Growth and Welfare Effects of Stabilizing Innovation Cycles[*]

Marta Aloi[†]
University of Nottingham

Laurence Lasselle[‡]
University of St Andrews

ABSTRACT

We consider a simple model of innovation where equilibrium cycles may arise and show that, whenever actual capital accumulation falls below its balanced growth path, subsidizing innovators by taxing consumers has stabilizing effects, promotes sustained growth and increases welfare. Further, if the steady state is unstable under laissez faire, the introduction of the subsidy can make the steady state stable. Such a policy has beneficial effects as it fosters output growth along the transitional adjustment path, and increases the welfare of current and future generations.

JEL Codes: E62; H32; O41
Keywords: Growth; Endogenous Cycles; Stabilization; Innovation; Subsidy; Welfare.

1   Introduction

It is well known that processes of growth based solely on factor accumulation cease at some point because of diminishing returns to scale of production. By contrast, introducing innovative products may create new opportunities on the production side. Innovation may be induced by the prospect of enjoying temporary monopoly profits. Such temporary monopoly power makes the law of diminishing returns to scale vanish and growth based on innovation sustainable [see e.g. the seminal papers by Romer (1986, 1990) and, for a general overview, Aghion and Howitt (1997)]. The stability properties of such a sustainable balanced growth path have been extensively studied in the literature. In particular, authors like Shleifer (1986), Aghion and Howitt (1992), Deneckere and Judd (1992), Matsuyama (1999) and Francois and Lloyd-Ellis (2003) have demonstrated the existence of equilibrium endogenous fluctuations and, more specifically, deterministic cycles within different models of innovation. In Shleifer (1986) equilibrium cycles arise due to strategic complementarities between innovators. In Aghion and Howitt (1992) the occurrence of cycles is linked to the negative dependence of current research upon future research. In Deneckere and Judd (1992) and Matsuyama (1999) equilibrium cycles arise due to the non linear dependence between incentives to innovate and the current level of innovations. That is, increases in variety of goods today induce imitation in later periods and these periods of imitation do not foster innovation; incentives for future innovation occur when goods' variety will start declining. More recently, Francois and Lloyd-Ellis (2003) have developed a model in which clustering of implementation and innovation is endogenous and a stable cyclical equilibrium may emerge along the balanced growth path. In their model, such a cyclical equilibrium trajectory has higher average growth, but lower welfare than the stationary equilibrium trajectory.

In this article, we draw on the insights of the above mentioned literature to investigate the stabilizing (destabilizing) effects of policies aimed at eliminating

cycles in innovative activity, and their implications on growth and welfare. Our work is closer in spirit to Mastuyama (1999) which focuses on the asynchronicity of innovation and investment activities. In his work, as in Deneckere and Judd (1992), what is crucial is the timing of entry of innovators into the market for new goods. To start up production innovators need to ensure that the market for their product is sufficiently large to recoup the costs of innovation, and since they enjoy monopoly rents only for one period, innovators introduce new products into the market at the same time as their competitors. Delaying entry would mean losing temporary monopoly rents and make innovation not profitable enough. Therefore, innovative activities take place at the same time, and prevail until competition among innovators builds up and monopoly rents drop. As the economy becomes more competitive, more resources are available to manufacturing activities, and both output and investment growth increase. Higher output and investment will, in turn, build up a larger resource base in the economy, which stimulates another period of innovative activity. In this model, under empirically plausible conditions, the balanced growth path is unstable and the economy achieves sustainable growth through cycles, perpetually moving back and forth between two phases. In one phase, when the growth rates of output and investment are higher, there is no innovation. The economy is then competitive and the evolution of this economy is the one pointed out by Solow. In the other phase, when the growth rates of output and investment are lower, there is innovation and the economy is "more monopolistic" as in the Schumpeterian endogenous growth approach. In the long run, the growth rates of innovation and investment are equal, however they do not follow the same evolution: they move over the cycle in an a-synchronized way. That is, the economy alternates between periods of high innovation and periods of high investment.[1] These phenomena should not be interpreted as short run

---

[1]Periods of high innovation are followed by periods of high investment, and in each phase of the economy either innovation or capital accumulation play the dominant role. Moreover, as shown by Matsuyama (1996, 1999) when the economy moves back and forth between the two phases growth is faster than along the (unstable) balanced growth path.

business cycles but, rather, as long-cycles induced by the clustering over time of entrepreneurial innovation.

The main contribution of our analysis, particularly with respect to the work of Matsuyama (1999) or Deneckere and Judd (1992), is to uncover both positive and normative implications of simple tax/subsidy policies to control for innovation driven cycles. We define stabilization policy as a policy that moves the economy from an unstable regime to a stable regime.[2] The focus on stabilization originates from the observation that nations and governments aim, when possible, at reaching high permanent growth without incurring into prolonged periods of low or no growth (i.e., slumps). We concentrate our attention on the issue of how subsidizing innovators (and taxing consumers) can generate stable sustained growth and higher welfare.

In this respect, our paper can also be seen as a reformulation in a macroeconomic context of the branch of the R&D literature that deals with public aid to innovation (see e.g. Aghion and Howitt (1992), Davidson and Segerstrom (1998) and Segerstrom (2000)). In this literature it is often claimed that there exists a role for public intervention both in subsidizing R&D activities and enforcing property rights to innovation as means to promote economic growth. We wish to add a new macroeconomic perspective to this claim: we show that in fast growing economies, in which high factor accumulation plays a crucial role alongside innovative sectors that enjoy temporary rents,[3] governments should follow an unorthodox approach when facing prolonged periods of slow growth. Namely, they should reallocate re-

---

[2]This macroeconomic concept of 'stabilisation policy' was pioneered by Grandmont (1985, 1986) and then developed by many authors. See, for instance, Judd and Deneckere (1992), Schmitt-Grohe and Uribe (1997), Guo and Lansing (1998), Christiano and Harrison (1999), Aloi et al. (2002).

[3]What we have in mind here are economies like Japan and East Asian newly industrialised countries (NICs) like South Korea, Hong Kong, Singapore and Taiwan. Japan experienced fast growth since the aftermath of the second World War until the early 1990s. Until the 1960s the main engine of growth was undoubtedly rapid factor accumulation; from the 1960s until the 1990s the main engine of growth of Japanese economy can be identified as the development and expansion of high technology industry (see Odagiri and Goto (1993)). As regards East Asian NICs, a large part of their growth until 1990 was driven by rapid factor accumulation (see Young (1995)). However, there is extensive evidence of the emerging role of high tech industry such as semiconductor industry in East Asian Economies (see Mathews and Cho (2000)).

sources from consumers to the innovative sectors. Our paper also demonstrates that a tax on innovators, aimed at raising resources available to consumers during economic slumps, is detrimental. On the one hand it can be destabilizing: precisely, if the steady state is globally stable under laissez faire, the introduction of the tax makes cycles possible. On the other hand, such a tax has negative welfare effects as it depresses output growth along the transitional adjustment path. These results are at odds with the general presumption that policies aimed at raising private saving (and, therefore, at expanding the resource base of the economy) foster higher growth.[4]

Although our model is rather stylized, it provides some insights on the role of stabilization policy in fast-growing economies falling into prolonged periods of low or no growth. Consider, for instance, the so-called East Asian tigers and Japan. In recent years these economies have experienced recurrent slumps and are still struggling to find a way out of stagnation. The issue of the policy requirements needed to resume sustained growth has been widely debated among economists (see e.g. Crafts (1999), Dornbusch (1995) and Ito (1996)). Most of these studies focus on the need for financial and structural reforms as pre-requisites to resume sustained growth in the long run. Also, there is a general agreement among researchers that the high saving rates of East Asian and Japanese economies are an impediment to their full economic recovery. Indeed, with profitability depressed very little of the relatively large share of income that is saved is eventually invested; moreover, high saving depresses domestic consumption. Also, standard expansionary fiscal and monetary policies seem unable to trigger enough stimulus in aggregate demand

---

[4]As mentioned earlier, there are few papers in the R&D literature that emphasise the role of subsidies to R&D in promoting economic growth. For instance, Davidson and Segerstrom (1998) distinguish between the role of innovative R&D and imitative R&D, and conclude that subsidising innovative R&D promote growth while subsidising imitative R&D can be detrimental to growth. General R&D subsidies, on the other hand, always exhert positive effects on growth. Similar conclusions are reached by Segerstrom (2000) where the emphasis is between vertical versus horizontal innovation. Even though our paper is not directly comparable with this strand of the literature, in that we adopt the most simple model of innovation and look at different issues (i.e. macroeconomic stabilization), it is interesting to note the similarities in the policy implications of the two approaches.

and investment. Our model suggests that transferring resources from consumers to innovators may help to overcome economic stagnation and resume sustained growth.

Finally, an interesting feature of the model developed here is that stabilization not only brings higher average growth, but also higher welfare. In Deneckere and Judd (1992) and Matsuyama (1999) no welfare analysis is provided, whereas Francois and Lloyd-Ellis (2003) find that, along equilibrium cycles, higher average growth is accompanied by lower welfare. In their set up, however, social welfare corresponds to the utility of an infinitely lived representative consumer, and the *stationary* (or acyclical) equilibrium trajectory falls in the 'Solow regime' region with factor accumulation and no innovation. In our model, stabilization brings higher welfare because the economy ends up in a stable equilibrium characterized by innovation and *sustained* growth (i.e. in a 'Romer regime' region), where the amount of resources available for consumption of both young and old is higher than under equilibrium cycles (and laissez faire).

