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# Real Exchange Rate Volatility and Asset Market Structure*

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**ABSTRACT**

We examine the influence of financial asset market structure for the volatility of the real exchange rate. Two-country models with low elasticities of substitution between home and foreign-produced traded goods, or models with non-traded distribution costs have been shown yield realistic levels of terms of trade and real exchange rate volatility. We argue that incomplete financial markets are a necessary condition for the terms of trade and real exchange rate to display realistic levels of volatility. We also illustrate that for some parameter values, how one models incomplete markets also matters for international business cycle properties of the these models.

**JEL Classification:** F31, F41.
**Keywords:** Real exchange rate volatility, financial market structure, non-traded goods, distribution costs.

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# 1 Introduction

What is the effect of financial asset market structure on the dynamics of the real exchange rate? More precisely, do models that assume the presence of a complete set of state-contigent claims at the international level yield different predictions about real exchange rate dynamics than incomplete markets models?[1] In the context of small open economy models, Schmidt-Grohe and Uribe (2003) argue that financial asset market structure does not matter for business cycle properties of key macroeconomic variables. In this paper, we examine if their findings hold for two-country, two-sector models in the Stockman and Tesar (1995) tradition. Engel (1999) has shown most of the variability of the real exchange rate comes not from the relative price of non-traded goods, but from deviations from the law-of-one-price. A theoretical model that illustrates this is Chari, Kehoe and McGrattan (2002). They show that a sticky price model where firms set prices in local currency can yield volatile real exchange rates if the model is driven by monetary shocks. Kollmann (2005) and Benigno and Thoenissen (2006) find that in their models, productivity shocks alone can not account for the volatility of the real exchange rate.

Following a seminal contribution by Burstein, Neves and Rebelo (2003), researchers have started to include a distribution sector into open economy macroeconomics model of the Stockman and Tesar (1995) tradition. The at first sight innocuous addition of a sector that distributes intermediate goods from suppliers to final goods producers has been shown to have remarkable effects on the volatility of the real exchange rate, as well as resolving a number of other international macroeconomics puzzles.

Dotsey and Duarte (2006) show that non-traded goods that can be used both as consumption as well as distribution goods can increase the volatility of both the real exchange rate and the terms of trade. Ovideo and Singh (2006) show that distribution costs can help to address the quantity anomaly. Preceding these works, using a flexible price, two-country two-sector model with non-traded consumption goods and distribution services, Corsetti, Dedola and Leduc (2004) are able to solve a number of the outstanding puzzles of international macroeconomics. Driven only by sector specific productivity shocks, their model is able to correctly predict the volatility of the real exchange rate, its correlation with relative consumption (Backus-Smith puzzle) and the terms of trade; as well as successfully addressing the quantity anomaly.

In this paper, we argue that as far as the volatility of the real exchange is concerned, the key factor in these models is not just the presence of distribution costs, but also the structure of the financial asset market. Distribution costs have been shown to lower the elasticity of substitution between home and foreign-produced goods. By lowering this elasticity, the terms of trade, defined as the relative price of foreign to home-produced goods, become more volatile in the presence of supply shocks, which translates into higher real exchange rate volatility. In the presence of a complete set of state-contingent claims at the international level, the magnitude and direction of terms of trade changes is constrained to what is necessary to completely share country-specific risk between the home and foreign country. In a bond economy, where agents are constrained to trade in a risk-less non state-contingent bond, this mechanism is absent. In Corsetti *et al* (2004) and Dotsey and Duarte (2006), it is the assumption of incomplete financial markets that generates high

[1] Specifically, we are considering simple bond economies Recent work by Devereux and Sutherland (2006) shows how to model incomplete asset market models with a richer asset structure.

real exchange rate volatility.

Schmidt-Grohe and Uribe (2003) show that as far as business cycle properties of small open economy models are concerned, financial asset market, whether compete or incomplete, and if so, how it is modelled, does not matter. In the two-country model context of this paper, we find only limited support for this proposition. We find that for certain values of the distribution cost parameter, $\psi$, and the elasticity of substitution between home and foreign-produced traded goods, $\theta$, a model with incomplete financial markets (a bond economy) generates significantly more exchange rate volatility than an equivalent model with complete financial markets. We also find that only some types of incomplete markets models (endogenous discount factor model, but not bond holding cost models) can generate a negative international transmission of supply shocks, whereas others (bond holding cost models) can not. Negative transmission is when the terms of trade appreciate in response to a positive supply shock, raising home purchasing power at the expense of foreign consumers.

So is it *veni, vedi, vici* for incomplete market models with distribution costs? Not quite. We show that the parameter space that allows these incomplete market models to generate realistic levels of real exchange rate volatility from supply side shocks alone is quite narrow. For slightly higher values of $\theta$ or lower values of $\psi$, both incomplete and complete financial markets models fail to generate any meaningful exchange rate volatility.

The remainder of the paper is structures as follows: section 2 sets out our baseline model and involves introducing a distribution sector into the model of Benigno and Thoenissen (2006). Section 3 sets out the calibration of structural parameters and shock processes used in our analysis. Sections 4 and 5 use impulse response functions to draw out and analyse the main mechanisms behind our results. Section 6 compares a set of second moments from our models, under different asset market structures to the data. Section 7 performs sensitivity checks on our results, while section 8 concludes.

## 2 A two-sector two-country model

The structure of this model follows closely Benigno and Thoenissen (2006), with the exception that we allow for a distribution, or retail sector that distributes goods from the intermediate goods sector to the final goods sector. This addition makes our model quite similar to that of Corsetti *et al* (2004). The basic structure of our model is related to the works of Chari, Kehoe and McGrattan (2002) and Stockman and Tesar (1995). There are three key modifications with respect to their baseline cases. Firstly we consider an incomplete market structure at the international level. Secondly, unlike Chari *et al.* (2002), but similar to Stockman and Tesar, we introduce non-tradeable intermediate inputs in the production process. Thirdly, we introduce a distribution sector as in Burstein, Neves and Rebelo (2003). Moreover, we focus on a perfectly competitive setting while Chari *et al* analyse an imperfectly competitive framework with staggered price setting behaviour.

### 2.1 Consumer Behaviour

We propose a two-country model with infinitely lived consumers. The world economy is populated by a continuum of agents on the interval $[0, 1]$. The population on the segment $[0, n)$ belongs to the country $H$ (Home), while the segment $[n, 1]$ belongs to $F$ (Foreign). Preferences for a generic Home-consumer are described by the following utility function:

$$U_t^j = \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s^j, (1 - h_s^j)) \tag{1}$$

where $\mathrm{E}_t$ denotes the expectation conditional on the information set at date $t$, while $\beta$ is the intertemporal discount factor, with $0 < \beta < 1$. The Home consumer obtains utility from consumption, $C^j$, and receives dis-utility from supplying labour, $h^j$.

In our baseline model, we assume that international asset markets are incomplete. The asset market structure in the model is relatively standard in the literature. We assume that Home individuals are assumed to be able to trade two nominal risk-less bonds denominated in the domestic and foreign currency. These bonds are issued by residents in both countries in order to finance their consumption expenditure. On the other hand, foreign residents can allocate their wealth only in bonds denominated in the foreign currency. [2] Home households face a cost (i.e. transaction cost) when they take a position in the foreign bond market. This cost depends on the net foreign asset position of the home economy as in Benigno (2001). In sections 4, 5, 6 and 7 we compare this baseline asset market structure with an asset market structure where a full set of state-contingent claims is assumed to exist, as well as an incomplete markets model with and endogenous discount factor, as described in Mendoza (1991). Domestic firms are assumed to be wholly owned by domestic residents, and profits are distributed equally across households. Consumer $j$ faces the following budget constraint in each period $t$:

$$P_t C_t^j + \frac{B_{H,t}^j}{(1 + i_t)} + \frac{S_t B_{F,t}^j}{(1 + i_t^*)\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)} = B_{H,t-1}^j + S_t B_{F,t-1}^j + P_t w_t h_t^j + \Pi_t^j \tag{2}$$

where $B_{H,t}^j$ and $B_{F,t}^j$ are the individual's holdings of domestic and foreign nominal risk-less bonds denominated in the local currency. $i_t$ is the Home country nominal interest rate and $i_t^*$ is the Foreign country nominal interest rate. $S_t$ is the nominal exchange rate expressed as units of domestic currency needed to buy one unit of foreign currency, $P_t$ is the consumer price level and $w_t$ is the real wage. $\Pi_t^j$ are dividends from holding a share in the equity of domestic firms obtained by agent $j$. All domestic firms are wholly owned by domestic agents and equity holding within these firms is evenly divided between domestic agents.

The cost function $\Theta(.)$ drives a wedge between the return on foreign-currency denominated bonds received by domestic and by foreign residents. We follow Benigno, P. (2001) in rationalising this cost by assuming the existence of foreign-owned intermediaries in the foreign asset market who apply a spread over the risk-free rate of interest when borrowing or lending to home agents in foreign currency. This spread depends on the net foreign asset position of the home economy. We assume that profits from this activity in the foreign asset market are distributed equally among foreign residents (see P. Benigno (2001)).[3]

[2] We want to highlight here the fact that this asymmetry in the financial market structure is made for simplicity. The results would not change if we allow home bonds to be traded internationally. We would need to consider a further arbitrage condition.

[3] Here we follow Benigno (2001) in assuming that the cost function $\Theta(.)$ assumes the value of 1 only when the

As in P. Benigno (2001), we assume that all individuals belonging to the same country have the same level of initial wealth. This assumption, along with the fact that all individuals face the same labour demand and own an equal share of all firms, implies that within the same country all individuals face the same budget constraint. Thus they will choose identical paths for consumption. As a result, we can drop the $j$ superscript and focus on a representative individual for each country.

The maximisation problem of the Home individual consists of maximising (1) subject to (2) in determining the optimal profile of consumption and bond holdings and the labour supply schedule. Households' equilibrium conditions (Home and Foreign) are described by the following equations:

$$U_C\left(C_t,(1-h_t)\right)=(1+i_t)\beta E_t\left[U_C\left(C_{t+1},(1-h_{t+1})\right)\frac{P_t}{P_{t+1}}\right] \tag{3}$$

$$U_C\left(C_t^*,(1-h_t^*)\right)=(1+i_t^*)\beta E_t\left[U_C\left(C_{t+1}^*,(1-h_{t+1}^*)\right)\frac{P_t^*}{P_{t+1}^*}\right] \tag{4}$$

$$U_C\left(C_t,(1-h_t)\right)=(1+i_t^*)\Theta\left(\frac{S_tB_{F,t}}{P_t}\right)\beta E_t\left[U_C\left(C_{t+1},(1-h_{t+1})\right)\frac{S_{t+1}P_t}{S_tP_{t+1}}\right] \tag{5}$$

$$\frac{U_l(C_t,(1-h_t))}{U_C\left(C_t,(1-h_t)\right)}=w_t \qquad \frac{U_l(C_t^*,(1-h_t^*))}{U_C\left(C_t^*,(1-h_t^*)\right)}=w_t^* \tag{6}$$

### 2.2 Producer behaviour

We let $Y$ be the output of final goods produced in the home country. Final goods producers combine home and foreign-produced intermediate goods, bought from the distribution sector, to produced $Y$ in the following manner:

$$Y\equiv C=\left[\omega^{\frac{1}{\kappa}}c_T^{\frac{\kappa-1}{\kappa}}+(1-\omega)^{\frac{1}{\kappa}}c_N^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}} \tag{7}$$

where $c_T$ and $c_N$ are the post-distribution intermediate traded and non-traded inputs and $\kappa$ is the elasticity of intratemporal substitution between traded and non-traded intermediate goods. The traded component is in turn produced using home and foreign-produced traded goods in the following manner:

$$c_T=\left[v^{\frac{1}{\theta}}c_H^{\frac{\theta-1}{\theta}}+(1-v)^{\frac{1}{\theta}}c_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{8}$$

where we denote with $c_H$ and $c_F$ the intermediate goods produced in the Home and Foreign countries respectively. $\theta$ is the elasticity of intratemporal substitution between home and foreign-produced intermediate goods.

net foreign asset position is at its steady state level, ie $B_{F,t}=\overline{B}$, and is a differentiable decreasing function in the neighbourhood of $\overline{B}$. This cost function is convenient because it allows us to log-linearise our economy properly since in steady state the desired amount of net foreign assets is always a constant $\overline{B}$. The expression for profits from financial intermediation is given by $K=\frac{B_{F,t}}{P_t^*(1+i_t^*)}\left[\frac{RS_t}{\Theta\left(\frac{S_tB_{F,t}}{P_t}\right)}-1\right]$.

