{\omega}$ is high enough that you receive nothing by waiting.

$$g(.)|_{\left(\widehat{\theta}-2e\leq\theta_i\leq\widehat{\theta}-\frac{2e\pi}{r}\right)} = \int_{\theta_i-e}^{\widehat{\theta}-e}\frac{1}{2e}*\frac{\rho}{\lambda r+(1-\lambda)r}u(r)\,dy + + \int_{\widehat{\theta}-e}^{\theta_i+e}\frac{1}{2e}*\frac{\rho}{\lambda r+(1-\lambda)\widetilde{\omega}(y)r}u(r)\,dy. \tag{14}$$

It is clear that $g(.)|_{\left(\widehat{\theta}-2e\leq\theta_i\leq\widehat{\theta}-\frac{2e\pi}{r}\right)} > 0$.

Furthermore in this range, as θ_i increases, the expected proportion of those wanting to withdraw goes down. i.e. the expected utility of running continuously increases.

$$\frac{dg(.)}{d\theta_i}/_{\left(\widehat{\theta}-2e\leq\theta_i\leq\widehat{\theta}-\frac{2e\pi}{r}\right)} = \frac{1}{2e}\left(\frac{\rho}{\lambda r+(1-\lambda)\left(\frac{\widehat{\theta}-\theta_i+2e}{2e}\right)r}-\frac{\rho}{\lambda r+(1-\lambda)r}\right)u(r)\,. \tag{15}$$

We know that $\frac{\widehat{\theta}-\theta_i+2e}{2e} < 1$.Therefore $\left.\frac{dg(.)}{d\theta_i}\right|_{\left(\widehat{\theta}-2e\leq\theta_i\leq\widehat{\theta}-\frac{2e\pi}{r}\right)} > 0$.

For the range, $\widehat{\theta}+2e-\frac{2e\pi}{r}\leq\theta_i\leq\widehat{\theta}+2e$, $\widetilde{\omega}(y)\leq\frac{\pi}{r}.\forall\, y$. Therefore the bank is expected not to crash $\forall\, y$. Those who withdraw early will receive r for sure. By waiting the agent i's expected utility is $\int_{\theta_i-e}^{\theta_i+e}\frac{1}{2e}\left(u\left(\pi+\frac{(1-\rho)y}{(1-\lambda)(1-\widetilde{\omega}(y))}\right)\right)dy$.

$$g(.)|_{\left(\widehat{\theta}+2e-\frac{2e\pi}{r}\leq\theta_i\leq\widehat{\theta}+2e\right)} = u(r)-\int_{\theta_i-e}^{\theta_i+e}\frac{1}{2e}\left(u\left(\pi+\frac{(1-\rho)y}{(1-\lambda)(1-\widetilde{\omega}(y))}\right)\right)dy. \tag{16}$$

In this range bank will not crash. i.e. r is low enough to make the reserves sufficient to cater to early demand. If early withdrawal is better than not withdrawing, everyone will definitely withdraw prompting the bank to crash. The fact that the bank does not crash in the range tells us that when $\widehat{\theta}+2e-\frac{2e\pi}{r}\leq\theta_i\leq\widehat{\theta}+2e$, we have $u(r)\leq\int_{\theta_i-e}^{\theta_i+e}\frac{1}{2e}\left(u\left(\frac{(1-\lambda)(\pi-\omega)+(1-\rho)y}{(1-\lambda)(1-\omega(y))}\right)\right)dy$.

Therefore $g(.)/_{\left(\widehat{\theta}+2e-\frac{2e\pi}{r}\leq\theta_i\leq\widehat{\theta}+2e\right)} < 0$.

Also note that in this range, when θ_i increases, the expected utility of running does not change, but remains at $u(r)$, but the expected utility of waiting increases.

$$\frac{dg(.)}{d\theta_i}/_{\left(\widehat{\theta}+2e-\frac{2e\pi}{r}\leq\theta_i\leq\widehat{\theta}+2e\right)} = -\frac{1}{2e}\left(\begin{array}{c} u\left(\pi+\frac{(1-\rho)(\theta_i+e)}{(1-\lambda)\left(1-\frac{\widehat{\theta}-\theta_i}{2e}\right)}\right) \\ -u\left(\pi+\frac{(1-\rho)(\theta_i-e)}{(1-\lambda)\left(1-\frac{\widehat{\theta}-\theta_i+2e}{2e}\right)}\right)\end{array}\right). \tag{17}$$

It is obvious that the utility when $y=\theta_i+e$ is greater than the utility when $y=\theta_i-e$.