The remainder of the paper is as follows. In Section 2 we describe the basic features of the model. In Section 3 we derive the equilibrium dynamics, while in Section 4 we present our main results. Section 5 concludes. Proofs of propositions are relegated in the Appendix.

2   The Model

Time is discrete, $t = 0, 1, 2, ...$ Agents live for two periods. When young they work and receive an income $w_t L$, and pay (or receive) a lump sum tax (subsidy) B_t, whereas when old they use all their savings for consumption. w_t represents the real wage, L is the exogenously fixed labor supply, and the population growth rate is zero. The utility function, U_t, of the representative consumer of each generation is given by $U_t = \ln c_{1t} + \frac{1}{1+\rho} \ln c_{2t+1}$ where $0 < \rho < 1$ is the subjective rate of time preferences, c_{1t} is the consumption when young and c_{2t+1} is the consumption

when old. Maximizing utility subject to the intertemporal budget constraint,[5] $c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} = w_t L + B_t$ yields, a simple saving function, S_t

$$S_t = s\left(w_t L + B_t\right).$$ (1)

where $s = \frac{1}{2+\rho}$ is the equilibrium saving rate.

Along the lines of Matsuyama (1999), we assume that in this economy there is one final good, taken as numeraire, which is competitively produced, and is either consumed or invested. The part invested is converted into a variety of differentiated intermediate products, and associated with labor (exogenously fixed) according to a Cobb-Douglas technology. The intermediate products are aggregated into a symmetric CES technology. The final goods production function is then given by

$$Y_t = \widehat{A}(L)^{\frac{1}{\sigma}} \left\{ \int_0^{N_t} (x_t(z))^{1-\frac{1}{\sigma}} \, dz \right\},$$ (2)

where $x_t(z)$ is the intermediate input of variety $z \in [0, N_t]$ and $\sigma \in (1, \infty)$ is the elasticity of substitution between each pair of intermediate goods. Notice that: the technology (2) satisfies constant returns to scale for a given availability of intermediates, N_t; and that the parameter $\frac{1}{\sigma}$ is the labor share of income, since $w_t L = \frac{Y_t}{\sigma}$. Turning to the specification of the intermediate goods, x_t^c represents the intermediate input produced in the competitive sector (with no innovation), and x_t^m the intermediate input produced by the monopolistic innovative sector. It is assumed that from period 0 to period $t-1$ only 'old' intermediate goods are available in the market. The variety $z \in [0, N_{t-1}]$ is produced under perfect competition by converting a units of capital into one unit of an intermediate, and is competitively sold at its marginal cost, i.e. $p_t^c = ar_t$. Between $t-1$ and t a range of 'new' intermediate goods $z \in [N_{t-1}, N_t]$ may be introduced. In this case, the variety $z \in [N_{t-1}, N_t]$ is produced and sold exclusively by the respective innovators in period t. Innovators operate under monopolistic competition with no barriers to entry, and enjoy

---

[5]The real interest rate is denoted by r.

monopoly rents only for one period. New intermediate goods are produced by using a units of capital per unit of output and require F units of capital per variety and are, therefore, sold at $p_t^m = \frac{ar_t}{1-\frac{1}{\sigma}}$. Since, in this model, capital is the un-consumed final good, and the final good is the numeraire, from now on we set $r_t = 1$.

We depart from Matsuyama (1999) by assuming that innovators can be either taxed or subsidized. Hence, the profit function of an innovator operating in period t is given by,

$$\Pi_t^m = p^m x_t^m - [ax_t^m + F] - T_{t-1}$$

where $T_{t-1} > 0$ $(T_{t-1} < 0)$ is a lump sum tax (subsidy) set by the government at the outset of period t (end of period $t-1$). Since there are no barriers to entry, net profit must be zero at all times, implying that 'new' intermediate products $(N_t > N_{t-1})$ are introduced if and only if $x_t^m \geq \frac{(\sigma-1)}{a}(F + T_{t-1})$. Note that if innovators receive a lump sum subsidy (or pay a lump sum tax) the effect is clearly to increase (or reduce) the incentive to enter by potential innovators. Demands for intermediate inputs come from the maximization of the final good profit function, taking into account that all intermediate goods enter symmetrically into the production of the final good, i.e. $x_t(z) \equiv x_t^c$ for $z \in [0, N_{t-1}]$ and $x_t(z) \equiv x_t^m$ for $z \in [N_{t-1}, N_t]$. Under these assumptions,

$$\frac{x_t^c}{x_t^m} = \left(\frac{p_t^c}{p_t^m}\right)^{-\sigma} = \left(1 - \frac{1}{\sigma}\right)^{-\sigma}.$$

The above implies that the demand for each intermediate input is,

$$x_t^c = \frac{1}{a}\theta\sigma F\left(1 + \frac{T_{t-1}}{F}\right), \qquad \theta \equiv \left(1 - \frac{1}{\sigma}\right)^{1-\sigma} \tag{3}$$

$$x_t^m = \frac{1}{a}(\sigma - 1)F\left(1 + \frac{T_{t-1}}{F}\right) \tag{4}$$

where $\theta \in (1, e = 2.71..)$. This is a parameter related to the monopoly margin of the innovator (i.e. $\frac{1}{\sigma-1}$). Thus, when σ is close to one $\theta \to 1$, and when $\sigma \to \infty$,

7

$\theta \to e$. Using the above relationships, the economy resource constraint on capital in period t is,[6]

$$K_{t-1} = N_{t-1}(ax_t^c) + (N_t - N_{t-1})(ax_t^m + F + T_{t-1}).\tag{5}$$

It then follows that when $N_t = N_{t-1}$ (i.e. no innovation), we have

$$\frac{K_{t-1}}{N_{t-1}} = ax_t^c\,,$$

whereas when $N_t > N_{t-1}$ (i.e. innovation), by substituting the expressions for demand, (3) and (4), into the economy resource constraint on capital, we get,

$$N_t = N_{t-1} + \frac{K_{t-1}}{\sigma F\left(1 + \frac{T_{t-1}}{F}\right)} - \theta N_{t-1}\,.$$

Letting $k_t \equiv \frac{K_t}{\theta \sigma F N_t}$ we obtain an expression governing the introduction of new products,

$$N_t = N_{t-1} + \max\left[0,\ \theta N_{t-1}\left(\frac{k_{t-1}}{1 + \frac{T_{t-1}}{F}} - 1\right)\right].\tag{6}$$

The critical point at which innovation starts to be profitable is $k_{cr} \equiv 1 + \frac{T_{t-1}}{F}$.

In a symmetric equilibrium, total output, as in (2), is equal to

$$Y_t = \widehat{A}(L)^{\frac{1}{\sigma}}\left[N_{t-1}(x_t^c)^{1-\frac{1}{\sigma}} + (N_t - N_{t-1})(x_t^m)^{1-\frac{1}{\sigma}}\right].$$

Using the demand for the intermediate inputs, (3) and (4), the reduced form aggregate production function for $N_t = N_{t-1}$ is

$$Y_t = \mathcal{A}_{t-1}\left(1 + \frac{T_{t-1}}{F}\right)^{\frac{1}{\sigma}}(k_{t-1})^{-\frac{1}{\sigma}}K_{t-1}\,,\tag{7}$$

while for $N_t > N_{t-1}$ the reduced aggregate production function becomes

$$Y_t = \mathcal{A}_{t-1}K_{t-1}\tag{8}$$

---

[6]K_{t-1} denotes the capital stock available at the beginning of period t. It corresponds to the amount of final goods left un-consumed at the end of period $t-1$ and carried over to period t.

where $\mathcal{A}_{t-1} \equiv \frac{\hat{A}}{a} \left(\frac{aL}{\theta \sigma F} \right)^{\frac{1}{\sigma}} \left(1 + \frac{T_{t-1}}{F} \right)^{-\frac{1}{\sigma}}$. Note that under laissez faire, $T_{t-1} = 0$, $\mathcal{A}_{t-1} = A \equiv \frac{\hat{A}}{a} \left(\frac{aL}{\theta \sigma F} \right)^{\frac{1}{\sigma}}$ (as in Matsuyama (1999)). The reduced form aggregate production of the final good is of the 'AK' type if $N_t > N_{t-1}$ as in expression (8). Therefore, if the resource base of the economy is large enough, $k_{t-1} \geq k_{cr}$, new products are introduced and the economy evolves according to a 'Romer regime'. If the resource base is not large enough, no innovation takes place and the aggregate production function (i.e. expression (7)) exhibits decreasing returns to capital. Hence the economy evolves according to a 'Solow regime'.

As shown in the section below, the equilibrium dynamics of the model simplifies to a non-linear one-dimensional difference equation, well defined in the forward direction of time. The analysis of this one-dimensional equation (evaluation of the steady state and its stability properties) allows us to study the evolution of the economy between the two regimes within a single growth process.