Final goods producers and producers of the composite traded goods are competitive and maximise their profits:

$$\max_{c_N, c_T} PC - P_T c_T - P_N c_N \tag{9}$$

$$\max_{c_H, c_F} P_T c_T - P_H c_H - P_F c_F \tag{10}$$

subject to (7) and (8) respectively. This maximisation yields the following input demand functions for the home economy (similar conditions hold for Foreign producers)

$$c_N = (1-\omega)\left(\frac{P_N}{P}\right)^{-\kappa} C, \tag{11}$$

$$c_H = \omega v \left(\frac{P_H}{P_T}\right)^{-\theta} \left(\frac{P_T}{P}\right)^{-\kappa} C \quad c_F = \omega(1-v)\left(\frac{P_F}{P_T}\right)^{-\theta} \left(\frac{P_T}{P}\right)^{-\kappa} C$$

Corresponding to the previous demand function we have the following prices indexes:

$$P_T^{1-\theta} = [vP_H^{1-\theta} + (1-v)P_F^{1-\theta}] \tag{12}$$

$$P^{1-\kappa} = [\omega P_T^{1-\kappa} + (1-\omega)P_N^{1-\kappa}] \tag{13}$$

### 2.2.1 Distribution sector

In the context of our model, the distribution sector is located between the intermediate goods producers of traded goods and the final goods sector. The distribution sector is competitive, and consists of firms combining intermediate traded goods with non-traded distribution services. In order to bring one unit of traded intermediate goods to the final goods producers requires $\psi$ units of distribution services, which are made up of non-traded goods. Non-traded goods themselves are passed to the final good producers without passing through the distribution sector.[4] The profit function of the distribution sector is as follows:

$$\max_{y_H} P_H y_H - \tilde{P}_H y_H - P_N \psi y_H$$

where $\tilde{P}_H$ is the wholesale price of home-produced intermediate goods, $P_H$ is the consumer price of home-produced intermediate good and $P_N$ is the price of the distribution service. Profit maximisation yields:

$$P_H = \tilde{P}_H + \psi P_N \tag{14}$$

which implies that the retail price equals the wholesale price plus the distribution margin. An analogous relation applies the the imported intermediate goods:

$$P_F = \tilde{P}_F + \psi P_N \tag{15}$$

so that the retail price of traded goods becomes market specific even in a competitive model where the law-of-one-price holds at the wholesale level. Having transformed the wholesale goods the distributor now sells the goods to the final goods producers.

[4]This structure follows Burstein, Neves and Rebello (2003). Mulraine (2006) puts forward a model in which distrubtion requires capital as well as labour, instead of non-traded goods.

### 2.2.2 Intermediate goods sectors

Firms in the traded intermediate goods sector produce goods using capital and labour services. Domestic firms are owned by domestic households. The typical firm producing traded goods maximises the expected discounted value of profit:

$$\max_{h_{H,t},x_{H,t}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_C(C_t,(1-h_t))}{U_C(C_0,(1-h_0))} \frac{P_0}{P_t} \left[P_{H_t} y_{H_t} - P_t w_t h_{H,t} - \tilde{P}_{H,t} x_{H,t}\right] \quad (16)$$

where $h_{H,t}$ is the total labour supply employed in the domestic traded intermediate sector, $x_{H,t}$ denotes investment in the traded domestic sector. Note that distribution costs do not apply to investment goods. Our maximisation problem is constrained by the production function and the law of motion of capital:

$$y_{H_t} = F(k_{H,t-1}, h_{H,t}) = A_t {h_{H,t}}^{\alpha} k_{H,t-1}^{1-\alpha} \quad (17)$$

$$k_{H,t} = (1-\delta) k_{H,t-1} + x_{Ht} - \phi\left(\frac{x_{Ht}}{k_{Ht-1}}\right) k_{Ht-1}$$

where $\phi(.)$ is the cost for installing investment goods.[5] The first-order conditions at a generic time $t$ are given by the following equations:

$$P_t w_t = \alpha \tilde{P}_{H,t} A_t (\frac{k_{H,t-1}}{h_{H,t}})^{1-\alpha} \quad (18)$$

$$\beta \mathrm{E}_t U_C(C_{t+1},(1-h_{t+1})) \times \quad (19)$$

$$\left\{ \frac{\tilde{P}_{H,t+1}}{P_{t+1}} \frac{\partial F(k_t,h_{t+1})}{\partial k_t} \phi'\left(\frac{x_{H,t}}{k_{H,t-1}}\right) + \frac{\tilde{P}_{H,t+1}}{P_{t+1}} \frac{\phi'\left(\frac{x_{H,t}}{k_{H,t-1}}\right)}{\phi'\left(\frac{x_{H,t+1}}{k_{Ht}}\right)} (\Omega_H) \right\}$$

$$= U_C(C_t,(1-h_t)) \frac{\tilde{P}_{H,t}}{P_t}$$

where $\Omega_H = (1-\delta) + \phi\left(\frac{x_{H,t+1}}{k_{H,t}}\right) - \phi'\left(\frac{x_{H,t+1}}{k_{Ht}}\right)\left(\frac{x_{H,t+1}}{k_{Ht}}\right)$

A similar problem holds for the non-traded goods sector[6]:

$$\max_{h_{N,t},x_{N,t}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_C(C_t,(1-h_t))}{U_C(C_0,(1-h_0))} \frac{P_0}{P_t} \left[P_{N_t} y_{N_t} - P_t w_t h_{N,t} - P_{H,t} x_{N,t}\right] \quad (20)$$

$$y_{N_t} = F(k_{t-1}, h_{N,t}) = A_{N,t} {h_{N,t}}^{\alpha} k_{N,t-1}^{1-\alpha} \quad (21)$$

$$k_{N,t} = (1-\delta) k_{N,t-1} + x_t - \phi\left(\frac{x_{N,t}}{k_{N,t-1}}\right) k_{N,t-1} \quad (22)$$

[5] The function $\phi(.)$ has the following properties: In the steady state, $\phi(.) = x/k$, $\phi'(.) = 1, \phi''(.) = b < 0$.

[6] Note that we made the assumption that the investment goods is obtained out of the intermediate tradeable good.

And the corresponding first order conditions are given by:

$$P_t w_t = \alpha P_{N,t} {A_{N,t}}^{\alpha} (\frac{k_{N,t-1}}{h_{N,t}})^{1-\alpha} \tag{23}$$

$$\begin{gathered}\beta \mathrm{E}_t U_C(C_{t+1}, (1-h_{t+1})) \times \\ \left\{ \frac{P_{N,t+1}}{P_{t+1}} \frac{\partial F(k_t, l_{t+1})}{\partial k_t} \phi' \left( \frac{x_{N,t}}{k_{N,t-1}} \right) + \frac{\tilde{P}_{H,t+1}}{P_{t+1}} \frac{\phi' \left( \frac{x_{N,t}}{k_{N,t-1}} \right)}{\phi' \left( \frac{x_{N,t+1}}{k_{N,t}} \right)} (\Omega_N) \right\} \\ = U_C(C_t, (1-h_t)) \frac{\tilde{P}_{H,t}}{P_t}\end{gathered} \tag{24}$$

where $\Omega_N = (1-\delta) + \phi \left( \frac{x_{N,t+1}}{k_{N,t}} \right) - \phi' \left( \frac{x_{N,t+1}}{k_{N,t}} \right) \left( \frac{x_{N,t+1}}{k_{N,t}} \right)$

### 2.3 Current account and resource constraints

One important implication of the incomplete market framework is that it allows us to characterise the dynamics of the current account. By aggregating the individual budget constraints in the home country, we obtain:

$$P_t C_t + \frac{S_t B_t^F}{(1+i_t^*)} \frac{1}{\Theta(\frac{S_t B_t^F}{P_t})} = S_t B_{t-1}^F + P_t w_l h_t + \Pi_t \tag{25}$$

where we have applied the assumption that home bonds are in zero net supply and only held by Home residents.

The following aggregate constraints apply in the home and an analogous set in the foreign economy:

1. All non-traded output is consumed or used for retail services, non-traded goods are not subject to distribution costs:
$$y_{N_t} = c_{N_t} + \psi c_{H_t} + \psi c_{F_t}$$
2. Investment is only undertaken with domestically produced traded goods and no distribution services are needed:
$$y_{H_t} = c_{H_t} + c_{H_t}^* + x_{H_t} + x_{N_t}$$
3. Labour supply is divided between the traded and non-traded goods sector:
$$h_t = h_{H_t} + h_{N_t}$$

The aggregate profits in the home economy are given by:

$$\Pi_t = \tilde{P}_{H_t} y_H - P_t w_t h_{H_t} - \tilde{P}_{H,t} x_{H_t} + P_{N_t} y_{N_t} - P_t w_t h_{N_t} - \tilde{P}_{H,t} x_{N_t}$$

From which substituting the economy-wide constraints (1) - (3) we obtain:

$$\frac{S_t B_t^F}{P_t(1+i_t^*)} \frac{1}{\Theta(\frac{S_t B_t^F}{P_t})} = \frac{S_t B_{t-1}^F}{P_t} + \frac{\tilde{P}_{H,t}}{P_t} c_{H_t}^* - \frac{\tilde{P}_{F,t}}{P_t} c_{F_t} \tag{26}$$

## 2.4 Terms of trade

There are two concepts of the terms of trade in this model. The terms of trade at the wholesale level: $\tilde{T} = \frac{\tilde{P}_F}{S\tilde{P}_H^*}$ and the terms of trade at the consumer level: $T = \frac{P_F}{P_H}$. When distribution costs are absent, the two concepts overlap. With distribution costs, $T$ can be expressed as a simple function of $\tilde{T}$ :

$$T = \tilde{T}\frac{\tilde{P}_H}{P_H}\frac{P_F}{\tilde{P}_F}$$

As long as the distribution margin is the same for home produced than for imported traded goods, we can express the above in the following log-linear form:

$$\hat{T} = \hat{\tilde{T}}\frac{1}{1+\psi}$$

where for any variable $x_t$, whose steady state value is $\bar{x}$, the expression $\hat{x}_t = \frac{x_t - \bar{x}}{\bar{x}}$ denotes the log-deviation from steady state.

## 2.5 Real exchange rate

The real exchange rate that we focus on in this paper is the consumption-based real exchange rate, defined as:

$$RS_t = \frac{S_t P_t^*}{P_t}$$

where $S_t$ is the domestic currency price of one unit of foreign currency. We can analyse through which channels the real exchange rate can deviate from purchasing power parity (PPP) by re-writing the real exchange rate as follows:

$$RS_t = \frac{S_t P_t^*}{P_t} = \underbrace{\frac{S_t P_{H_t}^*}{P_{H_t}}}_{LOOP} \times \underbrace{\frac{P_{H_t}}{P_{T_t}}\frac{P_{T_t}^*}{P_{H_t}^*}}_{ToT} \times \underbrace{\frac{P_{T_t}}{P_t}\frac{P_t^*}{P_{T_t}}}_{B-S}$$

Given the consumption-based price indices, we can decompose potential deviations of the real exchange rate from PPP into three components: **LOOP** - deviations from the law-of-one-price for traded goods due to the presence of distribution costs. These arise because the law-of-one-price for traded goods holds at the wholesale, but not at the retail level; **ToT** - deviations from PPP due to changes in the terms of trade, brought about by consumption home-bias; and **B-S** - deviations from PPP due to the presence of non-traded consumption goods, the familiar Balassa-Samuelson effect.

## 2.6 Monetary policy

Since we are characterizing a nominal model we need to specify a monetary policy rule. In what follows we assume that the monetary authorities in both countries follow a strategy of setting consumer price inflation equal to zero.

### 2.7 Solution technique

Before solving our model, we log-linearise around the steady state to obtain a set of equations describing the equilibrium fluctuations of the model. The log-linearisation yields a system of linear difference equations which can be expressed as a singular dynamic system of the following form:

$$\mathbf{A}\mathrm{E}_t\mathbf{y}(t+1 \mid t) = \mathbf{B}\mathbf{y}(t) + \mathbf{C}\mathbf{x}(t)$$

where $\mathbf{y}(t)$ is ordered so that the non-predetermined variables appear first and the predetermined variables appear last, and $\mathbf{x}(t)$ is a martingale difference sequence. There are four shocks in $\mathbf{C}$: shocks to the Home traded and non-traded intermediate goods sectors' productivity and shocks to the Foreign traded and non-traded intermediate goods sectors' productivity. The variance-covariance as well as the autocorrelation matrices associated with these shocks are described in table 2. Given the parameters of the model, which we describe in the next section, we solve this system using the King and Watson (1998) solution algorithm.

# 3 Calibration

In this section, we outline our choice of structural parameters and shock processes. We assume that utility is separable across time and that consumption and leisure are additively separable. Specifically, we assume the following functional form:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{C_s^{1-\rho}}{1-\rho} + \chi \frac{(1-h_s)^{1-\eta}}{1-\eta} \right] \tag{27}$$

where $\beta$ is the subjective discount factor, $\chi$ is a parameter chosen to ensure that in the steady state agents allocate 1/3 of their time endowment to work, and $\rho$ and $\eta$ are the constant relative risk aversion parameters (inverse of the intertemporal elasticity of substitution) associated with work and leisure, respectively.