Therefore $\left.\frac{dg(.)}{d\theta_i}\right|_{\left(\widehat{\theta}+2e-\frac{2e\pi}{r}\leq\theta_i\leq\widehat{\theta}+2e\right)} < 0$.

For the range, $\theta_i \geq \widehat{\theta}+2e$, $\widetilde{\omega}(y)=0 \ \forall\ y$. This means,

$$g(.)|_{\left(\theta_i\geq\widehat{\theta}+2e\right)} = u(r) - \int_{\theta_i-e}^{\theta_i+e}\frac{1}{2e}\left(u\left(\pi+\frac{(1-\rho)\,y}{1-\lambda}\right)\right)dy. \tag{18}$$

$\int_{\theta_i-e}^{\theta_i+e}\frac{1}{2e}u\left(\pi+\frac{(1-\rho)y}{1-\lambda}\right)dy > u(r)$ when $\theta_i \geq \widehat{\theta}+2e$. Therefore $g(.)/_{\theta_i>\widehat{\theta}+2e} < 0$.

Furthermore

$$\left.\frac{dg(.)}{d\theta_i}\right|_{\theta_i\geq\widehat{\theta}+2e} = -u(\pi+\frac{(1-\rho)(\theta_i+e)}{1-\lambda})+u(\pi+\frac{(1-\rho)(\theta_i-e)}{1-\lambda}). \tag{19}$$

$\left.\frac{dg(.)}{d\theta_i}\right|_{\theta_i\geq\widehat{\theta}+2e} < 0$ because $u(\pi+\frac{(1-\rho)(\theta_i+e)}{1-\lambda}) > u(\pi+\frac{(1-\rho)(\theta_i-e)}{1-\lambda})$.

We can now conclude that, if there is a point at which $g(.)=0$, it will be in the range $\widehat{\theta}-\frac{2e\pi}{r}\leq\theta_i\leq\widehat{\theta}+2e-\frac{2e\pi}{r}$.

Now we have to show there is only one such point.

This is so if in this range the function $g(.)$ is concave. In this range the function $g(.)$ can be split into two parts: $\widehat{\theta}-\frac{2e\pi}{r}\leq\theta_i\leq\widehat{\theta}$ and $\widehat{\theta}\leq\theta_i\leq\widehat{\theta}+2e-\frac{2e\pi}{r}$.

In the range $\widehat{\theta}-\frac{2e\pi}{r}\leq\theta_i\leq\widehat{\theta}$,

$$
\begin{aligned}
g(.)|_{\left(\widehat{\theta}-\frac{2e\pi}{r}\leq\theta_i\leq\widehat{\theta}\right)} &= \int_{\theta_i-e}^{\widehat{\theta}-e}\frac{1}{2e}*\left(\lambda+(1-\lambda)\frac{\pi}{r}\right)u(r)\,dy+ \\
&+\int_{\widehat{\theta}-e}^{\widehat{\theta}-\frac{2e\pi}{r}}\frac{1}{2e}*\frac{\rho}{\lambda*r+(1-\lambda)*\widetilde{\omega}(y)*r}u(r)\,dy \\
&+\int_{\widehat{\theta}-\frac{2e\pi}{r}}^{\theta_i+e}\frac{1}{2e}u(r)\,dy \\
&-\int_{\widehat{\theta}-\frac{2e\pi}{r}}^{\theta_i+e}\frac{1}{2e}*u\left(\pi+\frac{(1-\lambda)(\pi-\omega)+(1-\rho)y}{(1-\lambda)(1-\widetilde{\omega})}\right)dy.
\end{aligned}
\tag{20}
$$