3 Equilibrium Dynamics and Steady State

To derive the dynamic equilibrium we need to specify how capital accumulation evolves over time. At equilibrium, saving equals investment, $S_t = K_t$, and the government balances the budget, $B_t = (N_t - N_{t-1}) T_{t-1}$. The latter expression implies that the lump sum tax on innovators at the outset of period t ($T_{t-1} > 0$) is redistributed to the consumers in the form of a lump sum subsidy. Equivalently, lump sum taxes levied on the consumers ($T_{t-1} < 0$) finance the subsidy distributed to the innovators. Hence, the saving function, (1), can be written as

$$ S_t = s \frac{Y_t}{\sigma} + s \left(N_t - N_{t-1} \right) T_{t-1} . $$

Substituting for $N_t - N_{t-1}$ from (6) and Y_t from either (7) or (8) into the expression above we obtain capital K_t as

$$ K_t = \frac{s}{\sigma} \cdot \mathcal{A}_{t-1} \max \left[\left(1 + \frac{T_{t-1}}{F} \right)^{\frac{1}{\sigma}} k_{t-1}^{-\frac{1}{\sigma}}, \ 1 \right] K_{t-1} $$

$$+sT_{t-1} \max \left[0, \ \theta N_{t-1} \left(\frac{k_{t-1}}{1 + \frac{T_{t-1}}{F}} - 1 \right) \right].$$

Dividing both sides of the above expression by $\theta \sigma F N_t$, the forward perfect foresight dynamics of the system can be expressed as a one-dimensional map in k, $\Lambda: \Re_+ \to \Re_+$,

$$k_t = \Lambda(k_{t-1}) \equiv G\,(k_{t-1})^{1-\frac{1}{\sigma}} \tag{9}$$

$$if \quad k_{t-1} \leq k_{cr}$$

$$k_t = \Lambda(k_{t-1}) \equiv G \left(1 + \frac{T_{t-1}}{F} \right)^{-\frac{1}{\sigma}} k_{t-1} \left[\frac{1}{1 + \theta \left(\frac{k_{t-1}}{1+\frac{T_{t-1}}{F}} - 1 \right)} \right] + \frac{sT_{t-1}}{\theta \sigma F} \left(1 - \frac{1}{1 + \theta \left(\frac{k_{t-1}}{1+\frac{T_{t-1}}{F}} - 1 \right)} \right)$$

$$\tag{10}$$

$$if \quad k_{t-1} \geq k_{cr}$$

where $G \equiv \frac{sA}{\sigma}$ represents the growth potential of the economy at the laissez faire equilibrium.

We proceed by first studying the equilibrium properties of the steady state under laissez faire.[7] Table 1 below summarizes the stability properties of the steady state under laissez faire.

	Solow	Romer
SS value	$k^* = G^\sigma < 1$	$k^{**} = \frac{G-1}{\theta} + 1 > 1$

Stability Properties of SS	*Monotonic convergence to SS*	$1 < G < \theta - 1$ *two − period cycles*
		$G > \theta - 1$ *Non monotonic convergence to SS*

Table 1 - Steady State (SS) properties under laissez faire

The occurrence of two-period cycles depends on technology parameter values. Note, in particular, the role played by the parameter $\theta \equiv \left(1 - \frac{1}{\sigma} \right)^{1-\sigma}$ implying $\theta \in (1, \ e)$.

---

[7]Setting $T_{t-1} = 0$ the dynamical system (9)-(10) is equivalent to that of Matsuyama (1996).

This parameter measures the extent to which the innovator loses market power if he or she waits until the goods that he or she is competing with become competitively priced. If intermediate goods are poor substitutes, the rate of obsolescence is high. This triggers innovation. As intermediate goods become more substitutable the economy may fluctuate between periods of positive innovation and zero innovation. As substitutability of intermediate inputs increases the system switches to a regime with no innovation at all. Low values of θ and sufficiently high values of G imply an oscillatory convergence to a balanced growth path with innovation, whereas sufficiently small values of G imply a monotonic convergence to a stationary path with factor accumulation and no innovation. For $\theta > 2$ (implying $\sigma > 2$) there exists a range of values of G such that the equilibrium growth path of the economy fluctuates between a phase of capital accumulation and no innovation and a phase of no factor accumulation and innovation.[8]

We are now well equipped to address the main issue of our investigation, that is, to evaluate the growth and welfare effects of policies aimed at eliminating fluctuations.

4 Stabilization

First, note that in the 'Solow regime' where the economy monotonically converges to the steady state there is clearly no scope for stabilization. We focus therefore on equilibrium dynamics situated in the 'Romer regime'.

Fluctuations are generally not seen as beneficial for the economy. Ideally, nations would like to avoid a growth pattern characterized by upswings and downturns in output growth. Moreover, they would aim at reaching high permanent growth. In other words, they would aim at reaching a balanced growth path where there is enough innovation and, at the same time avoid cycles. In our model, this implies ensuring that: (i) the economy will be situated in the Romer regime where inno-

---

[8]The empirical plausibility of the conditions for cycles is discussed in Matsuyama (1999), and we refer the reader to Section 4 of his paper.

vation is the engine of growth, rather than in the Solow regime with no innovation and factor accumulation; (ii) in the event of cycles, policy should aim at bringing the system on a balanced growth path where the economy grows ($G > 1$), rather than on a balanced growth path where the economy stays stationary ($G < 1$).

To achieve a different dynamic allocation with respect to the case of laissez faire, the government has, in principle, a variety of policy options. We choose to focus on the non-distortive option of re-allocating resources between agents by implementing an appropriate system of subsidies to the innovators financed by lump-sum taxes on the consumer or vice versa. This, combined with the assumption of balanced budget, implies that we can focus on purely stabilizing/destabilizing effects of policy and not on policy that affects the steady state as well. In particular, we assume that policy makers follow the following simple stabilization principle,

$$T_{t-1} = \gamma(k^{**} - k_{t-1}), \qquad if \quad k_{t-1} < k^{**} \qquad (11)$$

where the parameter γ represents the size of the government intervention. This principle implies that the government intervenes only in the case of recessions, and can be interpreted as a stylized representation of a countercyclical policy rule. In particular if, at the end of period $t - 1$ (i.e. outset of period t), the government observes a deviation of k from its long run trend, it may decide either to redistribute income to the consumer by means of a lump sum tax on the innovators ($\gamma > 0$) or, to subsidize the innovators by taxing the consumer ($\gamma < 0$).[9]

Suppose that $k_{t-1} < k^{**}$. The government then decides that in order to increase output growth in the final good sector, innovators are to be subsidized by levying a lump sum tax on the young consumers. The rationale for such a move is to create

[9]Note that our policy rule (11) does not aim at managing short run recessions, as our set up is concerned with low frequency movements in macroeconomic variables. (11) implies that, whenever the economy experiences a long wave of low or no growth (i.e. a slump), the government intervenes by implementing a simple balanced-budget tax rule. Considering that the length of the cycle implied by our model is twenty/twenty five years, the assumption of a balanced budget rule is not implausible. Indeed, it reflects the behaviour of many governments that aim at balancing the budget over the medium-run, while they may allow budget deficits to control short run business cycles.

an incentive for the innovators to produce new intermediate products which foster higher production of the final good. This implies in turn higher consumption of the representative consumer once old. Similarly, the case of a tax levied on innovators and redistributed to the young consumer reduces the potential growth of the final good. This implies in turn lower consumption by the representative consumer once old. To verify the validity of this conjecture in this particular economy we need to study the effects of implementing the rule above on the stability properties of the equilibrium.

When $k_{t-1} \leq k_{cr}$ (i.e. 'Solow regime') the dynamics remains the same as described by (9). When $k_{cr} \leq k_{t-1} \leq k^{**}$ (i.e. 'Romer regime') by substituting T_{t-1} by the proposed rule, i.e. (11),[10] into (10) the dynamics becomes,

$$
\begin{aligned}
k_t = \Lambda(k_{t-1}) \equiv & G\left(\left(1 + \frac{\gamma}{F}(k^{**} - k_{t-1})\right)^{-\frac{1}{\sigma}}\right) k_{t-1} \left(\frac{1}{1 + \theta\left(\frac{k_{t-1}}{1 + \frac{\gamma}{F}(k^{**} - k_{t-1})} - 1\right)}\right) \\
& + \frac{s}{\theta\sigma F}\gamma(k^{**} - k_{t-1})\left(1 - \frac{1}{1 + \theta\left(\frac{k_{t-1}}{1 + \frac{\gamma}{F}(k^{**} - k_{t-1})} - 1\right)}\right).
\end{aligned}
\tag{12}
$$

When $k_{t-1} \geq k^{**}$ the dynamics remains the same as in (10).

By construction, the steady state solution of (12) gives the same steady state value of k as in the laissez faire equilibrium, i.e. k^{**}. The dynamics of adjustment, on the other hand, differs. Note that the expression (12) cannot be differentiated at k^{**} (indeed Λ has a kink at k^{**}), implying that $\Lambda'(k^{**})$ does not exist. However, we can always compute the value of $\Lambda'(k)$ when k tends to k^{**} from the left, i.e.,

$$
\Lambda'_{-}(k^{**}) = \frac{\gamma}{F}\left(\left(\frac{1}{\sigma} - 1\right)k^{**} - \frac{s}{\theta\sigma}\left(\frac{G-1}{G}\right)\right) + \frac{1-\theta}{G}\left(1 + \frac{\gamma}{F}k^{**}\right)
\tag{13}
$$

The value of $\Lambda'(k)$ when k tends to k^{**} from the right is equal to $\Lambda'_{LF}(k^{**})$, i.e. $\Lambda'_{+}(k^{**}) = \Lambda'_{LF}(k^{**}) = \frac{1-\theta}{G}$. Therefore, for any infinitely small neighborhood of the

---

[10]Note that, under the specified rule, the critical point at which innovation starts to be profitable becomes $k_{cr} = \frac{1 + \frac{\gamma}{F}k^{**}}{1 + \frac{\gamma}{F}}$.

steady state the slope of the dynamics changes. Indeed, as discussed in Section 2, the introduction of a lump sum tax/subsidy changes the critical point at which the economy moves from growth driven by factor accumulation to growth driven by innovation. Taxing innovators and subsidizing the consumer lowers the growth potential of the final output. Figure 1 gives an example of the changes in the transitional adjustment path. It also illustrates that this policy, which is aimed at stabilizing the economy, may on the contrary exert a destabilizing effect. If (as in Figure 1) the steady state is globally stable under laissez faire, the introduction of a tax makes cycles possible.