We put forward two calibrations: the first, is highly stylised and designed to facilitate the analysis of our impulse response functions. Our aim here is to isolate the contributions of distribution costs and asset market structure to the dynamics of the real exchange rate. Table 1 summarises this calibration. By setting all elasticities to 1, eliminating consumption home-bias, equalising the consumption shares of traded and non-traded goods as well assuming no capital accumulation, our incomplete markets model replicates the complete market allocation in the absence of distribution costs. As we increase the distribution margin, we can analyse how the complete and incomplete markets models diverge.

Having analysed the models' impulse response functions, section 6 considers a selection of second moments generated by the model under different asset market structures. For this purpose, we choose a more general calibration, based on the work of Benigno and Thoenissen (2006) and Corsetti, Dedola and Leduc (2004). The calibration is summarised in table 2, and discussed in more detail in section 7.

Table 1: Parameters and shock processes: impulse responses

| | |
|---|---|
| Preferences | $\beta = 1/1.04, \rho = \eta = 1, \bar{h} = 1/3$ |
| Final goods tech | $\omega = \omega^{*} = v = (1 - v^{*}) = 0.5$, $\theta = \kappa = 1$ |
| Intermediate goods tech | $\alpha = 1$ |
| Distribution sector | $\psi = varies$ |
| Financial markets | $\varepsilon = 0.01$ |
| Shocks | $\Omega = \begin{bmatrix} 0.84 & 0 & 0 & 0 \\ 0 & 0.84 & 0 & 0 \\ 0 & 0 & 0.30 & 0 \\ 0 & 0 & 0 & 0.30 \end{bmatrix}$ |
| | $V[\mu] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |

# 4 Impulse responses - some special cases

In this section, we use impulse response functions to analyse how our model reacts to supply shocks under complete and incomplete financial markets, for various degrees of distribution costs. Figures 1 - 8 show the response of the real exchange rate, the terms of trade (at the consumer level), the relative price of non-traded to traded goods, the trade balance, relative consumption and relative GDP to productivity shocks coming from the traded and non-traded sectors. To best illustrate the effects of market incompleteness and distribution costs, we propose a very specific calibration for this exercise. Table 1 shows all inter and intra-temporal elasticities are set to unity, there is no home bias and the share of labour in the production function is unity.

With this calibration, and in the absence of distribution costs, the incomplete markets model reproduces the complete markets allocation, as figures 1 and 2 illustrate. In both cases, we observe complete risk sharing, since the real exchange rate and relative consumption are perfectly correlated. We also know from Obstfeld and Rogoff (1996) that when $\rho = \kappa = \theta$ the current account does not react directly to non-traded sector supply shocks. When the economy is hit by a unit traded sector productivity shock, as in figure 1, because we assume that $\theta$, the elasticity of substitution between home and foreign traded goods, is equal to one, the terms of trade (the relative price of foreign to home-produced traded goods) depreciate by exactly the amount of the shock and thus shares risk completely, leaving relative consumption and the real exchange rate unchanged. When the supply shock originates from the non-traded sector, it is the relative price of non-traded to traded goods that adjusts to share risk between countries. Because we assume that $\kappa$, the elasticity of substitution between non-traded and traded goods is unity, a 1 percent rise in the supply of non-traded goods leads to a 1 percent fall in the relative price of non-traded goods. Even though the response of the relative price of home non-traded to traded goods completely isolates the foreign

Table 2: Parameters and shock processes: moment matching

| | |
|---|---|
| Preferences | $\beta = 1/1.04, \rho = \eta = 2, \bar{h} = 1/3$ |
| Final goods tech | $\omega = \omega^{*} = 0.55, \upsilon = (1 - \upsilon^{*}) = 0.835, \theta = 1.15, \kappa = 0.74$ |
| Intermediate goods tech | $\alpha = 0.585, \delta = 0.1, \left(\phi''\delta\right)/\phi' = -0.2$ |
| Distribution sector | $\psi = 1.09$ |
| Financial markets | $\varepsilon = 0.01$ |
| Shocks | $\Omega = \begin{bmatrix} 0.84 & 0 & 0.22 & 0 \\ 0 & 0.84 & 0 & 0.22 \\ 0 & 0 & 0.30 & 0 \\ 0 & 0 & 0 & 0.30 \end{bmatrix}$ |
| | $V[\mu] = \begin{bmatrix} 3.76 & 1.59 & 0.72 & 0.44 \\ 1.59 & 3.76 & 0.44 & 0.72 \\ 0.72 & 0.44 & 0.51 & 0.21 \\ 0.44 & 0.72 & 0.21 & 0.51 \end{bmatrix}$ |

economy's consumption, the real exchange rate adjusts to share risk, equating the marginal utility of income across countries.

Figures 1 - 2, the models without distribution costs, serve as our baseline against which to compare specifications with distribution costs. In figures 3 and 4, the solid (dotted) lines refer to the impulse responses of the incomplete (complete) financial markets model with a distribution cost parameter, $\psi = 0.5$, which yields a steady-state distribution margin of 33%. Note that in both asset market specifications the the real exchange rate is more volatile compared to the baseline specification. For both shocks, the incomplete markets model shows a significantly larger real exchange rate depreciation than the complete markets model. In the complete markets model, risk sharing implies that the real depreciation be linked with a rise in relative consumption. In the incomplete markets model, risk sharing only restricts the expected growth rates of the real exchange rate and relative consumption, not their levels, as figure 3 illustrates. Since $\rho = \kappa = \theta$, the response of the current account depends only on the size of the supply shock, not its sectoral location, as figures 3 - 8 show.[7]

Figures 5 and 6, as well as 7 and 8 repeat the above exercise for higher values of the distribution margin. In figures 5 and 6, we assume a distribution margin of 48.7%, which corresponds to $\psi = 0.95$. Under this calibration, the model with incomplete markets now generates significant real exchange rate volatility. A 1 percent standard deviation increase in productivity leads to a 3 to 4 percent standard deviation depreciation of the real exchange rate, depending on the sectoral

[7] The terms of trade in figures 1 - 8 is the terms of trade at the consumer level. In comparing the terms of trade response in figures 1 and 3 suggest that introducing distribution costs into the incomplete markets model actually reduces the volatility of the terms of trade. The terms of trade at the wholesale level does however rise, since $\hat{T} = \tilde{T}(1+\psi)^{-1}$.

location of the shock. Under complete markets, the real exchange rate response is very small and similar to the one in figures 3 and 4. In figures 7 and 8, we raise the distribution margin from 48.7% to 49.7% and find that, in the incomplete markets model, the response of the real exchange rate to supply side shocks rises by a factor of five! As before, the complete markets model displays very little real exchange rate volatility. We also note at this point, that given our baseline calibration, the higher the distribution costs the slower the incomplete markets model appears to return to equilibrium. Mean reversion appears to be minimal in figures 5 - 8.

Under our special symmetric calibration, we can write the log-linearised expression for the real exchange rate simply as:

$$\widehat{RS}_t = -\left(\frac{1-\omega+\psi}{1+\psi}\right)[\hat{q}_t - \hat{q}_t^*] + (1-\omega+\psi)\,\hat{T}_t \tag{28}$$

such that the real exchange rate depends on $q = \frac{P_N}{\tilde{P}_H}$, $q^* = \frac{P_N^*}{\tilde{P}_F^*}$ , the relative price of non-traded to domestically produced traded goods at home and abroad, and $T = \frac{P_F}{P_H}$, the terms of trade at the consumer level. Since $q$ and $q^*$ are determined only by the shock processes in our special case calibration, the difference in the dynamics of the real exchange rate in the complete and incomplete markets model is due to different dynamics of the terms of trade. Figures 3 -8 show that in the presence of distribution costs, the terms of trade in the incomplete markets model are more volatile than in the complete markets model. Reasons why the terms of trade are more volatile under incomplete than under complete markets are discussed in the next section.

# 5 Real exchange rate volatility and asset market structure

A key finding from the above impulse response analysis is that the structure of the financial asset market matters for the volatility of the real exchange rate. We find that adding distribution costs to our model raises the volatility of the real exchange rate, but does so more under incomplete financial markets than under complete financial markets.

Given the calibration used in section 4, we can decompose movements in the real exchange rate into movements in the relative price of non-traded to home-produced traded goods at home relative to abroad, $[\hat{q}_t - \hat{q}_t^*]$ and movements in the terms of trade (at the consumer level). An increase in traded sector TFP raises $q$, due to the familiar Balassa-Samuelson effect, and depeciates (raises) the terms of trade. The term of trade depreciate in order to raise relative demand for home-produced goods, both at home and in the foreign economy. It is well known that the magnitude of the terms of trade depreciation depends on the elasticity of substitution between home and foreign-produced traded goods, $\theta$. The larger is $\theta$ the more home and foreign-produced traded goods can be substituted for one another and the less the terms of trade have to adjust following a supply shock. From equation (28) we can see that these two effects are (partially) offsetting. The Balassa-Samuelson effect contributes towards a real appreciation, whereas the terms of trade effect contributes towards a real depreciation.

If the productivity shock occurs in the non-traded goods sector, $q$ falls as the price of non-traded goods declines. This contributes towards a real depreciation. The response of the terms of trade depends on the elasticity of substitution between traded and non-traded goods. If traded and non-traded goods are compliments ($\kappa < 1$), home-produced traded goods supply rises along with non-traded output (in response to higher domestic demand), and the terms of trade depreciate (rise) as the price of imports rises. If traded and non-traded goods are substitutes ($\kappa > 1$) the demand for traded goods will fall and the terms of trade will appreciate (fall). Empirically, the case ($\kappa < 1$) seems more plausible (see Menodza (1991) and Stockman and Tesar (1995)). In this case both the terms of trade and the relative price of non-traded to traded goods contributes towards a real depreciation.

From equation (28) we can see that, for a given $[\hat{q}_t - \hat{q}_t^*]$ and $\hat{T}_t$, introducing distribution costs increases both the contributions of (coefficients on) $\hat{T}_t$ and $[\hat{q}_t - \hat{q}_t^*]$ to the dynamics of of the real exchange rate. We can also see, that as $\psi$ rises, the contribution of $\hat{T}_t$ increases by more than the contribution of $[\hat{q}_t - \hat{q}_t^*]$, whose magnitude is determined only by the relative shock processes and not $\psi$. As long as the real exchange rate depreciates in response to a supply shock, raising $\psi$ raises the volatility of the real exchange rate.[8]

In the previous paragraphs, we analysed what happens to the real exchange rate if we increase the distribution margin, holding the response of the terms of trade constant. When we assume that the share of labour in output is unity, then $q = \frac{P_N}{\tilde{P}_H} = \frac{A_H}{A_N}$ and $q^* = \frac{P_N^*}{\tilde{P}_F^*} = \frac{A_F}{A_N^*}$, the dynamics of $[\hat{q}_t - \hat{q}_t^*]$ are not affected by the introduction of distribution costs.[9] The terms of trade, on the other hand, is a relative price that is determined in general equilibrium. Figures 3 - 8 suggest that the response of the terms of trade, at the consumer level, increases with $\psi$. In general, an increase in productivity, located either in the traded or the non-traded goods producing sector raises (depreciates) the terms of trade. If home traded sector productivity ($A_H$) rises, output of home-produced traded goods increases. To clear the market for traded goods, their price must fall, depreciating the terms of trade and raising demand for home-produced goods both at home and abroad, which assumes that the substitution effect outweighs the income effect. In the presence of distribution costs, a rise in ($A_H$) results in both higher output of home-produced goods, as well as greater distribution costs in the home market, due to the Balassa-Samuelson effect on non-traded goods prices. To clear the market, the terms of trade have to rise by more so as to partially off-set the increased distribution costs with a lower wholesale price and to stimulate extra demand for home-produced goods from abroad.

When the shock occurs in the non-traded goods sector the price of non-traded goods falls, and so do distribution costs, lowering the consumer price of traded goods. This raises the demand for home and foreign-produced traded goods. In the home country, this complementarity in demand leads to a positive output response from the traded sector. Therefore, the relative price of foreign to home-produced traded goods, $T$ increase. Corestti, *et al* (2004) show that raising $\psi$ has a similar effect to lowering $\theta$, and indeed to reducing the value of $\kappa$. Therefore distribution costs increase the volatility of the real exchange rate by increasing the volatility of the terms of trade.

We have now seen that for a given $[\hat{q}_t - \hat{q}_t^*]$ and $\hat{T}_t$, distribution costs can increase the volatility of the real exchange rate. We have also seen that distribution costs amplify the response of $\hat{T}_t$ to

[8] This assumes a 'positive transmission' of productivity shock across countries, whereby the terms of trade depreciate following a traded sector TFP shock. When the terms of trade appreciate to yield a 'negative transmission', i.e. transfering purchasing power to the country that experiences a shock, a rise in $\psi$ always increases exchange rate volatility since the terms of trade and the Balassa-Samuelson effects work in the same direction.

[9] In general, when $\alpha < 1$ the dynamics of $q$ depend on the evolution of the capital labour ratio in the two production sectors.

supply shocks. The next step is to analyse why we get so much much more volatility of the real exchange rate under incomplete than under complete financial markets.