$$
\left.\frac{dg}{d\theta_i}\right|_{\widehat{\theta}-\frac{2e\pi}{r}\leq\theta_i\leq\widehat{\theta}}=\frac{1}{2e}\left(\begin{array}{c} u(r)(1-\lambda)\left(1-\frac{\pi}{r}\right) \\ -u\left(\pi+\frac{(1-\lambda)\left(\pi-\frac{\widehat{\theta}-\theta_i}{2e}r\right)+(1-\rho)(\theta_i+e)}{(1-\lambda)\left(1-\frac{\widehat{\theta}-\theta_i}{2e}\right)}\right)\end{array}\right). \tag{21}
$$

$$
\frac{d^2g}{d\theta_i^2}/_{\widehat{\theta}-\frac{2e\pi}{r}\leq\theta_i\leq\widehat{\theta}}=-\frac{1}{2e}u'(.)\,\frac{(1-\lambda)\left(\frac{r-\pi}{2e}\right)+(1-\rho)}{(1-\lambda)\left(\frac{1-\widehat{\theta}-\theta_i}{2e}\right)^2}<0. \tag{22}
$$

Therefore in the range $\widehat{\theta}-\frac{2e\pi}{r}\leq\theta_i\leq\widehat{\theta}$, $g(.)$ is concave.

In the range $\widehat{\theta}\leq\theta_i\leq\widehat{\theta}+2e-\frac{2e\pi}{r}$,

$$
\begin{aligned}
g(.)|_{\left(\widehat{\theta}\leq\theta_i\leq\widehat{\theta}+2e-\frac{2e\pi}{r}\right)} &= \int_{\theta_i-e}^{\widehat{\theta}+e-\frac{2e\pi}{r}}\frac{1}{2e}*\frac{\rho}{\lambda r+(1-\lambda)\widetilde{\omega}r}u(r)\,dy \\
&+\int_{\widehat{\theta}+e-\frac{2e\pi}{r}}^{\theta_i+e}\frac{1}{2e}*u(r)\,dy \\
&-\int_{\widehat{\theta}+e-\frac{2e\pi}{r}}^{\theta_i+e}\frac{1}{2e}*u\left(\pi+\frac{(1-\lambda)(\pi-\widetilde{\omega})+(1-\rho)y}{(1-\lambda)(1-\widetilde{\omega})}\right)dy.
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
\left.\frac{dg}{d\theta_i}\right|_{\left(\widehat{\theta}\leq\theta_i\leq\widehat{\theta}+2e-\frac{2e\pi}{r}\right)} &= \frac{1}{2e}\left(\frac{(1-\lambda)\left(\frac{\widehat{\theta}-\theta_i}{2e}r+r-\pi\right)}{\lambda r+(1-\lambda)\left(\frac{\widehat{\theta}-\theta_i}{2e}+1\right)r}\right)u(r) \\
&-\frac{1}{2e}u\left(\pi+\frac{(1-\lambda)\left(\pi-\frac{\widehat{\theta}-\theta_i}{2e}r\right)+(1-\rho)(\theta_i+e)}{(1-\lambda)\left(1-\frac{\widehat{\theta}-\theta_i}{2e}\right)}\right).
\end{aligned}
\tag{24}
$$

In this range, as θ_i increases, the expected returns from running increases with θ_i which is given by the first term.. This is because now the range over which you surely receive r increases and the probability of receiving r increases. Let this term be A. This increases at a decreasing rate. The second term refers to the expected returns from not running which increases with θ_i. Let this be B. This increase at an increasing rate.

$$\left.\frac{d^2g}{d\theta_i^2}\right|_{\left(\widehat{\theta}\leq\theta_i\leq\widehat{\theta}+2e-\frac{2e\pi}{r}\right)} = \frac{dA}{d\theta_i} - \frac{dB}{d\theta_i} < 0. \tag{25}$$

Therefore we can conclude that there exists a unique θ^* such that above which, $g < 0$ and below which, $g > 0$ and thereby a patient agent would not withdraw if $\theta_i > \theta^*$ and would withdraw if $\theta_i < \theta^*$. ■

A2. Proof of Proposition 2 - effects on the probability of bank runs

Recall equation (3).