Turning to the welfare properties, we consider a social welfare function linear in the discounted lifetime utilities of the $n+1$ current and future generations, i.e.

$$SW_t = \frac{1}{1+\rho} \ln c_{20} + \sum_{t=0}^{n-1} \left(\frac{1}{1+\rho}\right)^t \left[\ln c_{1t} + \left(\frac{1}{1+\rho}\right) \ln c_{2t+1}\right],$$

where the utility of the old generation at time zero is taken as exogenous. Replacing c_{1t} and c_{2t+1} by the consumer optimal choices, $c_{1t} = \left(\frac{1-s}{s}\right) S_t = (1+\rho) S_t$ and $c_{2t+1} = (1+r) S_t = 2S_t$, the above social welfare function can be re-written as,

$$SW_t = \Omega + \sum_{t=0}^{n-1} \left(\frac{1}{1+\rho}\right)^t \ln S_t + \sum_{t=0}^{n-1} \left(\frac{1}{1+\rho}\right)^{t+1} \ln S_t \qquad (14)$$

where $\Omega \equiv \frac{1}{1+\rho} \ln c_{20} + \sum_{t=0}^{n-1} \left(\frac{1}{1+\rho}\right)^t \ln(1+\rho) + \sum_{t=0}^{n-1} \left(\frac{1}{1+\rho}\right)^{t+1} \ln 2$. As shown in the Appendix, if we evaluate the social welfare function, (14), at the laissez faire equilibrium and along the equilibrium cycle, it turns out that a subsidy to consumers financed by a tax on innovators reduces social welfare.

The following propositions summarize the results related to the implementation of the tax on innovators.

Proposition 1 *(Stability properties of the steady state under tax on innovators). If $G > \theta - 1$ there is a $\gamma^* > 0$, such that for any $\gamma > \gamma^*$ and for any $k_{cr} < k_{t-1} < k^{**}$, $\Lambda'_-(k^{**}) < -1$. The steady state is unstable and there are equilibrium cycles of period two.*
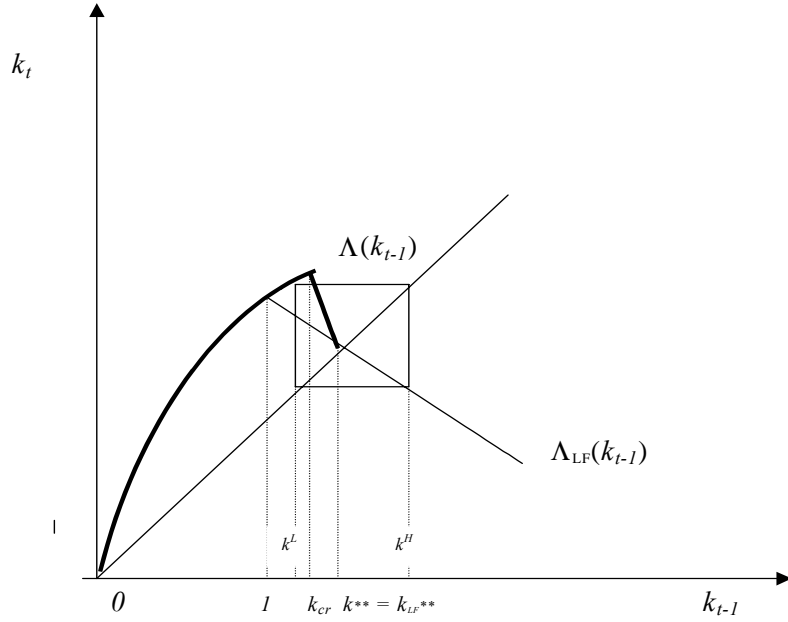
Figure 1: The graph of $\Lambda(k_{t-1})$ for $\gamma > 0$ and $G > \theta - 1$

Proof: See appendix.

Increasing γ as a means to stabilize downward fluctuations in output leads to equilibrium cycles. Hence it is highly destabilizing. In particular, as γ crosses γ^*, a flip bifurcation occurs: the steady state looses stability and a stable cycle of period two appears.[11]

Proposition 2 *(Growth and Welfare properties under tax on innovators)*
(i) The average growth rate of the economy over the two-period cycles under a tax on innovators is lower than the average growth rate of the economy under laissez faire in the 'Romer regime'. (ii) Welfare under a tax on innovators is lower than welfare under laissez faire in the 'Romer regime'.

Proof: See appendix.

---

[11]To check for the stability properties of the two-period cycle we have simulated the model. These simulations suggest that the cycle remains stable for a wide range of parameter values.

Propositions 1 and 2 establish that implementing a tax on innovators whose receipts are redistributed to the consumer is detrimental for the economy. The basic intuition is that reallocating resources from the innovators to the consumer affects the balance between the two engines of growth, i.e. factor accumulation and innovation. As the subsidy to the consumer financed through the lump sum tax on the innovators increases, the economy moves from a situation in which innovation is highly profitable to one in which innovation is less profitable. This implies that, as the economy moves closer to a regime with no innovation and stationary growth it can be trapped in a phase where it cycles between high innovation (and low factor accumulation) and low innovation (and high factor accumulation). This depresses output growth along the equilibrium adjustment path and, in turn, reduces social welfare.

However, if the government chooses the alternative option of taxing the young consumer and redistributing the receipts to the innovators as a lump sum subsidy the results are reversed. Figure 2 gives an example of the changes of the transitional adjustment path. It illustrates that this policy has a stabilizing effect on the economy. Under laissez faire the steady state is unstable and a two-period cycle emerges between the two growth regimes. As shown, the introduction of a subsidy stabilizes the economy in that the system converges towards the steady state. Fluctuations disappear and output growth is high along the equilibrium adjustment path.[12]

The next propositions summarize the results related to the implementation of the subsidy on innovators.

Proposition 3 *(Stability properties of the steady state under subsidy on innovators).*
If $G < \theta - 1$ there is a $\gamma^* < 0$, such that for any $-\frac{F}{k^{**}} < \gamma < \gamma^*$ and for any $k_{cr} < k_{t-1} < k^{**}$, $0 < \Lambda'_-(k^{**}) < 1$. The economy monotonically converges towards the steady state in the 'Romer regime'.

---

[12]To ensure $0 < k_{cr} < 1$ we impose $\gamma > \frac{-F}{k^{**}}$ (cf. footnote 10).
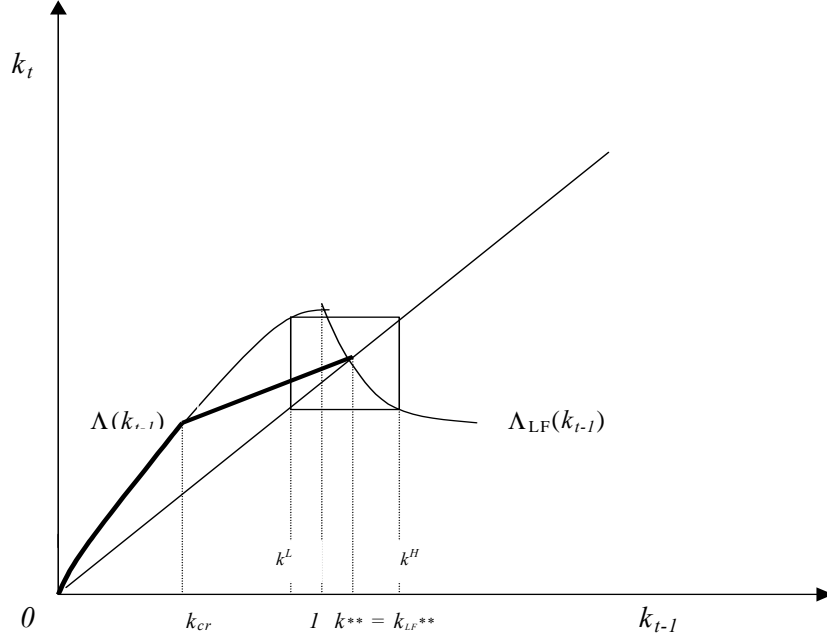
Figure 2: The graph of $\Lambda(k_{t-1})$ for $\gamma < 0$, $1 < G < \theta - 1$ and $\Lambda(k_{cr}) < \Lambda(k^{**})$.

Proof: See appendix.