Figures 1 - 8 compare the complete and incomplete financial markets model under different distribution margins. Recall from above that in all these experiments, the dynamics of $[\hat{q}_t - \hat{q}_t^*]$ are invariant to the asset market structure. Consequently, our analysis will focus only on the response of $\hat{T}_t$ to changes in productivity. We observe from figures 1 - 8 that in the presence of distribution costs, $T$ is more volatile under incomplete than under complete markets.

To see why, let us consider the role of the terms of trade in sharing risk between countries. The log-linearised risk sharing conditions of the complete and incomplete markets model are:

$$\widehat{RS}_t = \rho \hat{C}_t^R \tag{29}$$

$$E_t \widehat{RS}_{t+1} - \widehat{RS}_t = \rho E_t \hat{C}_{t+1}^R - \rho \hat{C}_t^R + \varepsilon \hat{b}_t \tag{30}$$

Condition (29), the log-linearised risk-sharing condition for the model with complete financial markets, links the log-deviation of the real exchange rate to the log-deviation of the relative marginal utility of consumption at home and abroad, where $\hat{C}_t^R = \hat{C}_t - \hat{C}_t$ and $\rho$ is the intertemporal elasticity of substitution. Condition (30), the log-linearised risk-sharing condition for the model with incomplete financial markets, links the expected growth rates of the real exchange rate with that of relative marginal utility of consumption at home and abroad. The incomplete markets risk-sharing condition is consistent with both positive as well as negative correlations between $\widehat{RS}_t$ and $\hat{C}_t^R$, thus as is shown in Benigno and Thoenissen (2006) such a model is able to address the consumption real exchange rate anomaly. Furthermore, the risk-sharing condition (30) does not place restrictions on the relative sizes or signs of $\widehat{RS}_t$ and $\hat{C}_t^R$, only on their paths. In the incomplete markets model, we can get real exchange rate responses that are much larger in magnitude than the response of relative consumption. Figures 3-8 confirm this.

For a given supply shock, a large response of the real exchange rate requires a large response of the terms of trade. Corsetti and Dedola (2005) have shown that distribution costs lower the elasticity of substitution between home and foreign produced traded goods. The higher the distribution margin, the lower the elasticity of substitution, and the lower the elasticity of substitution, the greater is the response of relative prices to a change in supply. We also know that the terms of trade shares risk between home and foreign consumers. *Ceteris paribus*, a depreciation of the terms of trade transfers purchasing power form home to foreign consumers.

Why is the real exchange rate not as volatile under complete markets? For a given supply shock, we need a large terms of trade depreciation to generate a large real exchange rate response. The large terms of trade depreciation in turn reduces relative consumption, even causing it to become *negative*. However, in the complete markets model, a large terms of trade depreciation would require a large *rise* in relative consumption, so we can see that in the complete markets model, high volatility of the real exchange rate can not arise because of high terms of trade volatility.

# 6 Second moments

Having analysed the impulse response functions of the model using the simplified calibration, in this section we employ a more realistic calibration of parameters and shock processes to compare our model to the data. As in our impulse response analysis, we assume the two countries are symmetric, as far as the structural parameters and the shock processes are concerned. Most of the parameter values are taken from Corsetti, Dedola and Leduc (2004). In their calibration the home country is the United States. Our calibration differs with respect to the degree of home-bias, $(v - v^*)$ where we have revised Corsetti *et al*'s value upwards, to bring it closer to the value suggested in Benigno and Thoenissen (2003). Initially, we close our incomplete markets model with a bond holding costs put forward by Benigno, P. (2001), which adds an extra parameter, $\varepsilon$, the spread of the domestic interest rate on foreign assets over the foreign rate. We set $\varepsilon = 0.01$, which implies a 100 basis point spread. Since we now consider the full model with capital accumulation, we also have to assume a depreciation rate of 10% per annum, as well as an adjustment cost parameter, $\left(\phi''\delta\right)/\phi'$ of -0.2.

The stochastic environment is determined by total factor productivity shocks to the traded and non-traded intermediate goods producing sectors. Our model is annual, because sectoral output and labour data, used to construct the Solow residual, are only available at annual frequency. Symmetric estimated variance co-variance (VCM) matrices for sectoral productivity shocks in two-sector two-country models such as ours are uncommon in the literature. A frequently used estimate of such a shock process is the one produced by Stockman and Tesar (1995). Their VCM is produced using logged and Hodrick-Prescott filtered Solow residuals, which tends to understate the persistence of the driving process. Two recent estimates for the US versus G-7 countries VCM, that use logged but not H-P filtered Solow residuals, can be found in Corsetti *et al* (2004) and Benigno and Thoenissen (2006). In this paper, we use the latter specification.[10]

We focus only on a small subset of second moments generated by our model. In particular, we are interested in the volatility of the real exchange rate and the terms of trade, the correlations between GDP on the one hand and the real exchange rate and the trade balance on the other. We also focus on three sets of second moments that have received particular attention in the recent open economy macroeconomics literature: the relative ranking of the cross-country correlations of GDP and consumption, which has become known as the quantity anomaly, the correlation between the real exchange rate and the terms of trade, as well as the correlation between the real exchange rate and relative consumption, sometimes know as the Backus-Smith puzzle.

Column 2 of table 3 contains selected second moments from the data. Columns 3 and 4 show the corresponding second moments generated by the incomplete markets model (ICM) and the complete markets model (CM), respectively. Both actual data and artificial economy data are of annual frequency, logged and Hodrick-Prescott filtered. Compared to the data, our models all over predict the volatility of GDP, yet all find that the terms of trade are less volatile than the real exchange rate. The most striking feature of table 3 is that whereas the incomplete markets model comes quite close to matching the relative volatility of the real exchange rate, the complete markets model, using an identical calibration, yields a real exchange rate that is less than 1/10 as volatile as the data. This suggests that it is not merely the presence of a distribution sector that generates a high volatility of the real exchange rate, but a combination of a distribution sector and incomplete financial markets.

Incomplete financial markets also help the model with distribution costs address the Backus-Smith puzzle, as previously shown by Corsetti *et al* (2004). Under both asset market structures,

[10] A discription of the estimation procedure and data sources can be found in Benigno and Thoenissen (2006). We performed sensitivity analyis using the Corsetti et al shocks and found the results to be very similar.

Table 3: Selected second moments: data and models

| | Data | ICM | CM | ICM $M^{`+'}$ | ICM $M^{`-'}$ | ICM $\theta$=1.5 | CM $\theta$=1.5 | ICM $M$,$\theta$=1.5 |
|---|---|---|---|---|---|---|---|---|
| Volatility | | | | | | | | |
| $\sigma_Y$ | 1.57 | 2.37 | 2.62 | 2.37 | 3.16 | 2.62 | 2.63 | 2.62 |
| $\frac{\sigma_{RS}}{\sigma_Y}$ | 4.39 | 3.26 | 0.31 | 3.39 | 3.39 | 0.34 | 0.31 | 0.32 |
| $\frac{\sigma_T}{\sigma_Y}$ | 2.12 | 1.85 | 0.23 | 1.90 | 1.55 | 0.26 | 0.22 | 0.25 |
| Correlations | | | | | | | | |
| (RS,GDP) | -0.09 | 0.28 | 0.07 | 0.27 | -0.69 | 0.17 | 0.06 | 0.15 |
| (TB,GDP) | -0.26 | 0.06 | -0.35 | 0.09 | -0.31 | 0.22 | 0.09 | 0.18 |
| (Y,Y$^*$) | 0.49 | 0.83 | 0.50 | 0.84 | 0.03 | 0.50 | 0.50 | 0.50 |
| (C,C$^*$) | 0.32 | 0.01 | 0.39 | -0.03 | -0.39 | 0.41 | 0.40 | 0.41 |
| (RS,C/C$^*$) | -0.45 | -0.66 | 1 | -0.67 | -0.85 | 0.96 | 1 | 0.97 |
| (RS,T) | 0.32 | 0.99 | 0.22 | 0.99 | 0.99 | 0.00 | 0.19 | 0.37 |
| Auto-correlations | | | | | | | | |
| $\rho(RS)$ | 0.78 | 0.44 | 0.20 | 0.44 | 0.47 | 0.19 | 0.20 | 0.19 |
| $\rho(Y)$ | 0.50 | 0.49 | 0.49 | 0.49 | 0.47 | 0.49 | 0.49 | 0.49 |

the model succeeds in addressing the quantity anomaly (see also Corsetti *et al* (2004) and Oviedo and Singh (2006) on this) but we come closer to the data in the complete than in the incomplete markets model. Both models also suggest a positive correlation between the real exchange rate and the terms of trade, in line with the data. The complete markets model also succeeds in generating a counter-cyclical trade balance and generates a real exchange rate that is less pro-cyclical than the one generated by the incomplete markets model. As our analysis of impulse response functions suggests, in the incomplete markets model, the real exchange rate is about twice as persistent as in the complete markets model.

Our calibrated model confirms the intuition we gleaned from our impulse response analysis. A flexible price two-sector model, driven only by productivity shocks that allows for a distribution sector can generate significant real exchange rate volatility. However, in order to do so one has to assume an incomplete international financial market structure. A model with international trade in state-contingent bonds can not generate significant real exchange rate volatility within this modelling framework.

The next section analyses how robust our findings are to changes in the elasticity of substitution between home and foreign-produced traded goods, to changes in the distribution margin, as well as to the way we eliminate the unit root in foreign bond holdings.

# 7 Sensitivity analysis

Table 3 suggests that a flexible price two-sector international real business cycle model with distribution costs and incomplete financial markets can succeed in addressing many of the puzzles of international macroeconomics. In this section we analyse how robust this finding is. In columns 6 and 7 (headed $ICM_{\theta=1.5}$ and $CM_{\theta=1.5}$) we report second moments for the incomplete and complete markets model under the assumption that elasticity of substitution between home and foreign-produced traded goods, $\theta = 1.5$ instead of 1.15, as in the benchmark calibration. We find that for both asset market structures, the model now grossly under predicts the volatility of the real exchange rate. We also find that for the slightly larger value of $\theta$, the incomplete markets model with distribution costs yields a correlation between the real exchange rate and relative consumption close to unity. The model's success at addressing the quantity anomaly is however robust to changes in $\theta$.

Next, we analyse how sensitive the relative volatility of the real exchange rate is to changes in the distribution margin. For figure 9, we use the baseline calibration to solve the model under incomplete and complete asset market structures for various values of the distribution cost parameter, $\psi$. Figure 9 plots the standard deviation of the real exchange rate relative to the standard deviation of GDP against $\psi$. We find that relative real exchange rate volatility rises with the distribution cost parameter, but does so more in the incomplete than in the complete markets model. We also note that in the neighbourhood of our baseline value of $\psi$ (1.09), the model becomes very sensitive to changes in $\psi$. Starting form this point, a small increase in $\psi$ leads to very large increases in the real exchange rate volatility in the incomplete markets model. Beyond $\psi = 1.14$, we get too many unstable roots and our algorithm is unable to solve the model.

Our sensitivity analysis points in the direction that, as far as real exchange rate volatility is concerned, our baseline results have a certain knife-edge quality. Small changes in either $\theta$ or $\psi$ can lead to large changes in the volatility of the real exchange rate, taking the model further away from the data along a number of dimensions.

### 7.1 Closing the model with an endogenous discount factor

In a seminal paper, Schmidt-Grohe and Uribe (2003) establish that for small open economy models, alternative ways of eliminating the unit root in bond holdings, be it via complete financial markets, bond holding or adjustment costs, or endogenous discount factors, all yield very similar results as far as real business cycle statistics are concerned. The purpose of this section is two-fold: first, we close the model using the endogenous discount factor approach of Mendoza (1991) to check if our results are dependent on how we have chosen to eliminate the unit root in bond holdings in our model; and second to verify if Schmidt-Grohe and Uribe's result holds in the context of our model and our selected moments.