The first two terms in $g(.)$ go down with θ^* because, as θ^* increases more people withdraw early, and the expected return by withdrawing early is less. The expected return by withdrawing late increases, so long as the bank survives. therefore on the whole $\frac{\partial g}{\partial\theta^*} < 0$.

$$\frac{\partial g}{\partial\theta^*} = \frac{1}{2e}\left(\begin{array}{c} -\left(\frac{(1-\lambda)\pi}{\lambda r}\right)u\left(r\right) - \rho u\left(r\right)\left(1-\lambda\right)\int_{\theta^*-e}^{\theta^*+e-\frac{2e\pi}{r}}\frac{dy}{(\lambda r+(1-\lambda)\widetilde{\omega}r)^2} \\ -\int_{\theta^*+e-\frac{2e\pi}{r}}^{\theta^*+e}\left(\frac{u'(.)}{2e(1-\lambda)(1-\widetilde{\omega})^2}\left(\left(1-\rho\right)y-\left(1-\lambda\right)\left(1-\pi\right)r\right)\right)dy \end{array}\right) < 0. \tag{26}$$

If the depositor withdraws early, his expected return increases with r. The return to those who do not withdraw goes down with r. Therefore $\frac{\partial g}{\partial r} > 0$. By the implicit function theorem $\frac{d\theta^*}{dr} > 0$, which means the. probability of bank runs increases with early return, r. ■

The second term in $g\left(.\right)$ is not affected by λ. In the first term, because $\frac{\pi}{r} < \widetilde{\omega} < 1$, an increase in λ will reduce the denominator more than the numerator. Hence, the whole term will go up with λ. This is because, when there are more impatient agents, it pays to withdraw early. The last term relates to the returns you get by not withdrawing. More the λ, less the return for those who wait. Therefore $\frac{\partial g}{\partial \lambda} > 0$. Therefore $\frac{d\theta^*}{d\lambda} > 0$. So the probability of bank run, increases with proportion of impatient agents, λ. ■

$$\left.\frac{\partial g}{\partial \pi}\right|_{\pi=0} = -\frac{1}{r} u\left(\frac{(1-\lambda r)\left(\theta^*|_{\pi=0} + e\right)}{(1-\lambda)}\right) - \frac{u(r)}{r}\log(\lambda) < 0. \tag{27}$$

Note that

$$g(\theta^*)|_{\pi \to 0} = \int_{\theta^*-e}^{\theta^*+e} \frac{1}{2e} * \frac{\lambda r}{\lambda r + (1-\lambda)\frac{\theta^*+y-e}{2e} r} u(r)\, dy, \tag{28}$$

Therefore $\theta^*|_{\pi=0}$ is very large, close to the upper dominant region. So we can say that $\left.\frac{\partial g}{\partial \pi}\right|_{\pi=0} < 0$. $\left.\frac{\partial g}{\partial \pi}\right|_{\pi \to r} \to -\infty < 0$.

$$\frac{d^2 g}{d\pi^2} = -\frac{1}{r}\left(\frac{u'(.)(1-\rho)(\theta^*-e)}{r\left(1-\frac{\pi}{r}\right)^2} + \frac{(1-\lambda)u(r)}{\lambda r + (1-\lambda)\pi} + u\left(\frac{1-\theta^*-e+\frac{2e\pi}{r}}{1-\frac{\pi}{r}}\right)\right) < 0. \tag{29}$$

Therefore we can conclude that $\frac{\partial g}{\partial \pi} < 0$. We know that $\frac{\partial g}{\partial \theta^*} < 0$, which means $\frac{d\theta^*}{d\pi} < 0$. So the probability of bank runs decreases with excess reserves, π. ■

A3. Proof of Proposition 3 - optimal early return

Using EU in equation (4), the first order condition for optimal r given by, $\frac{\partial EU}{\partial r} = 0$,

$$\left.\frac{\partial EU}{\partial r}\right|_{r=0} = (1-\lambda)\left(\begin{array}{c} \frac{\pi}{r^{2-\alpha}} + u\left(\pi + \frac{(1-\pi(1-\lambda))\theta^*}{1-\lambda}\right)\frac{d\theta^*}{dr} + \\ -\int_{\theta^*}^{\overline{\theta}}\left(u'\left(\frac{(1-\lambda)\pi+(1-\pi(1-\lambda))\theta}{1-\lambda}\right)\left(\frac{\lambda\theta}{1-\lambda}\right)\right)d\theta \end{array}\right). \tag{30}$$

Since the first term goes to $+\infty$, $\left.\frac{\partial EU}{\partial r}\right|_{r=0} > 0$. Therefore $r^* > 0$.

$$\left.\frac{\partial EU}{\partial r}\right|_{r=1} = \frac{1}{\overline{\theta} - \underline{\theta}} * \left(\begin{array}{c} -(1-\lambda)\pi(1-\alpha)\theta^* \\ -(1-\lambda)\left[u(\pi+(1-\pi)\theta^*) - \pi\right]\frac{d\theta^*}{dr} \\ -\int_{\theta^*}^{\overline{\theta}} u'\left(\frac{(1-\lambda)\pi+(1-\lambda-\pi(1-\lambda))\theta}{1-\lambda}\right)\lambda\theta d\theta + \lambda\alpha\overline{\theta} \end{array}\right). \tag{31}$$

It is obvious that $\left.\frac{\partial EU}{\partial r}\right|_{r=1} < 0$. We know that $\frac{\partial^2 EU}{\partial r^2} < 0$ and $r^* > 0$. Therefore we can conclude that $0 < r^* < 1$. ■

We compare the first-order conditions for r^*_{FB} and r^*. The first-order condition for r^*

can be written as follows:

$$u'(r) - \frac{1}{\left(\overline{\theta} - \theta^*\right)} \int_{\theta^*}^{\overline{\theta}} \theta u' \left(\frac{(1-\lambda r)\theta}{1-\lambda} + \pi(1-\theta) \right) d\theta \tag{32}$$
$$= \frac{(1-\lambda)\left[u\left(\frac{(1-\lambda)\pi + (1-\lambda r - \pi(1-\lambda))\theta^*}{1-\lambda} \right) - \frac{\pi}{r} u(r) \right] \frac{d\theta^*}{dr} - \int_{\underline{\theta}}^{\theta^*} \rho \left(\frac{ru'(r) - u(r)}{r^2} \right) d\theta}{\lambda\left(\overline{\theta} - \theta^*\right)}.$$

$u\left(\frac{(1-\lambda)\pi + (1-\lambda r - \pi(1-\lambda))\theta^*}{1-\lambda} \right) > \frac{\pi}{r} u(r)$. i.e. utility when no patient agent withdraws is more than the utility when everyone withdraws. We know that $\frac{d\theta^*}{dr} > 0$. We also know that $\rho\left(\frac{ru'(r) - u(r)}{r^2} \right) < 0$. This means that the RHS is positive. So for the first order condition to hold,

$$u'(r^*) - \frac{1}{\left(\overline{\theta} - \theta^*\right)} \int_{\theta^*}^{\overline{\theta}} \theta u' \left(\frac{(1-\lambda r^*)\theta}{1-\lambda} + \pi(1-\theta) \right) d\theta > 0. \tag{33}$$

Now consider the expected utility of an agent under the first best condition:

$$EU_{FB} = \lambda u(r) + (1-\lambda) \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{\overline{\theta} - \underline{\theta}} u \left(\frac{(1-\lambda r)\theta}{(1-\lambda)} \right) d\theta. \tag{34}$$

The first-order condition for optimal early return r^*_{FB},

$$u'(r^*_{FB}) - \frac{1}{\overline{\theta} - \underline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \theta u' \left(\frac{(1-\lambda r^*_{FB})\theta}{1-\lambda} \right) d\theta = 0. \tag{35}$$

In the first-best, no unnecessary reserves are kept and therefore more can be invested long term. This means,

$$\int_{\theta^*}^{\overline{\theta}} \theta u' \left(\frac{(1-\lambda r^*)\theta}{1-\lambda} - \pi(\theta - 1) \right) d\theta > \int_{\underline{\theta}}^{\overline{\theta}} \theta u' \left(\frac{(1-\lambda r_{FB})\theta}{1-\lambda} \right) d\theta. \tag{36}$$

Therefore it must be that $u'(r^*) > u'(r_{FB})$ which leads to the conclusion that $r^* < r^*_{FB}$. ■

A4. Proof of Proposition 4 - optimal reserve level

The first-order condition for π^* is given by, $h(.) : \frac{\partial EU}{\partial \pi} = 0$.

$$\left.\frac{\partial EU}{\partial \pi}\right|_{\pi=0}=\frac{1-\lambda}{\overline{\theta}-\underline{\theta}}*\left(\begin{array}{c} -u\left(\frac{(1-\lambda r)\,\theta^{*}|_{\pi=0}}{1-\lambda}\right)*\left.\frac{d\theta^{*}}{d\pi}\right|_{\pi=0}+\frac{u(r)\,\theta^{*}|_{\pi=0}}{r}+ \\ +\int_{\theta^{*}|_{\pi=0}}^{\overline{\theta}}u'\left(\frac{(1-\lambda r)\theta}{1-\lambda}\right)(1-\theta)\,d\theta \end{array}\right). \tag{37}$$

Since $\left.\frac{\partial\theta^{*}}{\partial\pi}\right|_{\pi=0}<0$ and $\theta^{*}|_{\pi=0}$ is large, it is obvious that $\left.\frac{\partial EU}{\partial\pi}\right|_{\pi=0}>0$. This shows that the optimal reserve is such that $\pi^{*}>0$. ■

Because we are maximising EU, $\frac{\partial h}{\partial\pi^{*}}<0$. Moreover, we know that EU goes down with λ: $\frac{\partial EU}{\partial\lambda}<0$. The reduction in EU when π increases, also increases when λ increases. This means, $\frac{\partial h}{\partial\lambda}>0$ and therefore $\frac{d\pi^{*}}{d\lambda}>0$. So, π^{*} increases with λ. ■

A5. Proof of Proposition 7

If we compare the equations (3) and (13), the depositor has a higher expected utility by not running when the bank has access to a lender. This is because, in the range $\left(\frac{\left(\left(\widehat{\theta}+e\right)r-2e\pi\right)(1-\lambda)l}{2e(1-\rho)+(1-\lambda)l},\widehat{\theta}+e-\frac{2e\pi}{r}\right)$, he does not get anything when there was no lender.

Further, the expected utility from running is higher because he gets r for sure with a lender, but r with a probability with no lender in the range $\left(\frac{\left(\left(\widehat{\theta}+e\right)r-2e\pi\right)(1-\lambda)l}{2e(1-\rho)+(1-\lambda)l},\widehat{\theta}+e-\frac{2e\pi}{r}\right)$. Therefore, for equation (3) to hold, $\theta^{l^{*}}<\theta^{*}$.

The first-order condition for optimal π^{l} is,

$$\begin{aligned} &\frac{\lambda r+(1-\lambda)\,\pi^{l^{*}}}{r}u(r)\frac{d\theta^{l^{*}}}{d\pi}+\int_{\underline{\theta}}^{\theta^{l*}}\frac{(1-\lambda)}{r}u(r)\,d\theta- \\ &\left(\lambda u(r)+(1-\lambda)\,u\left(\frac{(1-\lambda)\,\pi^{l^{*}}+\left(1-\lambda r-\pi^{l^{*}}(1-\lambda)\right)\theta^{l^{*}}}{1-\lambda}\right)\right)\frac{d\theta^{l^{*}}}{d\pi}+ \\ &\int_{\theta^{l*}}^{\overline{\theta}}\left((1-\lambda)\,u'\left(\frac{(1-\lambda)\,\pi^{l^{*}}+\left(1-\lambda r-\pi^{l^{*}}(1-\lambda)\right)\theta}{1-\lambda}\right)(1-\theta)\right)d\theta=0. \end{aligned} \tag{38}$$

If we compare equation (38) and equation (3), $\frac{d\theta^{l*}}{d\pi}>\frac{d\theta^{*}}{d\pi}$, the first term is bigger in here. The third term is smaller because $\frac{d\theta^{l*}}{d\pi}>\frac{d\theta^{*}}{d\pi}$ and $\theta^{l*}<\theta^{*}$ Because the third term is negative, the whole equation is bigger because of the third term. The last term is over a bigger range, so the last term is bigger. The only effect that makes the equation smaller is the second term, which compared to the other effects is not significant. Therefore $\pi^{l^{*}}<\pi^{*}$.■

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