Recall that, in this set up, the subsidy payed to innovators corresponds to a tax levied on young consumers. Hence, the term $-\gamma < F/k^{**}$ (where $\gamma < 0$) in proposition 3 identifies an upper limit to the size of the government intervention and, therefore, to the size of the tax/subsidy implemented. Substituting for k^{**}, as given in Table 1, such an upper bound corresponds to $-\gamma < \frac{F}{\frac{G-1}{\theta}+1}$, where $1 < G < \theta - 1$ and $\theta \equiv \left(1 - \frac{1}{\sigma}\right)^{\sigma-1}$ with $\theta > 2$. It can be easily computed[13] that the lowest level of this upper bound is $F/2$. The latter is higher the higher the cost of innovation, F. When F is high, the profitability of innovation activity is low. To bring the economy out of a low growth cycle into a high growth innovation-driven equilibrium path, the government will need to implement relatively large tax/subsidy. If, on the contrary, F is relatively low, the size of the intervention

---

[13]As $1 < G < \theta - 1$ and $\theta > 2$, then $1 < G < \theta - 1 \Leftrightarrow \frac{F}{2\left(1-\frac{1}{\theta}\right)} < \frac{F\theta}{G-1+\theta} < F$. Also, since $\theta > 2$, $\frac{1}{2} < 1 - \frac{1}{\theta} < 1 \Longleftrightarrow \frac{F}{2} < \frac{F}{2\left(1-\frac{1}{\theta}\right)} < F$. Hence, $-\gamma < \frac{F}{2}$.

does not need to be too large.

Turning now to welfare, a policy of subsidizing innovators (although financed by taxing young consumers) has positive effects in that it promotes output growth along the transitional adjustment path which, in turn, leads to higher social welfare.

Proposition 4 *(Growth and Welfare properties under subsidy on innovators)*
For any $-\frac{F}{k^{**}} < \gamma < \gamma^*$, (i) The average growth rate of the economy under subsidy on innovators is higher than the average growth rate of the economy under laissez faire;(ii) Welfare under subsidy on innovators is higher than welfare under laissez faire in the 'Romer regime'.

Proof: See appendix.

The basic intuition is as follows. Consider that the economy is in the 'Romer regime' and that, under laissez faire, it exhibits equilibrium two-period cycles between the two growth regimes. Hence a period of low growth (i.e. $k_L < k^{**}$), corresponding to an equilibrium $\{k_L, k_H\}$ situated in the Solow regime and to an increase in saving (i.e. $\Lambda(k_L) > \Lambda(k^{**})$), is followed by a period of high growth (i.e. $k_H > k^{**}$) corresponding to an equilibrium in the 'Romer' regime. This is due to the a-synchronization between innovation and investment activities that characterizes this model (see Introduction). In fact when saving increases, while the equilibrium is in the low growth regime (i.e. the 'Solow' regime), more resources are directed to the production of 'old' intermediate goods to the detriment of the innovative sector which becomes less profitable. This effect is reversed in the following period. If, however, saving were directed towards innovation, prolonged sustained growth is possible. Therefore, a policy aimed at taxing the young and distributing the proceeds to the innovators allows the economy to remain in the 'Romer' regime and achieve stability. It also increases the amount of goods available for both young and old. As a result, total welfare in the economy is higher. In other words, in our economy reallocating resources from the consumer to innovators affects the properties of the balanced growth path of the final output. More precisely, the economy

switches from an equilibrium in which it cycles between high innovation (and low factor accumulation) and low innovation (and high factor accumulation), to the regime with innovation and sustained growth. In the latter, resources would not be devoted to the production of 'old' products and the economy would smoothly converge to higher sustained (stable) growth.

5 Conclusions

In this paper we have presented an OLG economy exhibiting sustained growth through the implementation of a simple redistribution principle (i.e., subsidies to innovative sectors financed by a lump sum tax on young consumers). The OLG structure makes it possible to: (i) account for the duration of innovation driven cycles, typically longer than high frequency short run business cycles, (ii) study the dynamic properties of the economy by use of a one-dimensional map, which makes the analysis simple and straightforward, (iii) explain how the reallocation of resources between sectors and consumers affects the generational exchanges.

Taxing the young and redistributing the proceeds to the innovative sectors brings stability and increases the amount of goods available for both generations; hence it also increases total welfare of the economy. This suggests that fast-growing economies, in which high factor accumulation plays a crucial role alongside innovative sectors that enjoy temporary monopoly rents, should follow rather unorthodox policies when they are facing economic slumps.

APPENDIX

Proof of Proposition 1

By direct inspection of (13): (i) $\Lambda'_-(k^{**}) < \Lambda'_{LF}(k^{**})$, and (ii) $\Lambda'_-(k^{**})$ is decreasing in γ.

To proof existence we follow Mastuyama (1999). Recall first that $\Lambda'_-(k^{**}) < -1 < \Lambda'_{LF}(k^{**})$. For a period-2 cycle to exist it suffices to show that $H(k) \equiv \Lambda^2(k) - k = 0$ has a solution other than $k = k^{**}$. Since (see Fig. 3 at the end of the Appendix), $[\Lambda^2(k_{cr}), \Lambda(k_{cr})]$ is in the trapping region, $H[\Lambda^2(k_{cr})] = \Lambda^4(k_{cr}) - \Lambda^2(k_{cr}) \geq 0$ and $H(k_{cr}) = \Lambda^2(k_{cr}) - k_{cr} < 0$, hence, $H(k_{cr}) = 0$ has a solution in $[\Lambda^2(k), k_{cr}]$.∎

Proof of Proposition 2

Assume $k_{t-2} = k_H$, $k_{t-1} = k_L$, $k_t = k_H$, $k_{t+1} = k_L$, $k_{t+2} = k_H$ and so on, where k_L is situated in the 'Romer regime' and k_H is situated in the 'Solow regime' (see, e.g., Figure 1 in Section 4). This assumption implies that under our policy rule, see (11), $T_{t-2} = 0$, $T_{t-1} = \gamma(k^{**} - k_L)$, $T_t = 0$, $T_{t+1} = \gamma(k^{**} - k_L)$ and so on.

(i) In view of the above, and given the dynamics as in (9)-(10), the rates of growth of all relevant variables when the economy fluctuates every other period between the two regimes are,

$$g_{N_{Solow}} \equiv \frac{N_t}{N_{t-1}} = 1$$

$$g_{N_{Romer}} \equiv \frac{N_{t+1}}{N_t} = 1 + \theta(k_H - 1) \quad if \ k_H \geq k^{**}$$

$$= 1 + \theta\left(\frac{k_H}{1 + \frac{\gamma}{F}(k^{**} - k_L)} - 1\right) \quad if \ k_H < k^{**},$$

$$g_{K_{Solow}} \equiv \frac{K_t}{K_{t-1}} = \frac{k_t}{k_{t-1}} \cdot \frac{N_t}{N_{t-1}} = \frac{k_H}{k_L} = \frac{G(k_L)^{1-\frac{1}{\sigma}}}{k_L} = G(k_L)^{-\frac{1}{\sigma}}$$

$$g_{K_{Romer}} \equiv \frac{K_{t+1}}{K_t} = \frac{k_{t+1}}{k_t} \cdot \frac{N_{t+1}}{N_t} = \frac{k_L}{k_H} \frac{N_{t+1}}{N_t} = \frac{k_L}{k_H} G \quad if \ k_H \geq k^{**}$$

$$= \frac{k_L}{k_H}\left(1 + \theta\left(\frac{k_H}{1 + \frac{\gamma}{F}(k^* - k_L)} - 1\right)\right) \quad if \ k_H < k^{**},$$

20

$$
\begin{aligned}
g_{Y_{Solow}} &\equiv \frac{Y_t}{Y_{t-1}} = G\left(k_L\right)^{-\frac{1}{\sigma}} \\
g_{Y_{Romer}} &\equiv \frac{Y_{t+1}}{Y_t} = G \quad if \ k_H \geq k^{**} \\
&= \frac{\mathcal{A}_t K_t}{\mathcal{A}_{t-1}\left(1+\frac{T_{t-1}}{F}\right)^{\frac{1}{\sigma}}\left(k_{t-1}\right)^{-\frac{1}{\sigma}}K_{t-1}} = \frac{\left(1+\frac{T_t}{F}\right)^{-\frac{1}{\sigma}}}{\left(k_L\right)^{-\frac{1}{\sigma}}} \cdot \frac{k_H}{k_L} \\
&= \left(1+\frac{\gamma}{F}(k^{**}-k_H)\right)^{-\frac{1}{\sigma}}.G \quad if \ k_H < k^{**}
\end{aligned}
$$

Under laissez faire, $k > k_{cr}$, and the rate of growth of output is $g_{Y_{Romer}} = G$; while under taxes on innovators, $k_L < k_{cr} < k_H$, and the average growth rate of output over the two-period cycle corresponds to,

$$
g_{Y_{Tax}} \equiv \left(g_{Y_{Solow}}.g_{Y_{Romer}}\right)^{\frac{1}{2}} = G(k_L)^{-\frac{1}{2\sigma}}.
$$

To demonstrate that the average growth rate is higher under laissez faire than over the cycle it suffices to show that $(k_L)^{-\frac{1}{2\sigma}} < 1$. This condition is always verified since, under the tax on innovators, the critical point moves from a value of 1 to a value strictly higher than one, implying $k_L > 1$ and, therefore, $(k_L)^{-\frac{1}{2\sigma}} < 1$. ∎

(ii) *Welfare under laissez faire in the 'Romer regime'.* It is given by the social welfare function (14) evaluated at the laissez faire equilibrium, $k^{**} = \frac{G-1}{\theta} + 1$, i.e.

$$
\begin{aligned}
SW_{LF} &= \Omega + \ln S^{**}\left(\frac{2+\rho}{1+\rho}\right)\sum_{t=0}^{n-1}\left(\frac{1}{1+\rho}\right)^t \\
&= \Omega + \ln S^{**}\left(\frac{2+\rho}{1+\rho}\right)\left[\sum_{t=0}^{n-1}\left(\frac{1}{1+\rho}\right)^{2t}+\sum_{t=0}^{n-1}\left(\frac{1}{1+\rho}\right)^{2t+1}\right],
\end{aligned}
$$

where $S^{**} = \frac{s}{\sigma}Y^{**}$ is evaluated at the steady state k^{**}.