We can eliminate the unit root in bond holdings by assuming that the subjective discount factor, $\beta$ in the previous specification, is a decreasing function of the average level of consumption and leisure. Agents are assumed to become more impatient the more they consume. Given the functional form of our utility function, the discount factor becomes:

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \left[ \frac{C_s^{1-\rho}}{1-\rho} + \chi \frac{(1-h_s)^{1-\eta}}{1-\eta} \right] \exp \left[ \sum_{\tau=0}^{s-1} -v \left( U \left[ C_s, (1-h_s) \right] \right) \right] \right\} \tag{31}$$

where

$$v \left( U \left[ C_s, (1-h_s) \right] \right) = \ln \left( 1 + \varphi \left[ C_s + \chi (1-h_s) \right] \right)$$

The corresponding first order conditions for domestic and foreign bond holdings become:

$$U_C \left( C_t, (1-h_t) \right) = \beta(C_t, h_t)(1+i_t) E_t \left[ U_C \left( C_{t+1}, (1-h_{t+1}) \right) \frac{P_t}{P_{t+1}} \right] \tag{32}$$

$$U_C\left(C_t^*,(1-h_t^*)\right)=\beta(C_t^*,h_t^*)(1+i_t^*)E_t\left[U_C\left(C_{t+1}^*,(1-h_{t+1}^*)\right)\frac{P_t^*}{P_{t+1}^*}\right] \tag{33}$$

$$U_C\left(C_t,(1-h_t)\right)=\beta(C_t,h_t)(1+i_t^*)E_t\left[U_C\left(C_{t+1},(1-h_{t+1})\right)\frac{S_{t+1}P_t}{S_tP_{t+1}}\right] \tag{34}$$

where $\beta(C_t, h_t) = \exp\{-v(.)\}$. The key difference between Menodza's and Benigno's approach is that in the former uncovered interest rate parity always holds since there are no costs associated with holding foreign currency-denominated bonds.[11]

The column headed $ICM_{M\text{`+'}}$ in table 3 gives second moments for the model solved under the endogenous discount factor model using the baseline calibration. Under this calibration, the terms of trade depreciate in response to traded sector productivity shocks, thus positively transmitting the productivity shock to the foreign country. The differences between our baseline incomplete markets model and the endogenous discount factor model are minimal. The endogenous discount factor model yields a slightly more volatile real exchange rate and terms of trade, with most other second moments unaltered. This suggests that the high real exchange rate volatility observed in our incomplete markets model, using our baseline calibration, does not depend on how we model market incompleteness. As far as the Schmidt-Grohe and Uribe's result is concerned, using our baseline calibration, we find that market incompleteness matters for the dynamics of the real exchange rate and its deteminants, but not how one models this incompleteness. The column headed $ICM_{M\text{`-'}}$ gives second moments for the model solved for $\theta = 1.0766$. This value of $\theta$ yields the same real exchange rate volatility, but following a traded sector productivity shock, the terms of trade appreciate, leading to a negative international transmission. Table 3 shows that in this calibration home and foreign consumption are strongly negatively correlated. Figure 10 plots the volatility of the real exchange rate generated by the complete markets model (dotted line) and the endogenous discount factor model (solid line) against $\psi$, the distribution cost parameter. We note that to the right of the peak in real exchange rate volatility, the endogenous discount factor model generates 'negative transmission' of productivity shocks through the terms of trade. This is one aspect in which the endogenous discount factor model differs from both the complete markets model and the incomplete markets model closed with a bond holding cost. In the former, we observe no drastic increase in exchange rate volatility and no negative transmission as $\psi$ rises. In the latter, negative transmission of supply shocks is not possible.

Why does our baseline incomplete markets model solved with a bond holding cost not allow for negative transmission? In this model, a bond holding cost that is related to the net foreign asset position of the home economy ensures stationarity. When an economy accumulates debt, its costs of borrowing in foreign currency-denominated bonds rises, making foreign borrowing less attractive, ensuring eventually that the economy returns to its initial net foreign asset position. Our analysis of impulse response functions suggests that the effectiveness of this mechanism is reduced as $\psi$ rises. For high values of $\psi$ the model displays very slow mean reversion. The model does not allow negative transmission of shocks, because in this case the bond holding cost mechanism would actually destabilise the model.

In the final columns of table 3, we set $\theta$, the elasticity of substitution between home and foreign-produced traded goods to 1.5 in the endogenous discount factor model and find that Schmidt-Grohe

[11] In log-linearising the model, we need to set $\chi$ to yield a steady-state level of labour supply of $\bar{h}$. The parameter $\varphi$ is then set so as to yield a steady-state interest rate of 4% given balanced trade in the steady state.

and Uribe's result is reconfirmed. The dynamics of the real exchange rate, as well as most other reported second moments appear not to change much across asset market specifications, whether a complete or incomplete asset market structure is assumed.

## 8 Conclusion

This paper highlights the role of financial asset market incompleteness in determining the volatility of the real exchange rate. We show that a two-country, two-sector model with non-traded consumption goods and distribution services, driven only by supply side shocks, can generate realistic levels of real exchange rate volatility. The volatility of the real exchange rate is driven by the volatility of the terms of trade. For low values of the demand elasticity of traded goods, the terms of trade become very volatile. Given the presence of non-traded goods and consumption home-bias, large terms of trade movements translate into large real exchange rate movements.

Since the terms of trade also act to share risk across countries, large movements in the terms of trade are associated with large transfers of purchasing power between countries. In an incomplete financial markets setting, risk does not have to be shared completely or at all, so that the terms of trade can move by more or, indeed, less than is warranted by complete risk-sharing. Or stated the other way around, the risk-sharing condition of the complete markets model limits the volatility of the terms of trade, and therefore the real exchange rate. The terms of trade move only as much as is required to transfer the appropriate amount of purchasing power between countries so as to equalise the marginal utilities of income between the home and foreign country.

These results suggest that asset market structure matters for the volatility of the real exchange rate. Schmidt-Grohe and Uribe (2003) show that the business cycle properties of small open economy models are not affected by the asset market structure. The same reasoning is often applied to two-country models. Our analysis suggest that for our type of model, and for a certain, but narrow parameter range, complete and incomplete asset market structure can have very different implications for the dynamics of the real exchange rate. It may also matter how one attempts to induce stationarity into an incomplete markets model. We show that an endogenous discount factor model is consistent with both positive as well as negative transmission of supply shocks, whereas a bond holding cost model can only be solved (using conventional solution techniques such as those by King and Watson (1998)) using a parameterisation that ensures a positive transmission of supply shocks across countries.

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# A Log-linearised model (baseline incomplete markets)

This appendix contains the linearised equations of our baseline model that are used to solve the model using King and Watson's solution algorythm for MATLAB.

Consumers' first-order conditions:

$$\rho E_t\hat{C}_{t+1}= \rho\hat{C}_t+\hat{\imath}_t-E_t\pi_{t+1} \tag{A1}$$

$$\rho E_t\hat{C}^*_{t+1}= \rho\hat{C}^*_t+\hat{\imath}^*_t-E_t\pi^*_{t+1} \tag{A2}$$

$$E_t\widehat{RS}_{t+1}-\widehat{RS}_t= \rho E_t[\hat{C}_{t+1}-\hat{C}^*_{t+1}] - \rho[\hat{C}_t-\hat{C}^*_t] + \varepsilon\hat{b}_t \tag{A3}$$

$$\hat{w}_t= \rho\hat{C}_t+\eta\frac{\bar{h}}{1-\bar{h}}\hat{h}_t \tag{A4}$$

$$\hat{w}^*_t= \rho\hat{C}^*_t+\eta\frac{\bar{h}}{1-\bar{h}}\hat{h}^*_t \tag{A5}$$

Bond equation:

$$\beta\hat{b}_t= \hat{b}_{t-1}+a_b\left(\theta(v-v^*)+\theta-1-\psi\right)\hat{T}_t-a_b\left[\hat{C}_t-\hat{C}^*_t\right]-\frac{a_b\kappa(1-\omega)}{(\omega(1+\psi)^{(1-\kappa)}+1-\omega)}\left[\hat{R}_t-\hat{R}^*_t\right] \tag{A6}$$

First-order conditions - home country firms

$$\begin{aligned}\rho E_t\hat{C}_{t+1} &= \rho\hat{C}_t+(1+\beta(\delta-1))\left[\widehat{mpk}_{H_{t+1}}\right]-\phi\delta\left[\hat{x}_{H_t}-\hat{k}_{H_{t-1}}\right]+\phi\delta\beta\left[\hat{x}_{H_{t+1}}-\hat{k}_{H_t}\right] \\ &\quad +(1-v)\left[\hat{T}_t-E_t\hat{T}_{t+1}\right]+\frac{\psi}{1+\psi}\left[\hat{q}_t-\beta(1-\delta)E_t\hat{q}_{t+1}\right]+\frac{(1-\omega)}{(\omega(1+\psi)^{(1-\kappa)}+1-\omega)}\left[R_t-E_tR_{t+1}\right]\end{aligned} \tag{A7}$$

$$\begin{aligned}\rho E_t\hat{C}_{t+1} &= \rho\hat{C}_t+(1+\beta(\delta-1))\left[\widehat{mpk}_{N_{t+1}}\right]-\phi\delta\left[\hat{x}_{N_t}-\hat{k}_{N_{t-1}}\right]+\phi\delta\beta\left[\hat{x}_{N_{t+1}}-\hat{k}_{N_t}\right] \\ &\quad +(1-v)\hat{T}_t-(1-v)\beta(\delta-1)E_t\hat{T}_{t+1}+\frac{(1-\omega)}{(\omega(1+\psi)^{(1-\kappa)}+1-\omega)}R_t \\ &\quad +\frac{\beta(\delta-1)+\omega}{(\omega(1+\psi)^{(1-\kappa)}+1-\omega)}E_tR_{t+1}+\frac{\psi}{1+\psi}\left[\hat{q}_t-\beta(1-\delta)E_t\hat{q}_{t+1}\right]\end{aligned} \tag{A8}$$

$$\hat{k}_{H,t}= (1-\delta)\hat{k}_{H,t-1}+\delta\hat{x}_{H_t} \tag{A9}$$

$$\hat{k}_{N,t}= (1-\delta)\hat{k}_{N,t-1}+\delta\hat{x}_{N_t} \tag{A10}$$

$$\hat{w}_t= \hat{A}_{H,t}+(1-\alpha)\hat{k}_{H,t-1}-(1-\alpha)\hat{h}_{H,t}-(1-v)\hat{T}_t-\frac{\psi}{1+\psi}\hat{q}_t-\frac{(1-\omega)}{(\omega(1+\psi)^{(1-\kappa)}+1-\omega)}\hat{R}_t \tag{A11}$$

$$\hat{w}_t= \hat{A}_{N,t}+(1-\alpha)\hat{k}_{N,t-1}-(1-\alpha)\hat{h}_{N,t}+\frac{\omega(1+\psi)^{(1-\kappa)}}{(\omega(1+\psi)^{(1-\kappa)}+1-\omega)}\hat{R}_t \tag{A12}$$

Firms' problem foreign country

$$\begin{aligned}\rho E_t\hat{C}^*_{t+1} \quad = \quad & \rho\hat{C}^*_t+(1+\beta(\delta-1))E_t\left[\widehat{mpk}_{F_{t+1}}\right]-\phi\delta\left[\hat{x}_{F_t}-\hat{k}_{F_{t-1}}\right]+\phi\delta\beta\left[E_t\hat{x}_{F_{t+1}}-\hat{k}_{F_t}\right]-v^*\left[\hat{T}_t-E_t\hat{T}_{t+1}\right] \\ & +\frac{(1-\omega)}{(\omega(1+\psi)^{(1-\kappa)}+1-\omega)}\left[\hat{R}^*_t-E_t\hat{R}^*_{t+1}\right]+\frac{\psi}{1+\psi}\left[\hat{q}^*_t-E_t\hat{q}^*_{t+1}\right]\end{aligned} \tag{A13}$$

$$\begin{aligned}\rho E_t\hat{C}^*_{t+1} \quad = \quad & \rho\hat{C}^*_t+(1+\beta(\delta-1))\left[\widehat{mpk}_{N^*_{t+1}}\right]-\phi\delta\left[\hat{x}_{N^*_t}-\hat{k}_{N^*_{t-1}}\right]+\phi\delta\beta\left[\hat{x}_{N^*_{t+1}}-\hat{k}_{N^*_t}\right] \\ & -v^*\hat{T}_t-v^*\beta(\delta-1)E_t\hat{T}_{t+1}+\frac{(1-\omega)}{(\omega(1+\psi)^{(1-\kappa)}+1-\omega)}R^*_t \\ & +\frac{\beta(\delta-1)+\omega}{(\omega(1+\psi)^{(1-\kappa)}+1-\omega)}E_tR^*_{t+1}+\frac{\psi}{1+\psi}\left[\hat{q}^*_t-\beta(1-\delta)E_t\hat{q}^*_{t+1}\right]\end{aligned} \tag{A14}$$

$$\hat{k}_{F,t}= (1-\delta)\hat{k}_{F,t-1}+\delta\hat{x}_{F_t} \tag{A15}$$

$$\hat{k}_{N^*,t}= (1-\delta)\hat{k}_{N^*,t-1}+\delta\hat{x}_{N^*_t} \tag{A16}$$

$$\hat{w}^*_t= \hat{A}_{F_t}+(1-\alpha)\hat{k}_{F_{t-1}}-(1-\alpha)\hat{h}_{F,t}+v^*\hat{T}_t-\frac{(1-\omega^*)}{(\omega^*(1+\psi)^{1-\kappa}+(1-\omega)}\hat{R}^*_t-\frac{\psi}{1+\psi}\hat{q}_t \tag{A17}$$

$$\hat{w}^*_t= \hat{A}_{N^*_t}+(1-\alpha)\hat{k}_{N^*_{t-1}}-(1-\alpha)\hat{h}_{N^*t}+\frac{\omega^*(1+\psi)^{1-\kappa}}{(\omega^*(1+\psi)^{1-\kappa}+(1-\omega)}\hat{R}^*_t \tag{A18}$$

Resource constraints

$$\begin{aligned}\theta\left((v-1)\frac{C_H}{Y_H}+(v^*-1)\frac{C^*_H}{Y_H}\right)\hat{T} \quad = \quad & \frac{\kappa(1-\omega)}{(\omega(1+\psi)^{(1-\kappa)}+1-\omega)}\left[\frac{C_H}{Y_H}\hat{R}_t+\frac{C^*_H}{Y_H}\hat{R}^*_t\right]+\frac{C_H}{Y_H}\hat{C}_t+\frac{C^*_H}{Y_H}\hat{C}^*_t \\ & +\frac{X_H}{Y_H}\hat{x}_{H_t}+\frac{X_N}{Y_H}\hat{x}_{N_t}-\hat{A}_{H,t}-(1-\alpha)\hat{k}_{H,t-1}-\alpha\hat{h}_{H,t}\end{aligned} \tag{A19}$$

$$\hat{A}_{N,t}+(1-\alpha)\hat{k}_{N,t-1}+\alpha\hat{h}_{N,t}=\hat{C}_t-\kappa\hat{R}_t\left[\frac{(1-\omega)(1+\psi)^{\kappa}}{[(1-\omega)(1+\psi)^{\kappa}+\psi\omega]}-\frac{(1-\omega)}{\omega(1+\psi)^{1-\kappa}+(1-\omega)}\right] \quad \text{(A20)}$$

$$\begin{aligned}\hat{A}_{F_t}+(1-\alpha)\hat{k}_{F_{t-1}}-(1-\alpha)\hat{h}_{F,t} &= -\theta\left(v\frac{c_F}{y_F}+v^*\frac{c_F^*}{y_F}\right)\hat{T}_t+\kappa\frac{(1-\omega)}{\omega(1+\psi)^{1-\kappa}+(1-\omega)}\frac{c_F}{y_F}\hat{R}_t+\frac{c_F}{y_F}\hat{C}_t \\ &\quad +\kappa\frac{(1-\omega^*)}{\omega^*(1+\psi^*)^{1-\kappa}+(1-\omega*)}\frac{c_F^*}{y_F}\hat{R}_t^*+\frac{c_F^*}{y_F}\hat{C}_t^*+\frac{x_F^*}{y_F}\hat{x}_F^*+\frac{x_N^*}{y_F}\hat{x}_N^*\end{aligned} \quad \text{(A21)}$$

$$\hat{A}_{N,t}^*+\alpha\hat{l}_{N,t}^*+(1-\alpha)\hat{k}_{N,t-1}^*=-\kappa\left[\frac{(1-\omega^*)(1+\psi^*)^{\kappa}}{[(1-\omega^*)(1+\psi^*)^{\kappa}+\psi^*\omega^*]}\hat{R}_t^*-\frac{(1-\omega^*)}{\omega^*(1+\psi^*)^{1-\kappa}+(1-\omega^*)}\hat{R}_t^*\right]+\hat{C}_t^* \quad \text{(A22)}$$

$$\hat{h}_t=\frac{h_H}{h}\hat{h}_{H,t}+\frac{h_N}{h}\hat{h}_{N,t} \quad \text{(A23)}$$

$$\hat{h}_t^*=\frac{h_F^*}{h^*}\hat{h}_{F,t}^*+\frac{h_N^*}{h^*}\hat{h}_{N,t}^* \quad \text{(A24)}$$

GDP

$$\hat{y}_t=\frac{(\omega-1)+\omega(1+\psi)^{1-\kappa}}{\omega(1+\psi)^{1-\kappa}+(1-\omega)}\hat{R}_t+(v-1)\hat{T}_t+\hat{y}_{H,t}+\hat{y}_{N,t} \quad \text{(A25)}$$

$$\hat{y}_t^*=\frac{(\omega-1)+\omega(1+\psi)^{1-\kappa}}{\omega(1+\psi)^{1-\kappa}+(1-\omega)}\hat{R}_t^*+(\omega-1)\hat{R}_t-\widehat{RS}_t+v^*\hat{T}_t+\hat{y}_{F,t}^*+\hat{y}_{N,t}^* \quad \text{(A26)}$$

Real exchange rate

$$\widehat{RS}_t=(v-v^*+\psi)\hat{T}_t-\frac{(1-\omega)}{(\omega(1+\psi)^{(1-\kappa)}+1-\omega)}\hat{R}_t+\frac{(1-\omega^*)(1+\psi^*)^{\kappa}}{[(1-\omega^*)(1+\psi^*)^{\kappa}+\psi^*\omega^*]}\hat{R}_t^*-\frac{\psi}{1+\psi}\left[\hat{q}_t-\hat{q}_t^*\right] \quad \text{(A27)}$$

$$\hat{T}_t=\hat{T}_{t-1}+(1+\psi)^{-1}\left[\Delta s_t+\tilde{\pi}_{F,t^*}-\tilde{\pi}_{H,t}\right] \quad \text{(A28)}$$

$$\tilde{\pi}_{H,t}=-\frac{1-\omega+\psi}{\omega v+1-\omega+\psi}\left[\hat{q}_t-\hat{q}_{t-1}\right]-\frac{\omega(1-v)}{\omega v+1-\omega+\psi}\left[\tilde{\pi}_{F,t^*}+\Delta s_t\right] \quad \text{(A29)}$$

$$\tilde{\pi}_{F,t^*}=-\frac{1-\omega^*+\psi}{\omega^*(1-v^*)+1-\omega+\psi}\left[\hat{q}_t^*-\hat{q}_{t-1}^*\right]-\frac{\omega v^*}{\omega(1-v^*)+1-\omega+\psi}\left[\tilde{\pi}_{H,t}-\Delta s_t\right] \quad \text{(A30)}$$

$$\hat{q}_t = (1+\psi)\hat{R}_t + (1-v)(1+\psi)\hat{T}_t \tag{A31}$$

$$\hat{q}_t^* = (1+\psi)\hat{R}_t^* - v^*(1+\psi)\hat{T}_t \tag{A32}$$

Steady-state ratios:

$$\begin{aligned}
a_b &= \frac{\omega(1-v)\,(1+\psi)^{-\kappa}}{[\omega(1+\psi)^{1-\kappa} + (1-\omega)]} \\
\frac{\bar{c}_N}{\bar{y}_N} &= \frac{(1-\omega)(1+\psi)^{\kappa}}{[(1-\omega)(1+\psi)^{\kappa} + \psi\omega]} \\
\frac{\bar{c}_H}{\bar{y}_N} &= \frac{\omega v}{[(1-\omega)(1+\psi)^{\kappa} + \psi\omega]} \\
\frac{\bar{c}_F}{\bar{y}_N} &= \frac{\omega(1-v)}{[(1-\omega)(1+\psi)^{\kappa} + \psi\omega]}
\end{aligned}$$

$$\begin{aligned}
\frac{x_H}{y_H} &= \frac{x_N}{y_N} = \delta\left(\frac{1-\alpha}{1/\beta - 1 + \delta}\right) \text{ two identical expressions apply to the foreign economy} \\
\frac{y_H}{c_H} &= \left(\frac{1}{v} + \frac{x_N}{y_N}\frac{[(1-\omega)(1+\psi)^{\kappa} + \psi\omega]}{\omega v}\right)\left[1 - \frac{x_H}{y_H}\right]^{-1} \\
\frac{y_H}{c_H^*} &= \frac{1}{1-v} + \frac{x_H}{y_H}\frac{y_H}{c_H}\frac{v}{1-v} + \frac{x_N}{y_N}\frac{[(1-\omega)(1+\psi)^{\kappa} + \psi\omega]}{\omega v}\frac{v}{1-v} \\
\frac{x_N}{y_H} &= 1 - \frac{c_H}{y_H} - \frac{c_H^*}{y_H} - \frac{x_H}{y_H}
\end{aligned}$$

$$\begin{aligned}
\frac{\bar{c}_N^*}{\bar{y}_N^*} &= \frac{(1-\omega)(1+\psi)^{\kappa}}{[(1-\omega)(1+\psi)^{\kappa} + \psi\omega]} \\
\frac{\bar{c}_H^*}{\bar{y}_N^*} &= \frac{\omega v^*}{[(1-\omega)(1+\psi)^{\kappa} + \psi\omega]} \\
\frac{\bar{c}_F^*}{\bar{y}_N^*} &= \frac{\omega(1-v^*)}{[(1-\omega)(1+\psi)^{\kappa} + \psi\omega]}
\end{aligned}$$

$$\begin{aligned}
\frac{y_F}{c_F^*} &= \left(\frac{1}{1-v^*} + \frac{x_N^*}{y_N^*}\frac{(1-\omega^*)(1+\psi^*)^{\kappa} + \psi^*\omega}{\omega^*(1-v^*)}\right)\left[1 - \frac{x_F^*}{y_F}\right]^{-1} \\
\frac{y_F}{c_F} &= 1 + \frac{1-v^*}{v^*}\left[1 + \frac{x_F^*}{y_F}\frac{y_F}{c_F^*} + \frac{x_N^*}{y_N^*}\frac{(1-\omega^*)(1+\psi^*)^{\kappa} + \psi^*\omega}{\omega^*(1-v^*)}\right]
\end{aligned}$$

$$\begin{aligned}
\frac{h}{h_N} &= 1 + \frac{\omega(1+\psi)^{-\kappa}}{(1-\omega) + \psi\omega(1+\psi)^{-\kappa}} + \frac{x_N}{y_N} + \frac{x_H}{y_H}\frac{y_H}{c_H}\frac{c_H}{y_N} \\
\frac{h^*}{h_N^*} &= 1 + \frac{\omega^*(1+\psi^*)}{(1-\omega^*) + \psi^*\omega^*(1+\psi^*)^{-\kappa}} + \frac{x_F^*}{y_F}\frac{y_F}{c_F^*}\frac{c_F^*}{y_N^*} + \frac{x_N^*}{y_N^*}
\end{aligned}$$

## B Log-linearised model (complete markets)

The only difference relative to the incomplete markets model is the risk sharing condition. The risk-sharing condition changes from:

$$E_t\widehat{RS}_{t+1} - \widehat{RS}_t = \rho E_t[\hat{C}_{t+1} - \hat{C}^*_{t+1}] - \rho[\hat{C}_t - \hat{C}^*_t] + \varepsilon\hat{b}_t$$

to

$$\widehat{RS}_t = \rho\hat{C}_t$$

The current account equation now become redundant and can be omitted.

## C Log-linearised model (endogenous discount factor)

Compared to the baseline model, the consumer's Euler equations change:

$$\rho E_t\hat{C}_{t+1} = \rho\hat{C}_t + \frac{\beta_c(C,h)\bar{C}}{\beta(C,h)}\hat{C}_t + \hat{\imath}_t - E_t\pi_{t+1}$$

$$\rho E_t\hat{C}^*_{t+1} = \rho\hat{C}^*_t + \frac{\beta_{c^*}(C^*,h^*)\bar{C}}{\beta(C^*,h^*)}\hat{C}^*_t + \hat{\imath}^*_t - E_t\pi^*_{t+1}$$

$$E_t\widehat{RS}_{t+1} - \widehat{RS}_t = \rho E_t[\hat{C}_{t+1} - \hat{C}^*_{t+1}] - \rho[\hat{C}_t - \hat{C}^*_t]$$

an appropriate adjustment also has to be done to the captial Euler equations. The current account equation now also becomes redundant and can be deleted.

## D Data Sources

1) The series for GDP, Consumption Investment and net exports in table 3 are from the Penn World Tables.

2) Terms of trade in table 3 are taken from Datasteam and the real exchange rate is the US real effective exchange rate series from the BIS.

3) Data to construct the Solow residual are taken from the Groningen Growth and Development Centre, 60-Industry Database. We construct the industry specific Solow residuals by taking a linear detrended of $\ln A^i_t = \ln y^i_t - \alpha \ln n^i_t$ where $i$ denotes the sectors. $y^i_t$ value added in sector $i$, $n^i_t$ is hours worked in sector $i$ and $\alpha = 0.67$ as in the calibration of the model. Further details on which sectors we classed as traded and which as non-traded are available from the authors' by request.

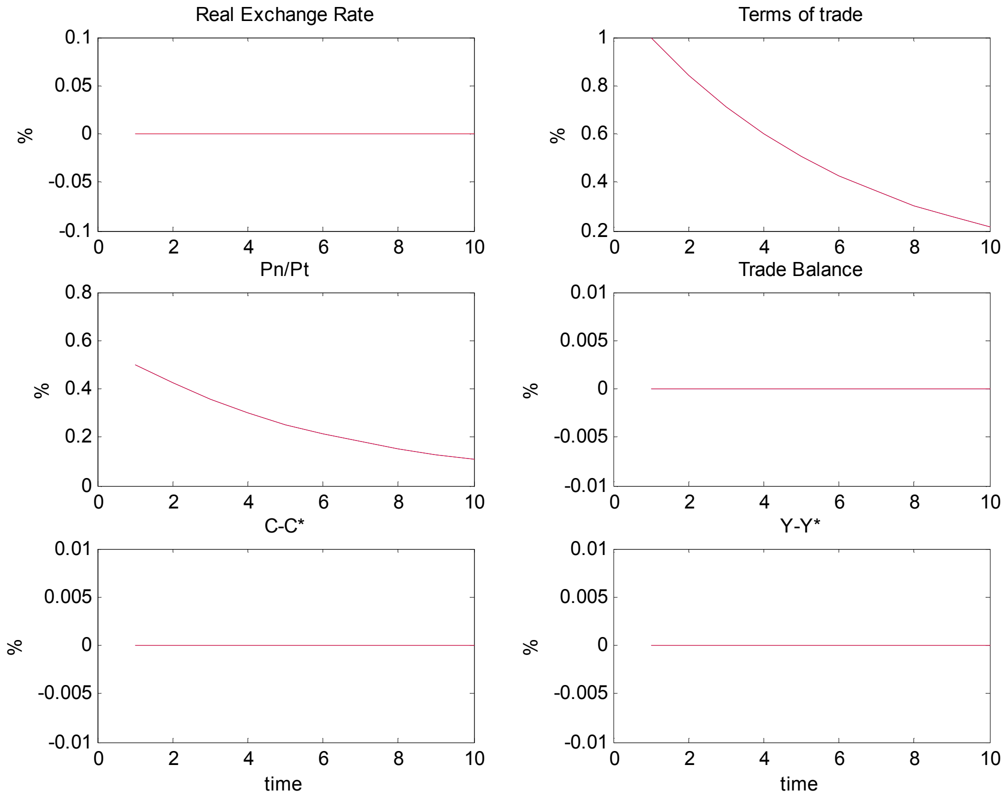


**Figure 1 A unit traded sector supply shock when ψ=0, θ = κ = 1 and the share of capital in production function is 0. All risk is shared through the terms of trade and complete and incomplete markets model yield the same allocation.**

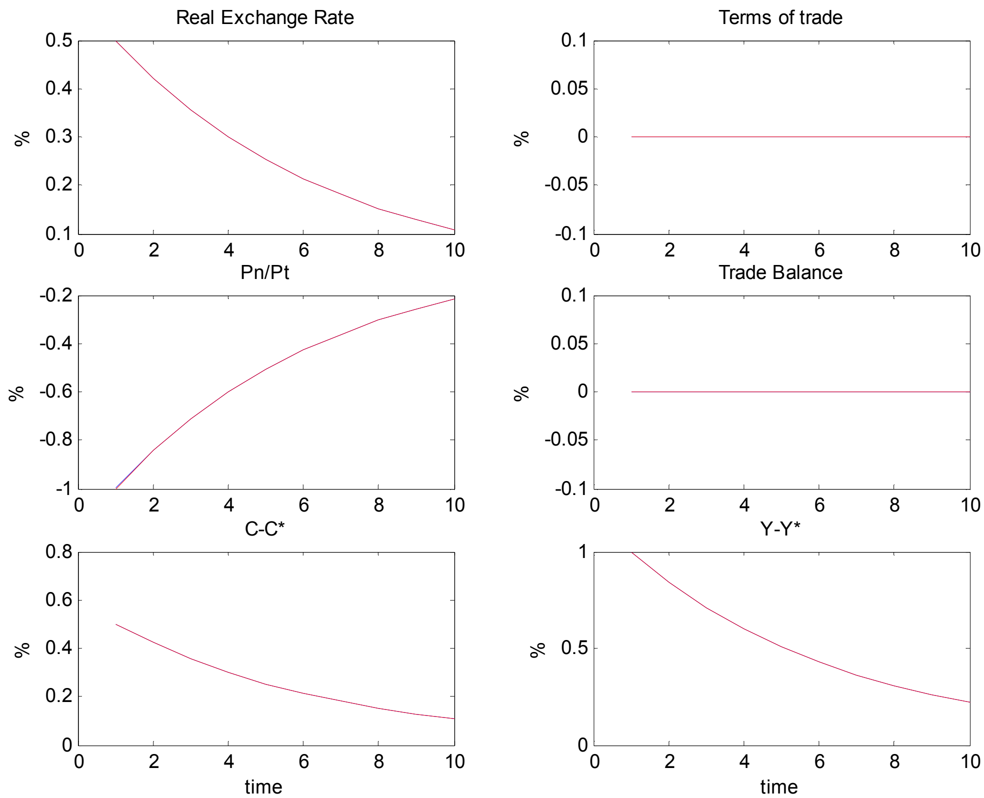


**Figure 2 A unit non-traded sector supply shock when ψ=0, θ = κ = 1 and the share of capital in production function is 0. Incomplete and complete markets model yield the same allocation. Relative price of non-traded goods adjusts so as to completely isolate the foreign economy from the shock – terms of trade and trade balance remain unaffected.**

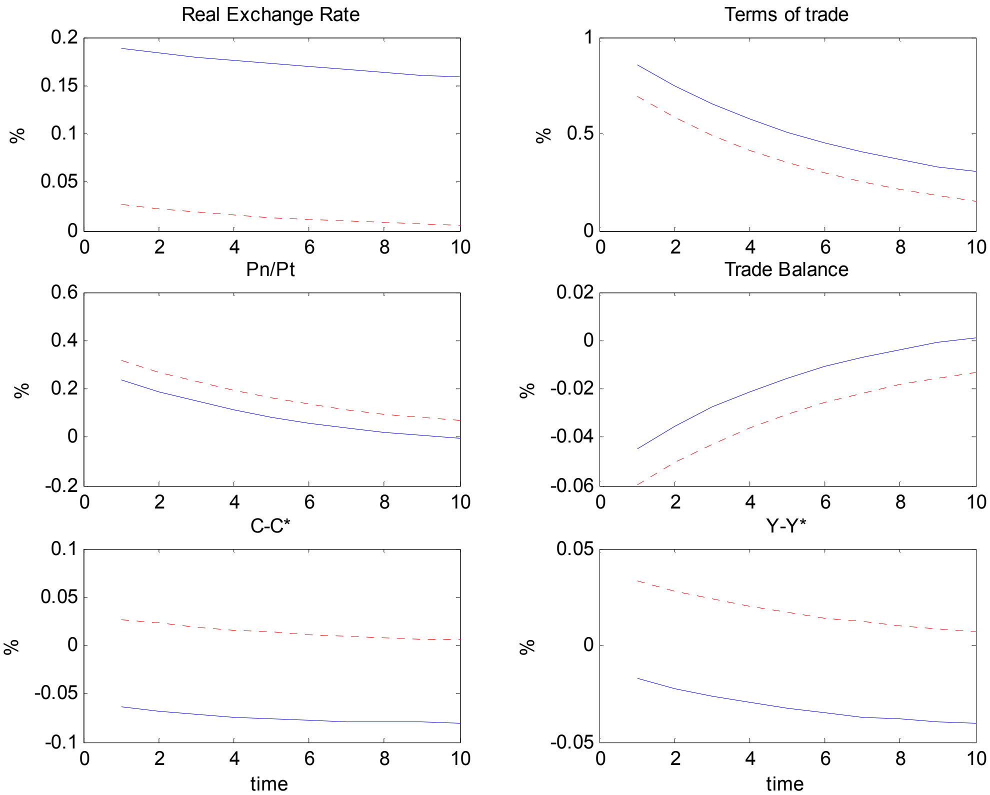


**Figure 3 A unit increase in traded sector productivity in the incomplete markets model (solid line) and complete markets model (dotted line), for a moderate distribution margin (33%).**

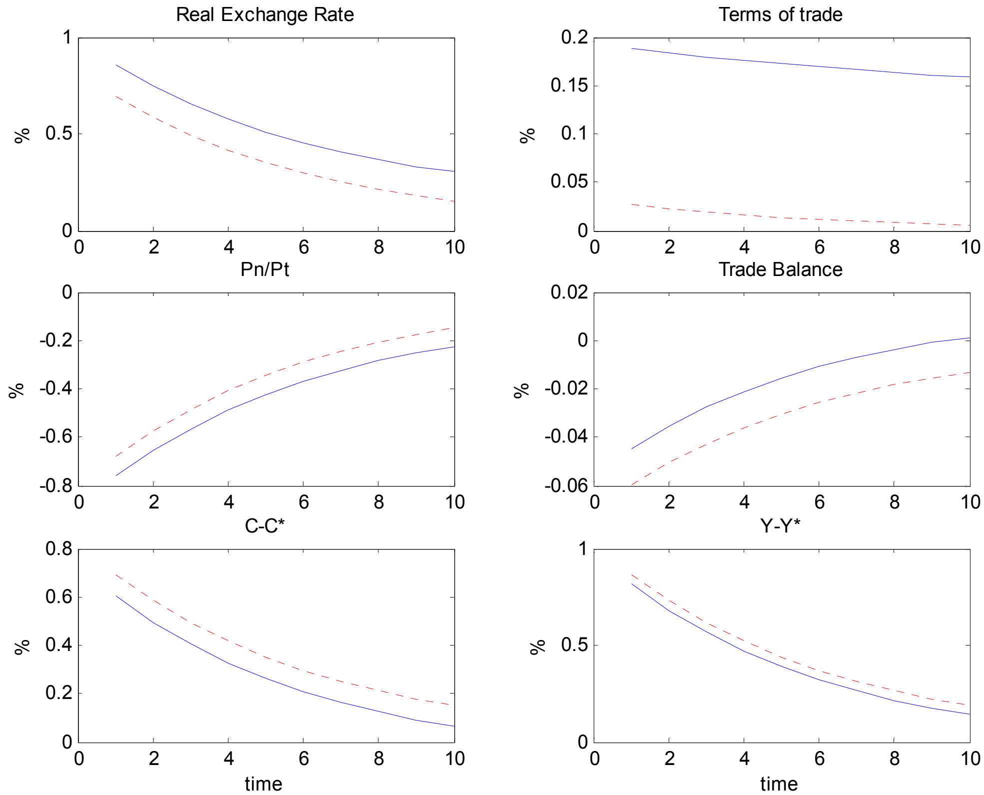


**Figure 4 A unit increase in non-traded sector productivity in the incomplete markets model (solid line) and complete markets model (dotted line), for a moderate distribution margin (33%).**

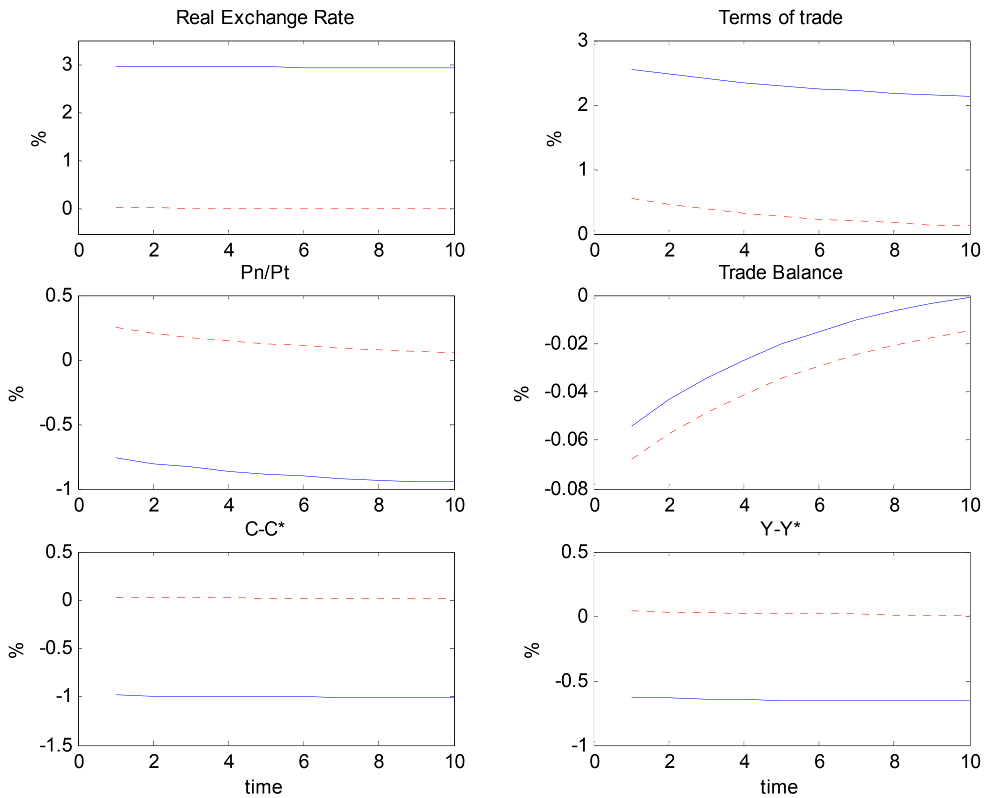


**Figure 5 A unit increase in traded sector productivity in the incomplete markets model (solid line) and complete markets model (dotted line), for a high distribution margin (48.7%).**

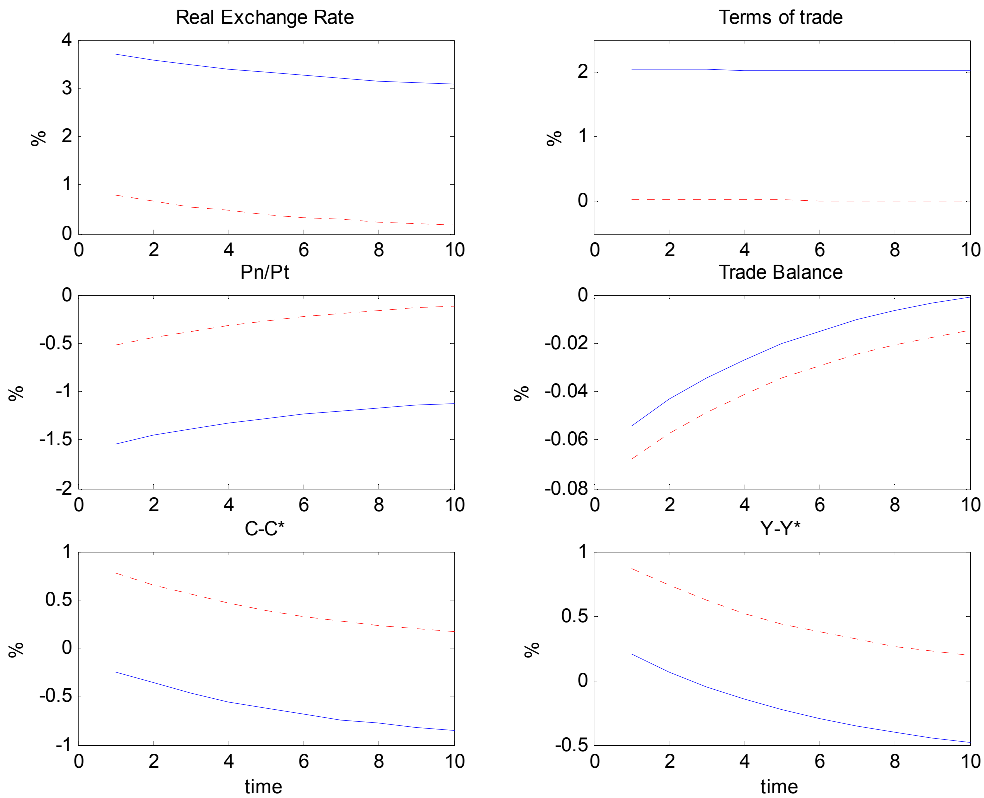


**Figure 6 A unit increase in non-traded sector productivity in the incomplete markets model (solid line) and complete markets model (dotted line), for a high distribution margin (48.7%).**

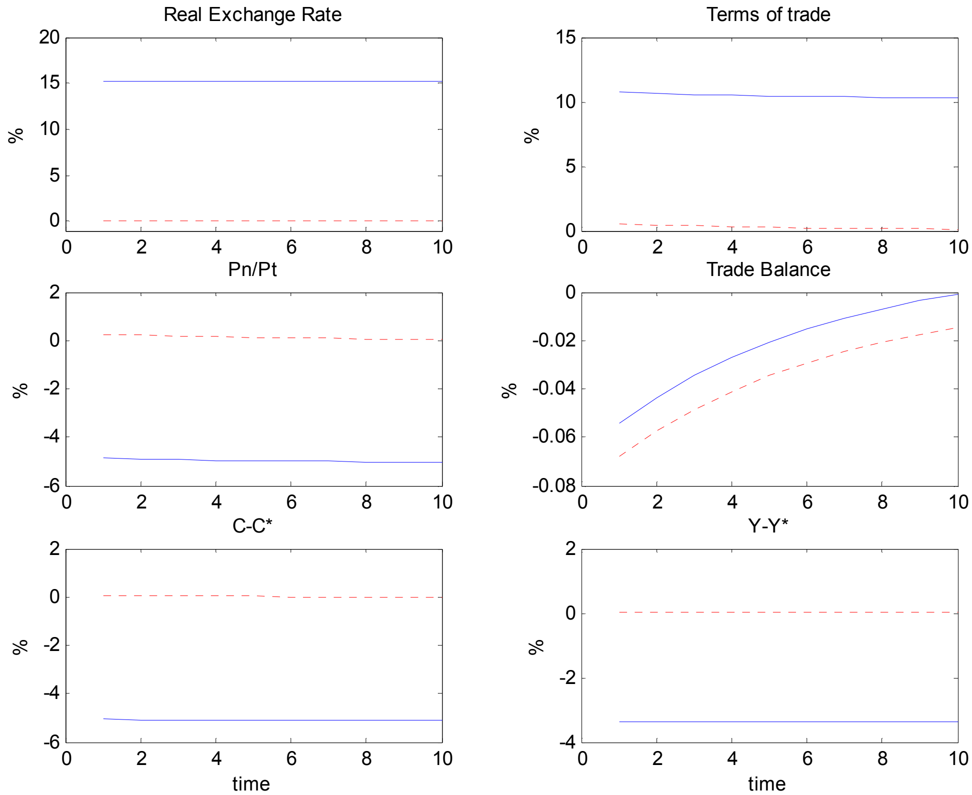


**Figure 7 A unit increase in traded sector productivity in the incomplete markets model (solid line) and complete markets model (dotted line), for a slightly higher distribution margin (49.7%).**

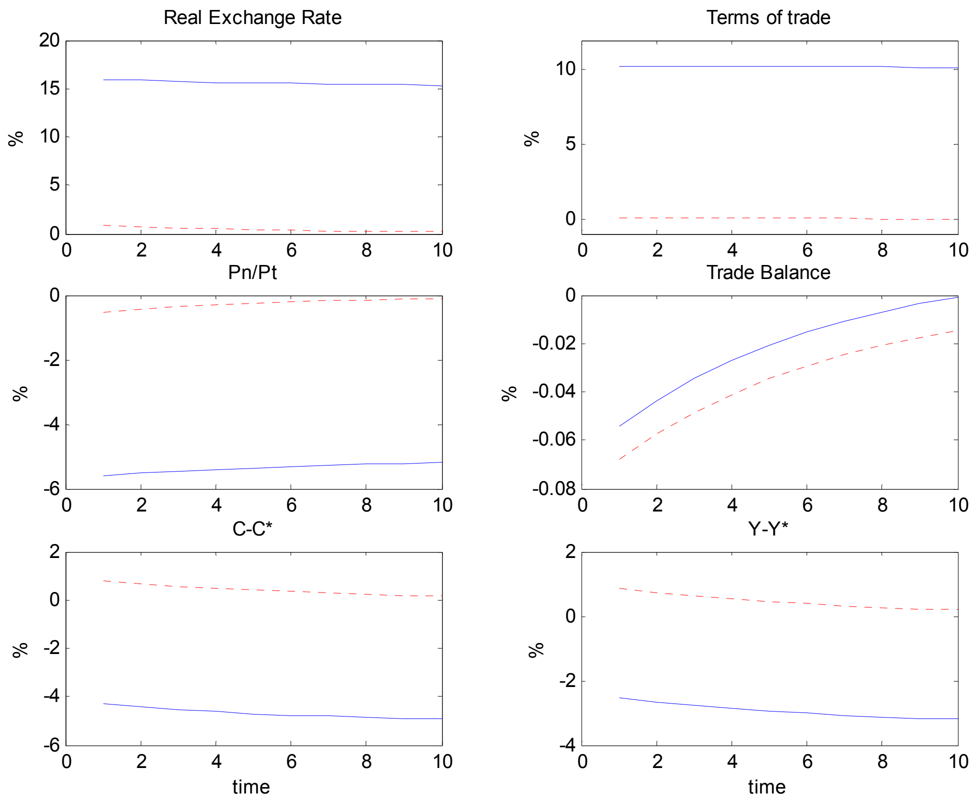


**Figure 8 A unit increase in non-traded sector productivity in the incomplete markets model (solid line) and complete markets model (dotted line), for a slightly higher distribution margin (49.7%).**

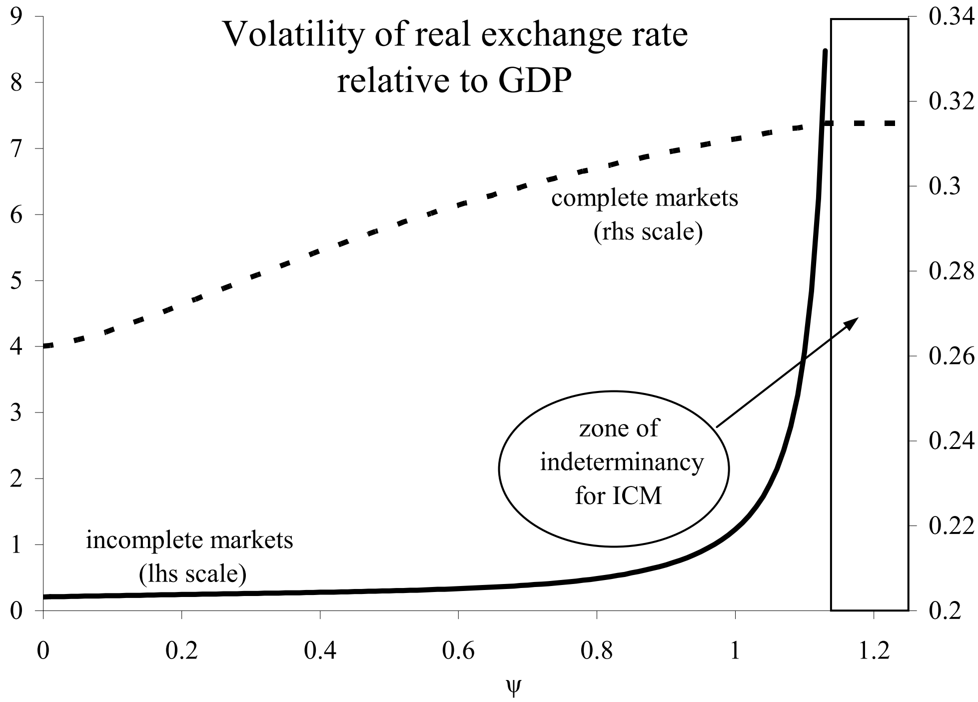


**Figure 9 Standard deviation of the real exchange rate relative to GDP for the complete markets model (dashed line, right hand side scale) and for the baseline incomplete markets model (solid line, left hand scale) for various values of the distribution cost parameter ψ, analysed under the baseline calibration. Beyond ψ = 1.13, the baseline incomplete markets model becomes indeterminate.**

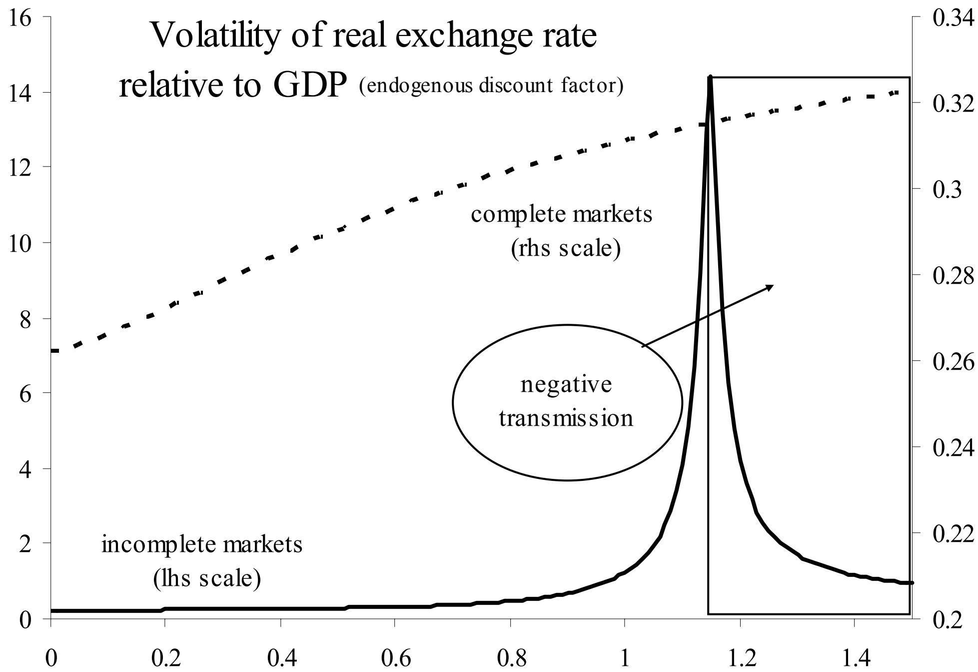


**Figure 10 Standard deviation of the real exchange rate relative to GDP for the complete markets model (dashed line, right hand side scale) and for the endogenous discount factor incomplete markets model (solid line, left hand scale) for various values of the distribution cost parameter $\psi$, analysed under the baseline calibration. Beyond $\psi = 1.13$, the real exchange rate and terms of trade appreciate following an increase in traded sector TFP – output shocks are negatively transmitted abroad.**

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