Welfare under taxes on innovators. Since the economy fluctuates every other period between the two regimes, the social welfare function (14) can be written as

$$
SW_{Tax} = \Omega+\ln S_{Romer}\left(\frac{2+\rho}{1+\rho}\right)\sum_{t=0}^{n-1}\left(\frac{1}{1+\rho}\right)^{2t}+\ln S_{Solow}\left(\frac{2+\rho}{1+\rho}\right)\sum_{t=0}^{n-1}\left(\frac{1}{1+\rho}\right)^{2t+1},
$$

where $S_{Solow} = \frac{s}{\sigma}Y^*$ and $S_{Romer} = \frac{s}{\sigma}Y^{**}$ are evaluated at their respective steady state, k^* and k^{**}.

Subtracting SW_{LF} and SW_{Tax} , and disregarding the positive constant $\left(\frac{2+\rho}{1+\rho}\right)$, gives,

$$SW_{LF} - SW_{Tax} = \left(\ln\frac{S^{**}}{S_{Romer}}\right)\sum_{t=0}^{n-1}\left(\frac{1}{1+\rho}\right)^{2t} + \left(\ln\frac{S^{**}}{S_{Solow}}\right)\sum_{t=0}^{n-1}\left(\frac{1}{1+\rho}\right)^{2t+1}.$$

The first term on the r.h.s. of this equality is positive since at equilibrium (with $k_H > k^{**} > k_{cr}$), $\ln\frac{S^{**}}{S_{Romer}} = \ln(g_{Y_{Romer}}) = \ln G > 0$. Since $k_L < k_{cr} < k^{**}$, the sign of the second term on the r.h.s. depends on the sign of the growth rate over the two-period cycle, i.e., $\ln\frac{S^{**}}{S_{Solow}} = \ln(g_{Y_{Tax}}) = \ln G (k_L)^{-\frac{1}{2\sigma}}$. The latter is positive as long as $G(k_L)^{-\frac{1}{2\sigma}} > 1$, which is equivalent to $k_L < G^{2\sigma}$. To demonstrate that the latter inequality is verified, first, recall that $\sigma \in (1,\infty)$ and that, under the tax on innovators, the critical point moves from a value of 1 to a value strictly higher than one implying, $1 < k_L < k^{**}$. Hence, if we can show that $k^{**} < G^2$, it also follows that $k_L < G^{2\sigma}$. By use of the expression for k^{**} we get, $k^{**} - G^2 = (G-1)(1/\theta - 1 - G) < 0$ since $G > 1$ and $\theta \in (1, 2.71)$. ∎

Proof of Proposition 3

Let us note that for any $\gamma > -\frac{F}{k^{**}}$, $k_{cr} < 1 < k^{**}$. Therefore, if Λ is increasing in k in the Romer regime, then $\Lambda(k_{cr}) < \Lambda(1) < \Lambda(k^{**})$. In addition, for any $k \in (k_{cr}, k^{**})$, $\Lambda(k) > k$ then $k_{t+1} = \Lambda(k_t) > k_t$ and the economy monotonically converges to the steady state- see Figure 2. Formally, it suffices to show that $0 < \Lambda'_-(k^{**}) < 1$. Using (13), and re-arranging terms, we have $\frac{\frac{\theta-1}{G}}{(1-\frac{1}{\sigma})k^{**}+(\frac{\theta-1}{G})k^{**}+\frac{s}{\theta\sigma}\left(\frac{G-1}{G}\right)} < -\frac{\gamma}{F} < \frac{1+\frac{\theta-1}{G}}{(1-\frac{1}{\sigma})k^{**}+(\frac{\theta-1}{G})k^{**}+\frac{s}{\theta\sigma}\left(\frac{G-1}{G}\right)}$ Since we assume $\gamma > -\frac{F}{k^{**}}$, then, $-\frac{\gamma}{F} < \frac{1}{k^{**}}$. It can be easily checked that $\frac{1}{k^{**}} < \frac{1+\frac{\theta-1}{G}}{(1-\frac{1}{\sigma})k^{**}+(\frac{\theta-1}{G})k^{**}+\frac{s}{\theta\sigma}\left(\frac{G-1}{G}\right)}$. In fact, by simple manipulations, and substituting for $\frac{G-1}{\theta} = (k^{**} - 1)$, the latter inequality reduces to $-\frac{1}{\sigma}\left(1-\frac{s}{G}\right)k^{**} - \frac{s}{G} < 0$, which is always true since $0 < s < 1$ and $G > 1$. Therefore, $\frac{\frac{\theta-1}{G}}{(1-\frac{1}{\sigma})k^{**}+(\frac{\theta-1}{G})k^{**}+\frac{s}{\theta\sigma}\left(\frac{G-1}{G}\right)} < -\frac{\gamma}{F} < \frac{1}{k^{**}}$ suffices to show that there is monotonic convergence towards the steady state. Indeed,

$\frac{1}{k^{**}} > \frac{\frac{\theta-1}{G}}{(1-\frac{1}{\sigma})k^{**}+(\frac{\theta-1}{G})k^{**}+\frac{s}{\theta\sigma}\left(\frac{G-1}{G}\right)} \iff (1-\frac{1}{\sigma})k^{**} + \frac{s}{\theta\sigma}\left(\frac{G-1}{G}\right) > 0$, which is always verified. ∎

Proof of Proposition 4

Here, we assume that under laissez faire the economy exhibits equilibrium two-period cycles, with $k_L < 1 < k^{**} < k_H$, and where k_L is situated in the 'Solow regime' and k_H is situated in the 'Romer regime'(see Figure 2 in section 4). As for the case of taxes on innovators (cf. proof of Proposition 2) we assume that the economy starts at $k_{t-1} = k_L$, implying that a subsidy $T_{t-1} = \gamma(k^{**} - k_L)$, with $\gamma > -\frac{F}{k^{**}}$, is implemented at the outset of period t.

(i) From the section devoted to the proof of Proposition 2, we know the expressions for g_Y under laissez faire and under subsidy/tax. Hence, $g_{Y_{LF}} - g_{Y_{Sub}} = (g_{Y_{Solow}} \cdot g_{Y_{Romer}})^{\frac{1}{2}} - g_{Y_{Sub}} = G\left[(k_L)^{-\frac{1}{2\sigma}} - \left(1 + \frac{\gamma}{F}(k^{**} - k_L)\right)^{-\frac{1}{\sigma}}\right]$. If $g_{Y_{LF}} - g_{Y_{Sub}} < 0$ then, $(k_L)^{-\frac{1}{2\sigma}} < \left(1 + \frac{\gamma}{F}(k^{**} - k_L)\right)^{-\frac{1}{\sigma}} \iff 1 + \frac{\gamma}{F}(k^{**} - k_L) < (k_L)^{\frac{1}{2}} \iff \frac{1+\frac{\gamma}{F}k^{**}}{1+\frac{\gamma}{F}} < \frac{k_L\left(k_L^{-\frac{1}{2}}+\frac{\gamma}{F}\right)}{1+\frac{\gamma}{F}}$ where $1 + \frac{\gamma}{F} > 0$. The l.h.s. of the latter inequality, $\frac{1+\frac{\gamma}{F}k^{**}}{1+\frac{\gamma}{F}}$, is always lower than one since $\gamma > -\frac{F}{k^{**}}$. Compare, now, the r.h.s of the inequality with k_L, knowing that $k_L > k_{cr} = \frac{1+\frac{\gamma}{F}k^{**}}{1+\frac{\gamma}{F}}$. Since we have assumed $G < \theta - 1$ under laissez faire, i.e. cycles of period two, we know that $k_L < 1$. Now, let us show that $\frac{k_L^{-\frac{1}{2}}+\frac{\gamma}{F}}{1+\frac{\gamma}{F}} > 1 \iff k_L^{-\frac{1}{\sigma}} > 1$, which is always true since $k_L < 1$. Therefore $g_{Y_{LF}} < g_{Y_{Sub}}$. ∎

(ii) To sign the welfare effects, it is convenient to derive first the expressions for the average growth rates over the two period cycle at the outset of period t. Recalling that $k_{t-1} = k_L$ and $T_{t-1} = \gamma(k^{**} - k_L)$, and by use of the expressions for the growth rates derived in the section devoted to the proof of Proposition 2, we obtain $g_{Y_{Romer}} = \sqrt{(G) \cdot \left(G \cdot \left(1 + \frac{\gamma}{F}(k^{**} - k_L)\right)^{-\frac{1}{\sigma}}\right)}$ and $g_{Y_{Solow}} = \sqrt{\left(G \cdot (k_L)^{-\frac{1}{\sigma}}\right) \cdot \left(G \cdot \left(1 + \frac{\gamma}{F}(k^{**} - k_L)\right)^{-\frac{1}{\sigma}}\right)}$.

Welfare under laissez faire in the 'Romer regime'. Under laissez faire the economy exhibits equilibrium two-period cycles, hence the social welfare function (14)

amounts to

$$SW_{LF} = \Omega + \ln S_{Romer} \left(\frac{2+\rho}{1+\rho}\right) \sum_{t=0}^{n-1} \left(\frac{1}{1+\rho}\right)^{2t} + \ln S_{Solow} \left(\frac{2+\rho}{1+\rho}\right) \sum_{t=0}^{n-1} \left(\frac{1}{1+\rho}\right)^{2t+1},$$

where $S_{Romer} = \frac{s}{\sigma} Y^{**}$ and $S_{Solow} = \frac{s}{\sigma} Y^{*}$ are evaluated at their respective steady state, k^{**} and k^{*}.

Welfare under subsidies on innovators. In this case, $k_{cr} < k_L < 1 < k^{**}$, and the economy monotonically converges towards the steady state in the 'Romer regime' (see Figure 2 in section 4). The social welfare function (14) becomes

$$
\begin{aligned}
SW_{Sub} &= \Omega + \sum_{t=0}^{n-1} \left(\frac{1}{1+\rho}\right)^{t} \ln S_{k_{L,t}} + \sum_{t=0}^{n-1} \left(\frac{1}{1+\rho}\right)^{t+1} \ln S_{k_{L,t}} \\
&= \Omega + \left(\frac{2+\rho}{1+\rho}\right) \sum_{t=0}^{n-1} \left(\frac{1}{1+\rho}\right)^{2t} \ln S_{k_{L,2t}} + \left(\frac{2+\rho}{1+\rho}\right) \sum_{t=0}^{n-1} \left(\frac{1}{1+\rho}\right)^{2t+1} \ln S_{k_{L2,t+1}},
\end{aligned}
$$

where $S_{k_{L,t}} = \frac{s}{\sigma} Y_t$, with $Y_t = \mathcal{A}_{t-1} K_{t-1}$ (cf. (8)) and $t = 0, 1, ..., n-1$.

Subtracting SW_{Sub} and SW_{LF}, and disregarding all terms in $\left(\frac{2+\rho}{1+\rho}\right)$, gives,

$$SW_{Sub} - SW_{LF} = \sum_{t=0}^{n-1} \left(\frac{1}{1+\rho}\right)^{2t} \left(\ln \frac{S_{k_{L,2t}}}{S_{Romer}}\right) + \sum_{t=0}^{n-1} \left(\frac{1}{1+\rho}\right)^{2t+1} \left(\ln \frac{S_{k_{L,2t+1}}}{S_{Solow}}\right).$$

To sign the above we proceed as follows. First, note that at $t = 0$, $\ln \frac{S_{k_L}}{S_{Romer}} = \ln \frac{Y_{k_L}}{Y_{Romer}} = \ln\left(\sqrt{G^2 \cdot \left(1 + \frac{\gamma}{F}(k^{**} - k_L)\right)^{-\frac{1}{\sigma}}}\right)$ and $\ln \frac{S_{k_{L+1}}}{S_{Solow}} = \ln \frac{Y_{k_{L+1}}}{Y_{Solow}}$

$= \ln\left(\sqrt{\left(G \cdot (k_L)^{-\frac{1}{\sigma}}\right) \cdot \left(G \cdot \left(1 + \frac{\gamma}{F}(k^{**} - k_L)\right)^{-\frac{1}{\sigma}}\right)}\right)$. The term $\ln \frac{Y_L}{Y_{Romer}}$ is positive if $G^2 \cdot \left(1 + \frac{\gamma}{F}(k^{**} - k_L)\right)^{-\frac{1}{\sigma}} > 1$. The l.h.s. of this inequality is always higher than one since $G > 1$ and $0 < 1 + \frac{\gamma}{F}(k^{**} - k_L) < 1$; hence, $\ln \frac{Y_L}{Y_{Romer}} > 0$. Moreover, since the economy monotonically converges towards the equilibrium k^{**}, $\ln \frac{Y_L}{Y_{Romer}} > 0$ is satisfied for any $k_{L,2t}$, with $t = 0, 1, ..., n-1$. Turning to $\ln \frac{Y_{k_{L+1}}}{Y_{Solow}}$, this is positive if $G^2 \cdot (k_L)^{-\frac{1}{\sigma}} \cdot \left(1 + \frac{\gamma}{F}(k^{**} - k_L)\right)^{-\frac{1}{\sigma}} > 1$. The l.h.s. of this inequality is always higher than one since $G > 1$, $k_L < 1$ and $0 < 1 + \frac{\gamma}{F}(k^{**} - k_L) < 1$; hence, $\ln \frac{Y_{k_{L+1}}}{Y_{Solow}} > 0$. The latter holds for any $k_{L,2t+1}$, with $t = 0, 1, ..., n-1$. ∎

k_t

$k_t = k_{t-1}$

$\Lambda(k_{t-1})$

$\Lambda(k_{cr})$
$\Lambda^3(k_{cr})$

$\Lambda^4(k_{cr})k_c$
$\Lambda^2(k_{cr})$

$\Lambda_{LF}(k_{t-1})$

$\Lambda^2(k_{cr})- k_{cr} > 0$

$1 \quad \Lambda^2(k_{cr}) \; k_{cr} \; k^{**} \quad \Lambda^3(k_{cr}) \; \Lambda(k_{cr})$

k_{t-1}

$\Lambda^2(k_{cr})- k_{cr} < 0$

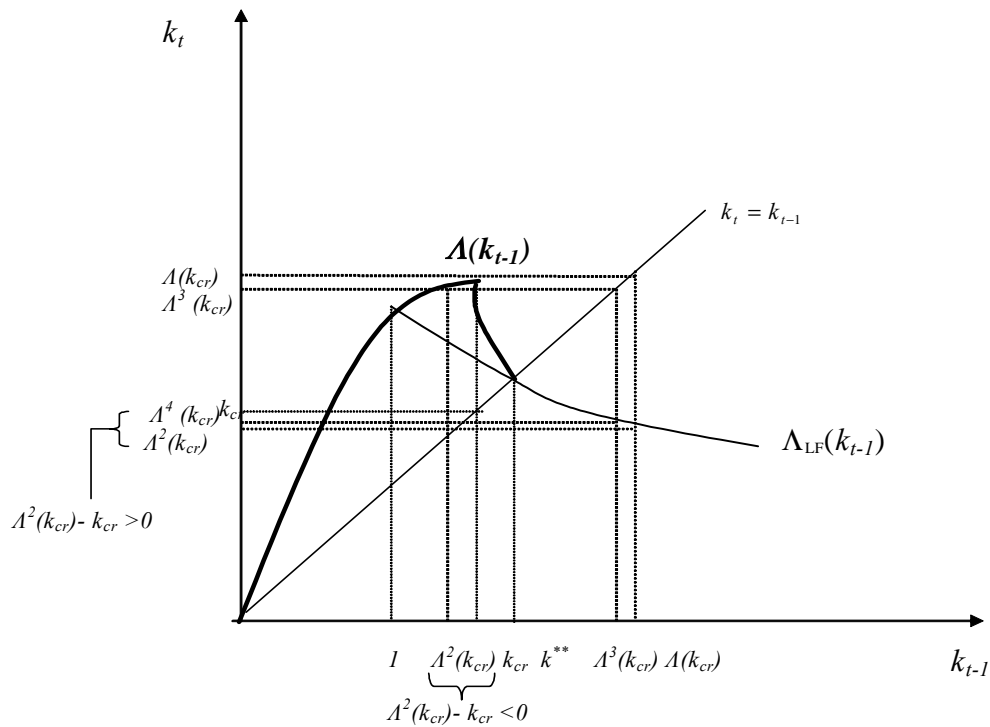Figure 3: Period-2 cycles, $\gamma > 0$ and $G > \theta - 1$

References

[1] Aghion, P. and P. Howitt (1992), ”A Model of Growth Through Creative De-struction”, *Econometrica* 60, 323-351.

[2] Aghion, P. and P. Howitt (1997), *Endogenous Growth Theory*, MIT Press.

[3] Aloi, M., T. Lloyd-Braga and H. J. Whitta-Jacobsen (2003), ”Endogenous Business Cycles and Systematic Stabilization Policies”, *International Economic Review* 44, 895-915.

[4] Crafts, N. (1999), ”Implications of financial crisis for East Asian trend growth”, *Oxford Review of Economic Policy* 15, 110-131.

[5] Christiano, L. and S. G. Harrison (1999), ”Chaos, sunspots and automatic stabilizers”, *Journal of Monetary Economics* 44, 3-31.

[6] Davidson, C. and P. Segerstrom (1998), "R&D subsidies and economic growth", *Rand Journal of Economics* 29, 548-577.

[7] Deneckere, R. and K. Judd (1992), "Cyclical and Cahotic Behavior in a Dynamic Equilibrium Model, with Implications for Fiscal Policy", in J. Benhabib (ed.) *Cycles and Chaos in Economic Equilibrium*, Princeton University Press.

[8] Dornbusch, R. (1995), "Why is Japan Shooting Itself in the Foot?", *Business Week*, 8 May.

[9] Francois, P. and H. Lloyd-Ellis (2003), "Animal Spirits Through Creative Destruction", *American Economic Review* 93, 530-550.

[10] Grandmont, J. M. (1985), "Endogenous Competitive Business Cycles", *Econometrica* 53, 741-783.

[11] Grandmont, J. M. (1986), "Stabilizing Competitive Buisness Cycles", *Journal of Economic Theory* 40, 57-76.

[12] Guo, J. T. and K. J. Lansing (1998), "Indeterminacy and Stabilization Policy", *Journal of Economic Theory* 82, 481-490.

[13] Ito, T. (1995), "Japan and the Asian Economics: A 'Miracle' in Transition", *Brookings Papers on Economic Activity* 2, 205-272.

[14] Mathews, J. A. and D-S Cho (2000), *Tiger Technology: The Creation of a Semiconductor Industry in East Asia*, Cambridge University Press.

[15] Matsuyama, K. (1996), "Growing through cycles", *CMS-EMS Working Paper* N. 1203, Northwestern University.

[16] Matsuyama, K. (1999), "Growing through cycles", *Econometrica* 67, 335-347.

[17] Odagiri, H. and A. Goto (1993), "The Japanese System of Innovation: Past, Present and Future", in R. Nelson (ed.) *National Innovation Systems*, Oxford University Press.

[18] Romer, P. M. (1986), "Increasing returns and long run growth" *Journal of Political Economy* 94, 1002-37.

[19] Romer, P. M. (1990), "Endogenous technological change" *Journal of Political Economy* 98, S71-S102.

[20] Segerstrom, P. (2000), "The Long-Run Effects of R&D Subsidies", *Journal of Economic Growth* 5, 277-305.

[21] Schmitt-Grohe, S. and M. Uribe (1997), "Balanced-Budget Rules, Distortionary Taxes and Aggregate Stability", *Journal of Political Economy* 105, 976-1000.

[22] Shleifer, A. (1986), "Implementation Cycles", *Journal of Political Economy* 94, 1163-1190.

[23] Young, A. (1995), "The Tiranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience", *Quarterly Journal of Economics* 110, 641-680.

ABOUT THE CDMA

The **Centre for Dynamic Macroeconomic Analysis** was established by a direct grant from the University of St Andrews in 2003. The Centre funds PhD students and facilitates a programme of research centred on macroeconomic theory and policy. The Centre has research interests in areas such as: characterising the key stylised facts of the business cycle; constructing theoretical models that can match these business cycles; using theoretical models to understand the normative and positive aspects of the macroeconomic policymakers' stabilisation problem, in both open and closed economies; understanding the conduct of monetary/macroeconomic policy in the UK and other countries; analyzing the impact of globalization and policy reform on the macroeconomy; and analyzing the impact of financial factors on the long-run growth of the UK economy, from both an historical and a theoretical perspective. The Centre also has interests in developing numerical techniques for analyzing dynamic stochastic general equilibrium models. Its affiliated members are Faculty members at St Andrews and elsewhere with interests in the broad area of dynamic macroeconomics. Its international Advisory Board comprises a group of leading macroeconomists and, ex officio, the University's Principal.

Affiliated Members of the School

Dr Arnab Bhattacharjee.
Dr Tatiana Damjanovic.
Dr Vladislav Damjanovic.
Dr Laurence Lasselle.
Dr Peter Macmillan.
Prof Kaushik Mitra.
Prof Charles Nolan (Director).
Dr Gary Shea.
Prof Alan Sutherland.
Dr Christoph Thoenissen.

Senior Research Fellow

Prof Andrew Hughes Hallett, Professor of Economics, Vanderbilt University.

Research Affiliates

Prof Keith Blackburn, Manchester University.
Prof David Cobham, Heriot-Watt University.
Dr Luisa Corrado, Università degli Studi di Roma.
Prof Huw Dixon, York University.
Dr Anthony Garratt, Birkbeck College London.
Dr Sugata Ghosh, Brunel University.
Dr Aditya Goenka, Essex University.
Dr Campbell Leith, Glasgow University.
Dr Richard Mash, New College, Oxford.
Prof Patrick Minford, Cardiff Business School.
Dr Gulcin Ozkan, York University.
Prof Joe Pearlman, London Metropolitan University.
Prof Neil Rankin, Warwick University.
Prof Lucio Sarno, Warwick University.

Prof Eric Schaling, Rand Afrikaans University.
Prof Peter N. Smith, York University.
Dr Frank Smets, European Central Bank.
Dr Robert Sollis, Durham University.
Dr Peter Tinsley, George Washington University and Federal Reserve Board.
Dr Mark Weder, University of Adelaide.

Research Associates

Mr Nikola Bokan.
Mr Michal Horvath.
Ms Elisa Newby.
Mr Qi Sun.
Mr Alex Trew.

Advisory Board

Prof Sumru Altug, Koç University.
Prof V V Chari, Minnesota University.
Prof John Driffill, Birkbeck College London.
Dr Sean Holly, Director of the Department of Applied Economics, Cambridge University.
Prof Seppo Honkapohja, Cambridge University.
Dr Brian Lang, Principal of St Andrews University.
Prof Anton Muscatelli, Glasgow University.
Prof Charles Nolan, St Andrews University.
Prof Peter Sinclair, Birmingham University and Bank of England.
Prof Stephen J Turnovsky, Washington University.
Mr Martin Weale, CBE, Director of the National Institute of Economic and Social Research.
Prof Michael Wickens, York University.
Prof Simon Wren-Lewis, Exeter University.

RECENT WORKING PAPERS FROM THE
CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS

Number	Title	Author(s)
CDMA05/08	Financial Market Analysis Can Go Mad (in the search for irrational behaviour during the South Sea Bubble)	Gary S. Shea (St Andrews)
CDMA05/09	Some Welfare Implications of Optimal Stabilization Policy in an Economy with Capital and Sticky Prices	Tatiana Damjanovic (St Andrews) and Charles Nolan (St Andrews)
CDMA05/10	Optimal Monetary Policy Rules from a Timeless Perspective	Tatiana Damjanovic (St Andrews), Vladislav Damjanovic (St Andrews) and Charles Nolan (St Andrews)
CDMA05/11	Money and Monetary Policy in Dynamic Stochastic General Equilibrium Models	Arnab Bhattacharjee (St Andrews) and Christoph Thoenissen (St Andrews)
CDMA05/12	Understanding Financial Derivatives During the South Sea Bubble: The case of the South Sea Subscription shares	Gary S. Shea (St Andrews)
CDMA06/01	Sticky Prices and Indeterminacy	Mark Weder (Adelaide)
CDMA06/02	Independence Day for the "Old Lady": A Natural Experiment on the Implications of Central Bank Independence	Jagjit S. Chadha (BNP Paribas and Brunel), Peter Macmillan (St Andrews) and Charles Nolan (St Andrews)
CDMA06/03	Sunspots and Monetary Policy	Jagjit S. Chadha (BNP Paribas and Brunel) and Luisa Corrado (Cambridge and Rome Tor Vergata)
CDMA06/04	Labour and Product Market Reforms in an Economy with Distortionary Taxation	Nikola Bokan (St Andrews and CEBR), Andrew Hughes Hallett (Vanderbilt and CEPR)
CDMA06/05	Sir George Caswall vs. the Duke of Portland: Financial Contracts and Litigation in the wake of the South Sea Bubble	Gary S. Shea (St Andrews)
CDMA06/06	Optimal Time Consistent Monetary Policy	Tatiana Damjanovic (St Andrews), Vladislav Damjanovic (St Andrews) and Charles Nolan (St Andrews)

CDMA06/07	Optimal Stationary Monetary and Fiscal Policy under Nominal Rigidity	Michal Horvath (St Andrews)
CDMA06/08	Bank Lending with Imperfect Competition and Spillover Effects	Sumru G. Altug (Koç and CEPR) and Murat Usman (Koç)
CDMA06/09	Real Exchange Rate Volatility and Asset Market Structure	Christoph Thoenissen (St Andrews)
CDMA06/10	Disinflation in an Open-Economy Staggered-Wage DGE Model: Exchange-Rate Pegging, Booms and the Role of Preannouncement	John Fender (Birmingham) and Neil Rankin (Warwick)
CDMA06/11	Relative Price Distortions and Inflation Persistence	Tatiana Damjanovic (St Andrews) and Charles Nolan (St Andrews)
CDMA06/12	Taking Personalities out of Monetary Policy Decision Making? Interactions, Heterogeneity and Committee Decisions in the Bank of England's MPC	Arnab Bhattacharjee (St Andrews) and Sean Holly (Cambridge)
CDMA07/01	Is There More than One Way to be E-Stable?	Joseph Pearlman (London Metropolitan)
CDMA07/02	Endogenous Financial Development and Industrial Takeoff	Alex Trew (St Andrews)
CDMA07/03	Optimal Monetary and Fiscal Policy in an Economy with Non-Ricardian Agents	Michal Horvath (St Andrews)
CDMA07/04	Investment Frictions and the Relative Price of Investment Goods in an Open Economy Model	Parantap Basu (Durham) and Christoph Thoenissen (St Andrews)
CDMA07/05	Growth and Welfare Effects of Stablizing Innovation Cycles	Marta Aloi (Nottingham) and Laurence Lasselle (St Andrews)

For information or copies of working papers in this series, or to subscribe to email notification, contact: