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Optimal Time Consistent Monetary Policy [*]

Tatiana Damjanovic[†]   Vladislav Damjanovic[‡]
University of St Andrews   University of St Andrews

Charles Nolan[§]
University of St Andrews

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ABSTRACT

We discuss the issue of time consistency of monetary policy. We develop a simple and intuitive procedure to derive analytically the unconditionally optimal (UO) policy in a general linear-quadratic set-up, a perspective stressed by Taylor (1979) and Whiteman (1986). We compare the UO perspective on optimal monetary policy with alternative approaches. We use our approach in simple backward- and forward-looking models and argue that the UO perspective is worthy of renewed interest.

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1. Introduction

Kydland and Prescott (1977) brought into sharp focus the issue of time inconsistency of optimal policy in macroeconomic models with forward-looking behavior and rational expectations. In so doing, they transformed the debate on how 'good' dynamic policy ought to be formulated and conducted. Many insights, and literatures, grew out of the Kydland and Prescott paper; a key insight, of course, was the desirability of rules in the conduct of policy. The question is: How should one form these rules? In this paper we take up a theme from Taylor (1979) and pursued in Whiteman (1986). They proposed searching for policies, under rational expectations, which maximize the unconditional expectation of the government's objective function. More recently, however, an alternative perspective has been adopted in theoretical research on time-consistent monetary policy. Woodford (2003), concerned with the formulation of credible policies, proposes a dynamic optimization-based method for solving for optimal policy, which he has labelled a "timeless perspective for optimal policy" (TP-policy). However, the debate continues as to the appropriate criterion for policymakers. For example, see the contributions of Soderlind (1999), Blake (2001), Jensen and McCallum (2002, 2006) and Walsh (2005); Currie and Levine (1993) is an early recognition and analysis of many of the issues that have arisen in the subsequent literature.

First, in section 2, we exposit different ways of addressing the time inconsistency issue. We first look at TP-policy strategies as this usefully sets out the key issues in forward-looking models. We also look briefly at some of the concerns with the TP approach. However, our main focus is to show how one can derive *analytically* policies that minimize the unconditional expectation of losses, what we refer to as unconditionally optimal policy or optimal unconditional

continuation policy (UO-policy). This is the first contribution of this paper. To accomplish this, as we shall see, involves taking expectations over all feasible initial conditions in constructing the optimal policy program. We derive these optimal continuation policies in a general linear-quadratic set-up. For concreteness, we then specialize these results to recover the policies advocated by Blake (2001) and Jensen and McCallum (2002) and proved to be optimal by Whiteman (1986). Whiteman's proof of optimality is somewhat algebraically involved whilst our approach is straightforward, intuitive and easy to implement and generalize. We also explain the sense in which consumers' discount rates do not matter when we construct UO-policy, an observation going back to Taylor's (1979) contribution but which has not been formally set out.

In section 3 we pursue further the potential rationale for adopting UO-policies. In formulating optimal policies it may seem natural to adopt as the criterion of policy the conditional expected discounted value of losses. However, as is well known, when there are forward-looking structural relationships the issue of time consistency is present. One approach is to minimize conditional losses subject to policy rules that have constant coefficients. However, as we show, such rules are what we call 'conditionally inconsistent' (i.e., the parameters of the optimal (simple) rule are dependent on initial conditions). This is a form of time-inconsistency but it is useful to give it a separate label to distinguish it from the case where the form of the optimal rule (e.g., the nature of the first-order conditions for a policy optimum) are different in some start-up period(s)[1].

One response to this difficulty is simply to 'ignore' the part of discounted expected losses that reflects these initial conditions; the timeless perspective offers

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[1]Some further examples may be useful. In a forward-looking model, the optimal policy is time-inconsistent but conditionally consistent, whilst the rule that minimizes the conditional discounted loss function is time consistent but conditionally inconsistent. So, conditional inconsistency is a form of time inconsistency.

a justification for this approach and, as a result, adopts as the criterion of policy the conditional variance of the arguments in the loss function. However, that is not the only response to this difficulty, and it may not be the most natural. If one adopts the unconditional value of losses as the criterion of policy, one recovers rules for policy that are time consistent, conditionally consistent and optimal within the class of policies under investigation.

In the case when the equations describing the economy are purely backward-looking it may seem that the UO-perspective has little to offer. In that case optimal policy, calculated using the discounted conditional loss function, is time-consistent (and conditionally consistent). However, even in this situation it may be possible to argue that minimizing unconditional losses is still desirable. We show that there is a trade-off between the best policy given the initial conditions and the 'most desirable' distribution from which the initial conditions are drawn. In addition, we show that the UO-policy converges to the one which maximizes the conditional loss function when the time discount rate tends to unity. In general, it is an interesting, and important, philosophical question whether we should discount the welfare of future generations, and how we should define the aggregate discount rate. Such issues were famously noted by Ramsey (1928) and more recently by Somers (1971) and Barro (1999).

Finally, in section 4 we briefly recap our key arguments and conclude.

2. Different ways to cope with time inconsistency

Most macroeconomic models that are useful for policy analysis seem to face the issue of time inconsistency. Hence, monetary policy analysis has for a long time recognized that one needs to address the incentives/criterion facing policymakers. Early suggestions included appointing a conservative central banker, proposed by Kenneth Rogoff, or the contracting approach urged by Carl Walsh. Woodford

(2003) proposes that policymakers should adopt the timelessly optimal policy; that is, the policy that would have been decided upon for the current period had such a binding decision been taken infinitely far in the past. The timelessly optimal rule emphasizes both commitment and flexibility; policymakers ought to implement policies to which it would have been optimal to commit, had a binding decision been made far in the past. However, that does not require policymakers to apply rules regardless of what other changes may occur in the economy. If there are structural changes, for instance, then policy ought to be employed as if that change had been known about infinitely far in the past. It is important to note that this perspective on optimal policy, like those employed below, remains time inconsistent in the sense of Kydland and Prescott (1977). However, they are time consistent in a more limited sense; the policy is sustainable, since it may perform better than discretionary policy, which the government will implement should it deviate from commitment. McCallum and Jensen (2006) emphasized this point and we take it up below.

2.1. The model

We turn now to formalize the timeless perspective in a general linear-quadratic framework. Consider a discounted quadratic loss function of the form

$$L_t = \frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j \left(x_{t+j} - x_{t+j}^* \right)' Q \left(x_{t+j} - x_{t+j}^* \right). \tag{2.1}$$

E_t is the expectations operator *conditional on information up through date t*, β is the time discount factor, x_t is a vector of target variables, x^* is a vector of target values which could depend on disturbance terms, and Q is a symmetric, positive definite matrix.

We define
$$x_t = \begin{bmatrix} Z_t \\ z_t \\ i_t \end{bmatrix}.$$

Here z_t is a vector of non-predetermined endogenous variables, the value of which may depend upon both policy actions and exogenous disturbances at date t, Z_t is a vector of predetermined endogenous variables (lags of variables that are included in z_t and i_t) and i_t is a vector of policy instruments, the value of which is chosen in period t.

We further assume that the evolution of the endogenous variables z_t and Z_t is determined by a system of simultaneous equations

$$\widehat{I} \begin{bmatrix} Z_{t+1} \\ E_t z_{t+1} \end{bmatrix} = A \begin{bmatrix} Z_t \\ z_t \end{bmatrix} + B i_t + C s_t, \tag{2.2}$$

where $B = \begin{bmatrix} 0 \\ B \end{bmatrix}$, $C = \begin{bmatrix} 0 \\ C \end{bmatrix}$ and s_t is a vector of exogenous disturbances.

The policy maker minimizes the loss function (2.1) subject to constraint (2.2) and given initial conditions (x_t, s_t). He searches for a policy rule of the general form

$$\phi_i' i_t + \phi_z' z_t + \phi_Z' Z_t + \phi_s' s_t = \overline{\phi}. \tag{2.3}$$

It will generally be the case that the vector $\phi = \left(\phi_i', \phi_z', \phi_Z', \phi_s', \overline{\phi} \right)$ will depend on time and/or initial conditions.

2.2. Timeless-perspective policy

One way to construct a policy which does not depend on time or initial conditions has been proposed in Woodford (2003). The following algorithm, which is well-known, will recover the optimal policy from a timeless perspective:

- Step 1: Write the conditionally expected discounted Lagrangian:

$$J_t = E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{1}{2} \left(x_{t+j} - x_{t+j}^* \right)' Q \left(x_{t+j} - x_{t+j}^* \right) + \mu_{t+j}' \left(\widetilde{A} x_{t+j} - \widetilde{I} x_{t+j+1} \right) \right],$$

(2.4)

where $\widetilde{A} := \begin{bmatrix} A & B \end{bmatrix}, \widetilde{I} := \begin{bmatrix} \widehat{I} & 0 \end{bmatrix}$, and μ_{t+j} is a vector of Lagrange multipliers associated with the constraints (2.2).

- Step 2: Write the first-order conditions with respect to the endogenous variables, x_{t+j}

$$\left(x_{t+j} - x_{t+j}^* \right)' Q + \mu_{t+j}' \widetilde{A} - \beta^{-1} \mu_{t+j-1}' \widetilde{I} = 0, \ for \ j > 0; \quad (2.5)$$

$$\left(x_t - x_t^* \right)' Q + \mu_t' \widetilde{A} = 0, \ for \ j = 0. \quad (2.6)$$

- Step 3: Ensure commitment to the policy program by 'ignoring' the first-order conditions for period zero (2.6) and replace them with (2.7):

$$\left(x_t - x_t^* \right)' Q + \mu_t' \widetilde{A} - \beta^{-1} \mu_{t-1}' \widetilde{I} = 0. \quad (2.7)$$

The following example demonstrates this algorithm in practice.

Example 2.1. *Following Clarida, Gali and Gertler (1999) consider the loss function*

$$L_t = E_t \sum_{j=0}^{\infty} \beta^j \left\{ \pi_{t+j}^2 + \alpha y_{t+j}^2 \right\}, \quad (2.8)$$

and a forward-looking Phillips curve given by

$$\pi_t = \beta E_t \pi_{t+1} + \lambda y_t + e_t, \quad (2.9)$$

where π_t is inflation at time t, y_t is the output gap, and e_t is a stationary identically

distributed shock process with finite[2] variance, σ^2. *The Lagrangian for the policy problem may be written as*

$$J_t = \sum_{j=0}^{\infty} \beta^j E_t \left\{ \left(\pi_{t+j}^2 + \alpha y_{t+j}^2 \right) + \mu_{t+j} \left[\pi_{t+j} - \beta E_t \pi_{t+1+j} - \lambda y_{t+j} - e_{t+j} \right] \right\}. \quad (2.10)$$

The commitment solution, or timelessly optimal solution is, in effect, to ignore the first-order conditions for $j = 0$. So, in any time period, we have the following pair of optimality conditions

$$\pi_t = -\frac{1}{2}\mu_t + \frac{1}{2}\mu_{t-1}; \quad (2.11)$$

$$y_t = \frac{\lambda}{2\alpha}\mu_t.$$

Hence,

$$\pi_t = -\frac{\alpha}{\lambda}(y_t - y_{t-1}). \quad (2.12)$$

(2.12) relates the path of inflation and output to one another in a manner that is commonly characterized as the timelessly optimal program.

TP-policy has an interesting property, it minimizes the conditional loss function in the case when the economy starts from steady state, $(x_t, s_t) = \phi(0, 0)$, for in this case $\mu_{t-1} = 0$ and (2.6) would be identical to (2.7). In this case, since the targeted variables can be represented as a linear combination of initial variables and future shocks, their expected value would be zero, $E_t x_{t+j} = 0$. Therefore, the conditional expectation of the loss function in this case would coincide with its conditional variance, as pointed out in Woodford (2003). Consequently, TP

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[2]In wider sense we assume that e_t can be represented as a linear combination of white noise processes, such that $e_t = \sum_{j=0}^{\infty} A_j u_{t-j}$, where $\{u_t\}$ are i.i.d. with zero mean and unit variance, such that $\sigma^2 := \sum_{j=0}^{\infty} A_j^2$ is finite.

policy would be optimal if the government were minimising conditional variance of the loss function ignoring its mean.

TP policy may be thought of as the 'opposite' of discretionary policy in the following sense. While discretionary policy gives the largest weight to utility in period zero, effectively ignoring the consequences for the future, the TP policy minimizes the discounted value of all future losses ignoring the value of initial conditions, the distribution of which depends on the policy adopted. In the following section we will discuss the policy which, from our perspective (see also Jensen and McCallum (2002, 2006)), represents a mixture of those two approaches as it minimizes the loss function "on average" or across all possible initial conditions. More precisely, it minimizes the integral of the loss function (2.1) over the distribution of initial values which is itself generated by the chosen policy.

2.3. Unconditionally optimal policy

Soderlind (1999) analyzes 'optimal simple rules' in a rational expectations model and argues that the optimal policy parameters depend on initial values. For instance, in the example just considered, Woodford's timeless perspective methodology always dominates when $y_{t-1} = 0$, for, in this case, the timeless perspective policy is the same as the optimal ('time inconsistent') policy. Jensen and McCallum (2002) and Blake and Kirsanova (2004), using the methodology described in Soderlind (1999), provide examples where, for particular initial conditions, there is a time consistent linear policy which results in smaller losses than TP-policy "on average". We shall return to this point in more detail below.

Since we cannot find, in general, time-consistent policy which is dominant for all initial conditions, it is natural to search for a class of policies which do well, in some sense, "on average", in effect treating initial conditions as a new random

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variable. The policy which we now seek to justify is one which minimizes the unconditional expectation of the loss function; this is equal to the expectation over *all possible initial states* of the economy (Taylor, 1979). More formally, then, the optimal policy from a timeless perspective that we are looking for can be defined as a policy rule $\phi' = (\phi_i', \phi_z', \phi_Z', \phi_s', \overline{\phi},)$ which minimizes the unconditional expectation ($\widetilde{E}$) of the loss function (2.1), subject to constraint (2.2):

$$\phi'^* = \arg\min \widetilde{E} L_t(\phi'). \tag{2.13}$$

We shall call such a policy "Unconditionally Optimal" and denote it UO-policy.

This basic approach to policy *evaluation*, focussing on the asymptotic variance of the arguments in a loss function, has been adopted many times recently (Rotemberg and Woodford (1997, 1998), Woodford (1999), Clarida, Gali and Getler (1999), Erceg, Henderson and Levin (2000) and Kollman (2002)). However, there is now much evidence to suggest that UO-policy and TP-policy are different.

First, in a little cited contribution, Whiteman (1986) has shown that, for precisely the economy considered in Example 2.1, the policy which minimizes the unconditional loss function is given by (2.14)

$$\pi_t = -\frac{\alpha}{\lambda}(y_t - \beta y_{t-1}), \tag{2.14}$$

rather than by (2.12) which corresponds to the TP-policy. Using a numerical algorithm Blake (2001) shows that policy (2.14) satisfies the first-order and second-order conditions for an unconditional optimum. Jensen and McCallum (2002) also make this point by computing the exact losses numerically for the case just analyzed.

Second, the Lagrangian constructed in Woodford's timeless perspective methodology depends on the consumers' discount factor, but the optimal policy which minimizes unconditional losses does not. The formal statement of that

result is provided in Proposition (2.2) which we ascribe to John Taylor as he is the first explicit reference (within the context of linear rational expectations models) to the issue of unconditionality emphasized above of which we are aware.

Proposition 2.2. *(Taylor, 1979) The time preference parameter in loss function (2.1) is not important for the UO policymaker. That is, the best UO policy minimizes losses (2.15) for all discount factors $\gamma \in (0,1)$*

$$\widetilde{E}L_t\left(\gamma\right) = \widetilde{E}E_t \sum_{j=0}^{\infty} \gamma^j l_{t+j}. \tag{2.15}$$

Here, l_t denotes the period loss function.

Proof. *It follows immediately that,*

$$\arg\min_{\phi'} \widetilde{E}L_t\left(\gamma\right) = \arg\min_{\phi'} \frac{1}{1-\gamma}\widetilde{E}l_t = \arg\min_{\phi'} \widetilde{E}l_t.$$

Hence, we have proved that the same policy is unconditionally optimal for $L_t\left(\gamma\right)$ for any $\gamma \in (0,1)$

Proposition (2.2) is additionally interesting as it demonstrates that the same policy is unconditionally optimal for all households, regardless of their individual time discount factors. For example, we may consider an overlapping generations economy populated with individuals whose life-time utility function has the form

$$U_t = -\sum_{j=0}^{n} \rho(j)l_{t+j},$$

where $\rho(j)$ represents the time discount factor for j years ahead. If we assume that the time discount rate does not depend on current welfare, the unconditionally optimal policy would not depend on the time-discounting function

$$\arg\max_{\phi'} \widetilde{E}U_t\left(\rho(.)\right) = \arg\min_{\phi'} \widetilde{E}l_t \left[\sum_{j=0}^{n} \rho(j)\right] = \arg\min_{\phi'} \widetilde{E}l_t.$$

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The 'best-on-average' criterion avoids the need for one to take a stand on what is the appropriate social discount rate; see the interesting discussions of these issues in Barro (1999) and Somers (1971).

Blake (2001), following the earlier approach of Taylor (1979), emphasizes that the unconditionally optimal time-consistent policy should coincide with a TP policy as the policymaker's discount factor approaches unity. However a formal proof of that assertion has not been provided so far[3]. We will provide one in the next section.

2.3.1. Formulating unconditionally optimal policies

In this section we shall show that a policymaker aiming to minimize unconditional losses should formulate an unconditional criterion to begin with and then calculate the first-order necessary conditions. In other words one should not try first to find the optimality conditions for a time inconsistent or conditional policy and then make the rule time-invariant by ignoring first period constraints. It would be as if one were trying to find an optimum of a composite function, that is $\arg \min f(g(x))$, by writing the first-order conditions for $g(x)$ only. From an unconditional perspective, the correct approach is to apply the unconditional expectations operator in formulating the policy Lagrangian and *then* derive the optimality conditions.

Hence, we propose the following methodology:

- Step 1: Write the conditional Lagrangian (2.4).

- Step 2: Re-formulate this as an unconditional Lagrangian:

$$J = \widetilde{E} J_t;$$

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[3]That is, with the exception of Whiteman (1986), who provided a formal proof for the model economy considered in example 2.1 with stationary and identically distributed shocks.

using the property of unconditional expectation: $\widetilde{E}x_t = \widetilde{E}x_{t+j}$, we can write

$$J = \frac{1}{1-\beta}\widetilde{E}\left(\frac{1}{2}\left(x_t - x_t^*\right)' Q\left(x_t - x_t^*\right) + \mu_t'\widetilde{A}x_t - \mu_{t-1}'\widetilde{I}x_t\right),$$

which corresponds to the Hamiltonian

$$H = \frac{1}{1-\beta}\left(\frac{1}{2}\left(x_t - x_t^*\right)' Q\left(x_t - x_t^*\right) + \mu_t'\widetilde{A}x_t - \mu_{t-1}'\widetilde{I}x_t\right)$$

- Step 3: Write the first-order conditions for the optimal timeless policy with respect to all endogenous variables;

$$\frac{\partial H}{\partial x_t} = \frac{1}{1-\beta}\left(\left(x_t - x_t^*\right)' Q + \mu_t'\widetilde{A}_t - \mu_{t-1}'\widetilde{I}\right) = 0. \qquad (2.16)$$

Condition (2.16) implies the following dynamics for the Lagrange multipliers

$$\left(x_t - x_t^*\right)' Q + \mu_t'\widetilde{A}_t - \mu_{t-1}'\widetilde{I} = 0. \qquad (2.17)$$

The general conclusion can be formulated in the following proposition.

Proposition 2.3. *The first order conditions (2.17) are the necessary conditions for problem 2.13, subject to 2.2.*

Proof.   The proof follows immediately by applying Pontryagin's Maximum Principle. ∎

We contrast equations (2.17) with (2.7) above. It is easy to see that TP-policy tends to UO-policy when discount rate tends to unity, $\beta \longrightarrow 1$.

As we shall see, Proposition 2.3 is exactly the justification required to demonstrate that the Blake-Jensen-McCallum result is the policy to minimize unconditional losses.

2.3.2. Example with forward looking Phillips Curve

Example 2.4. *We search for an unconditionally optimal policy which minimizes the loss function*

$$L_t = E_t \sum_{j=0}^{\infty} \beta^j \left\{ \pi_{t+j}^2 + \alpha y_{t+j}^2 \right\}, \tag{2.18}$$

subject to the constraint

$$\pi_t = \beta E_t \pi_{t+1} + \lambda y_t + e_t. \tag{2.19}$$

We formulate the time-dependent Lagrangian

$$J_t = E_t \sum_{j=0}^{\infty} \beta^j \left(\left(\pi_{t+j}^2 + \alpha y_{t+j}^2 \right) + \mu_{t+j} \left(\pi_{t+j} - \beta E_t \pi_{t+j+1} - \lambda y_{t+j} - e_{t+j} \right) \right).$$

Since we search for the unconditionally optimal policy, we need to minimize the "unconditional" Lagrangian, which means we must formulate the problem using the unconditional expectation of the Lagrangian, J_t:

$$J = \widetilde{E} J_t = \widetilde{E} \left(E_t \sum_{j=0}^{\infty} \beta^j \left(\left(\pi_{t+j}^2 + \alpha y_{t+j}^2 \right) + \mu_{t+j} \left(\pi_{t+j} - \beta E_t \pi_{t+j+1} - \lambda y_{t+j} - e_{t+j} \right) \right) \right).$$
$$\tag{2.20}$$

The unconditional expectations operator has the following property $\forall t, j$, $\widetilde{E} x_t = \widetilde{E} x_{t+j}$ which implies that $\widetilde{E} E_t \mu_{t+j} \pi_{t+j+1} = \widetilde{E} \pi_t \mu_{t-1}$. The unconditional Lagrangian may then be rewritten as

$$J = \frac{1}{1-\beta} \widetilde{E} \left[\left(\pi_t^2 + \alpha y_t^2 \right) + \mu_t \left(\pi_t - \lambda y_t - e_t \right) - \widetilde{E} \beta \pi_t \mu_{t-1} \right].$$

The corresponding Hamiltonian is

$$H_t = \frac{1}{1-\beta} \left[\left(\pi_t^2 + \alpha y_t^2 \right) + \mu_t \left(\pi_t - \lambda y_t - e_t \right) - \beta \pi_t \mu_{t-1} \right].$$

The first order conditions follow:

$$(1-\beta) \frac{\partial H_t}{\partial \pi_t} = \left(2\pi_t + \mu_t - \beta \mu_{t-1} \right) = 0;$$

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$$(1 - \beta)\frac{\partial H_t}{\partial y_t} = (2\alpha y_t - \lambda \mu_t) = 0.$$

These relations can be written as

$$\pi_t = -\frac{\alpha}{\lambda}y_t + \beta\frac{\alpha}{\lambda}y_{t-1}. \tag{2.21}$$

This is the optimal program proposed by Blake-Jensen-McCallum, and proved to be optimal by Whiteman (1986).

Let us consider further the origins of equations such as (2.21) in a slightly more general setting. Consider the problem of minimizing the unconditional expectation of a variable, z, which depends on an endogenous policy variable, p, and exogenous realisation of the fundamental i.i.d. shocks history $u_{t-} = \{u_{t-k}\}_{k=0}^{\infty}$, where shocks e_t can be expressed as $e_t = \sum_{j=0}^{\infty} A_j u_{t-j}$. The unconditional expectations operator can be represented in Lebesgue integral form as follows

$$\widetilde{E}z_t = \int z(p, u_{t-})d\mu,$$

where $d\mu$ is the Cartesian product of $(du_{t-k})_{k=0}^{\infty}$, and where the u_t are the basic i.i.d. shocks with zero mean and unit second moment.

We emphasize that $d\mu$ is given exogenously and does not change with policy. To maximize the integral we need to maximize the corresponding Hamiltonian, which is the expression *under* the integral, $z(p)$. Intuitively this is plausible as the policy which minimizes the objective in every state of nature (the components of the sum), will also minimize the expectation (i.e., the sum or integral). Hence, we employ the first order conditions for the Hamiltonian, $\frac{\partial z(p)}{\partial p} = 0$.

For instance, if we assume that the $e_t = u_t$, where shocks u_t are (serially) uncorrelated in example 2.4, the information space at time t will be described by the pair (y_{t-1}, e_t). It follows that the dynamic relation for the output gap is given by

$$y_t = \xi y_{t-1} + \gamma e_t, \tag{2.22}$$

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where γ and ξ are endogenously chosen policy parameters. Equation (2.22) in turn implies the following relation between the output gap and shocks

$$y_t = \sum_{i=0}^{\infty} \gamma \xi^i u_{t-i} \qquad (2.23)$$

Therefore, the loss function can be explicitly expressed in terms of the policy parameters and the realization of shocks, $L_t(\gamma, \xi, u_{t-})$. The unconditional expectation then will be represented as

$$L_u(\gamma, \xi) = \int L_t(\gamma, \xi, u_{t-}) d\mu. \qquad (2.24)$$

Instead of calculating the explicit expression (2.24) for unconditional losses in terms of policy parameters and shocks, we may employ the Lagrange method and write the unconditional Lagrangian in the form of (2.25)

$$
\begin{aligned}
J \;=\; & \int E_t \sum_{j=0}^{\infty} \beta^j \left(\pi_{t+j}^2 + \alpha y_{t+j}^2 \right) + \mu_{t+j} \left(\pi_{t+j} - \beta \pi_{t+j+1} - \lambda y_{t+j} - e_{t+j} \right) \\
& + \theta_{t+j} \left(-y_{t+j} + \xi y_{t+j-1} + \gamma e_{t+j} \right) d\mu.
\end{aligned} \qquad (2.25)
$$

The first order conditions with respect to policy parameters, ξ and γ, will reveal that the Lagrange multiplier in period $t + 1$, θ_{t+1} is uncorrelated with the information set at time t, $(E_t \theta_{t+1} e_t = 0, E_t \theta_{t+1} y_t = 0)$, and hence it is expected to be zero, $E_t \theta_{t+j} = 0$. Therefore, expression (2.22) is not binding and expression (2.25) can be simplified as in (2.20).

Finally, we note that in general measure $d\mu$ has the following property: If x_t and y_t can be represented as linear combinations of shocks, $x_t = \sum_{i=0}^{\infty} A_i u_{t-i}$ and $y_t = \sum_{i=0}^{\infty} B_i u_{t-i}$, then $\widetilde{E} x_t y_t = \sum_{i=0}^{\infty} A_i B_i \sigma^2$, which always exists when $\sum_{i=0}^{\infty} A_i^2$ and $\sum_{i=0}^{\infty} B_i^2$ are bounded. For further details see Hamilton (1999).

3. Some Arguments in Favour of Unconditional Expectation

In this section we will argue that a credible policy should satisfy certain key properties. If one aims to construct a policy program which is the result of maximization of government objectives, then that policy should have the following properties: First, it should be time consistent; second, the same rules should be defined for all initial conditions; and finally, it should maximize the government objective function on the class of policies under consideration. Formally, the optimal policy should satisfy the following:

Definition 3.1. *Policy ϕ is "time consistent" iff $\forall j > 0$, $\phi(t) = \phi(t + j)$.*

Definition 3.2. *Policy ϕ is "conditionally consistent" iff it does not depend on initial condition, $\forall x_t, s_t$, $\phi(x_t, s_t) = \phi(0, 0)$.*

Definition 3.3. *Policy ϕ is sustainable with respect to loss function L in the class $\Phi, \phi \in \Phi$, iff it is the best one in this class, $\forall \phi' \in \Phi$, $L(\phi) \leq L(\phi')$.*

In other words, one requires that the policy which maximizes a given objective function should depend neither on time (this property is, of course, simply time consistency) nor on initial conditions (we call this "conditional consistency"). If "conditional consistency" is violated, the government has an incentive to revise the policy rule as initial conditions change and this imposes a credibility problem similar to the one created by the violation of time consistency.

To illustrate the definitions, we can say that in a pure (linear) backward-looking model (a model lacking non-predetermined endogenous variables, z_t) the optimal policy is both time consistent and conditionally consistent. Kydland and Prescott (1977) have shown that in models with forward looking variables policy which minimizes the conditional loss function is time inconsistent. In view of

our analysis, we may add that it is "conditionally consistent" and sustainable with respect to the conditional loss function on the class of linear policies. TP-policy is both time and conditionally consistent by construction, however it does not minimize the conditional loss function for almost all initial conditions and therefore it is not sustainable with respect to the conditional loss function.

3.1. The principle difference between forward-looking or mixed and pure backward-looking constraints

We now underline an important principal difference between forward looking and backward looking models. It is well known that the optimal policy in an economy with only backward looking constraints is time consistent in contrast to models which contain some forward-looking components. In this section we will show that while the best conditional policy for a pure backward-looking economy is the same for all initial conditions, this is not the case for time consistent policies in forward-looking economies.

In particular, the policy parameters of the time consistent policy which minimize the conditional discounted losses depend on initial conditions. This was noted in Soderlind (1999) and demonstrated numerically in Blake and Kirsanova (2004), where Woodford's timeless perspective policy is shown to be conditionally optimal when the economy starts from steady state, $(y_{t-1} = 0)$. However, clearly that is a special case. Therefore, conditionally optimal time consistent policy is subject to the same time inconsistency problem as the best time inconsistent policy itself; as conditions change, the policymaker has an incentive to revise the policy-rules and the credibility issue is not eradicated.

3.2. Example with pure Backward Looking Phillips Curve

In the following example, the Phillips curve is purely backward-looking. In this example, the optimal conditional policy is time and conditionally consistent and sustainable with respect to the conditional loss function, which makes this policy credible.

Example 3.4. *The problem, then, is described by the loss function (3.1) and the simple backward-looking Phillips relation (3.2)[4]:*

$$L_t = (1 - \beta) E_t \sum_{j=0}^{\infty} \beta^{t+j} \frac{1}{2} \left(y_{t+j}^2 + \pi_{t+j}^2 \right);$$ (3.1)

$$y_t = \pi_t - \pi_{t-1} + e_t.$$ (3.2)

The unconditional Lagrangian may be formulated following the approach outlined earlier,

$$\widetilde{E}\mathcal{L}_t = \widetilde{E}\left[\frac{1}{2} \left(y_t^2 + \pi_t^2 \right) + \theta_t \left(y_t - \pi_t + \pi_{t-1} - e_t \right) \right];$$

$$= \widetilde{E}\left[\frac{1}{2} \left(y_t^2 + \pi_t^2 \right) + \theta_t \left(y_t - \pi_t - e_t \right) + E_t \theta_{t+1} \pi_t \right].$$

The expression under the integral is thus

$$z(y_t, \pi_t) = \frac{1}{2} \left(y_t^2 + \pi_t^2 \right) + \theta_t \left(y_t - \pi_t - e_t \right) + E_t \theta_{t+1} \pi_t,$$ (3.3)

and so we write the first order conditions

$$\frac{\partial z}{\partial y_t} = y_t + \theta_t = 0;$$

$$\frac{\partial z}{\partial \pi_t} = \pi_t - \theta_t + E_t \theta_{t+1}.$$

We now compare two policies: The first which is conditionally *the best*

$$p_c: \quad \pi_t = -y_t + \beta E_t y_{t+1};$$ (3.4)

---

[4]To simplify we assume $\lambda = 1$.

and the second, which is *unconditionally* the best

$$p_u : \pi_t = -y_t + E_t y_{t+1}. \tag{3.5}$$

For simplicity we consider the case when the e_t shocks are i.i.d. with zero mean and finite second moment, σ^2. The calculations are relatively straightforward and we relegate them to the appendix. As we show there, the best conditional and the best unconditional outturns for inflation and output may be represented as follows:

$$\begin{aligned} \pi_t &= d\left(\pi_{t-1} - e_t\right); \\ y_t &= \frac{d-1}{d}\pi_t, \end{aligned}$$

where $d = d_u = \frac{3-\sqrt{5}}{2}$, for the best unconditional policy, and $d = d_c = \frac{2+\beta-\sqrt{\beta^2+4}}{2\beta}$ for the best conditional policy. The distribution of *initial* inflation can then be shown to be given by

$$\pi_t \sim N\left(0; \frac{d^2}{1-d^2}\sigma^2\right).$$

It follows that inflation has lower dispersion when the government implements the unconditionally optimal policy.

That is, the unconditional loss function is given by

$$\widetilde{E}L_t = \left(1 + \left(\frac{d-1}{d}\right)^2\right)\frac{d^2}{1-d^2}\sigma^2 \tag{3.6}$$

and this attains a minimum when $d = d_u$.

Similarly, the conditional loss function can be shown to be

$$E_t L_t = \frac{d^2 + (1-d)^2}{1-\beta d^2}\left(\sigma^2 + (1-\beta)\pi_{t-1}^2\right)$$

which attains its minimum when $d = d_c$ and it does not depend on initial condition, π_{t-1}. When $\beta \longrightarrow 1$, it is easy to see that $E_t L_t \longrightarrow \widetilde{E}L_t$.

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In this example the asymptotic variance of output and inflation is lower under the unconditionally optimal plan, as compared with the plan that minimizes the conditional discounted loss function. This is clear from equation (3.6). So, even in this simple example, where optimal policy is time consistent, depending on one's perspective on the appropriate criterion of government (and the appropriateness of discounting) one could still offer a justification for unconditionally optimal programs.

Policy (3.4), denoted p_c, minimizes $L_t(\pi_{-1})$ for any given initial inflation, $\pi_{t-1} = \pi_{t-1}(p, e_{t-})$, which depends on the policy adopted and the history of realized shocks, (e_{t-}), while policy (3.5), denoted p_u, minimizes $L_u = \int L_t(\pi_{t-1})dF(\pi_{t-1})$, where $dF(\pi_{t-1})$ is a measure of initial inflation rates; in other words, it minimizes the loss function "on average". The conventional wisdom is that $L_t(\pi_{t-1}, e_{t-}, p_c) < L_t(\pi_{t-1}, e_{t-}, p), \forall p, \pi_{t-1}, e_{t-}$. What we have argued above is that $L_u(p_u) < L_u(p), \forall p$. The difference is explained by considering what may be thought of as 'externalities'; that is, policy will influence inflation and output in a certain way so that the policymaker determines the distribution of initial conditions, $F(\pi_{t-1})$. So, if past policymakers had implemented the best *conditional* policy rule, the current generation would face a less favorable distribution of initial conditions, $F(\pi_{t-1}, e_{t-})$, than if the government had implemented the best *unconditional* policy. Thus, there is a trade-off between the best policy and the most desirable distribution from which the initial conditions are drawn. On average the economy is better off when the government implements the unconditionally optimal policy.

Conditionally optimal policy p_c is credible if government minimizes the conditional loss function, $E_t L_t$, as is unconditionally optimal policy, p_u, if government minimizes the unconditional loss function, $\widetilde{E}L_t$. Which policy the government should adopt is a rather philosophical question as is the value of the

social discount factor (see for example Somers (1971) and Barro (1999)). However, although in this example both policies are credible and sustainable, that is not so when forward looking structural equations are present. We now turn to that case.

3.3. Example with Forward-looking Phillips curve

We consider an optimal policy for problem (2.18), (2.19) and policy in the form of (3.7)[5]

$$\pi_t = -y_t + cy_{t-1}, \tag{3.7}$$

which nests the timeless perspective policy when $c = 1$, and the unconditionally optimal policy when $c = \beta$. For simplicity, let e_t be i.i.d. with zero mean and finite dispersion, σ^2.

At any time period t the information is described by the pair (y_{t-1}, e_t), and without loss of generality the output gap, y_t, may be written as

$$y_t = dy_{t-1} + \gamma e_t, \tag{3.8}$$

which in combination with (3.7) results in a dynamic relation for inflation as follows:

$$\pi_t = (c - d) y_{t-1} - \gamma e_t; \tag{3.9}$$
$$E_t \pi_{t+1} = (c - d) y_t.$$

The Phillips curve (2.19) implies $y_t = \pi_t - \beta (c - d) y_t + e_t$, which together with (3.9) yields

$$y_t = \frac{(c - d)}{(1 + \beta c - \beta d)} y_{t-1} + \left(\frac{1 - \gamma}{1 + \beta c - \beta d} \right) e_t. \tag{3.10}$$

We may now solve for coefficients d and γ combining (3.8) with (3.10)

---

[5] We consider this class of policy for simplicity. The general policy would have the form $\pi_t = -my_t + cy_{t-1}$. However, to prove that a particular policy is not the optimal one, it is enough to show that there is a dominant policy from class (3.7).

$$d = \frac{(c-d)}{(1 + \beta c - \beta d)} =$$

$$d = \frac{2 + \beta c - \sqrt{4 + (\beta c)^2}}{2\beta} > 0;$$

$$\gamma = \frac{1 - \gamma}{1 + \beta c - \beta d} = \frac{d}{c}.$$

Following Blake (2001) we can calculate the unconditional expectation of the loss function

$$Ey_{t+k}^2 = \gamma^2 \sigma^2 \frac{1}{1 - d^2}$$

$$EL_t = E\pi_t^2 + E_t y_{t+k}^2 = \gamma^2 \sigma^2 \left[\frac{(d-c)^2 + 2 - d^2}{1 - d^2} \right], \qquad (3.11)$$

which achieves its minimum when $c = \beta$.

Now we will calculate the conditional expected discounted value of losses. Integrating forward on our expression for output we get that

$$y_{t+k} = d^{k+1} y_{t-1} + \gamma \sum_{i=0}^{k} d^i e_{t+k-i}, \qquad (3.12)$$

which allows us to compute the conditional second moments of the output gap terms

$$E_t y_{t+k}^2 = d^{2(k+1)} x_{t-1}^2 + \gamma^2 \sigma^2 \frac{1 - d^{2(k+1)}}{1 - d^2}. \qquad (3.13)$$

It follows then that:

$$
\begin{aligned}
L_t &= \left(\frac{d}{c}\right)^2 \left(1 + \frac{\beta (c-d)^2 + 1}{1 - \beta d^2}\right)\left(e_t^2 + \frac{\beta}{1 - \beta}\sigma^2\right) \\
&\quad + \left[(c-d)^2 + a^2 \left(\beta (c-d)^2 + 1\right) \frac{1}{1 - \beta a^2}\right] x_{t-1}^2 \qquad (3.14) \\
&\quad + 2\frac{a}{c}\left[(c-d) + a \left(\beta (c-d)^2 + 1\right) \frac{1}{1 - \beta a^2}\right] x_{t-1} e_t.
\end{aligned}
$$

If $x_{t-1} = 0$, the conditional expectation (3.14) reduces to (3.15)

$$L_t = \left(\frac{d}{c}\right)^2 \left(1 + \frac{\beta(c-d)^2 + 1}{1 - \beta d^2}\right)\left(e_t^2 + \frac{\beta}{1 - \beta}\sigma^2\right) \qquad (3.15)$$

which achieves its minimum when $c = 1$, which corresponds to Woodford's timeless perspective policy.

However, in general the best policy, c, depends on the three variables (e_t, x_{t-1}, σ) and is not invariant to initial conditions. Below we provide the numerical calculation for optimal c.
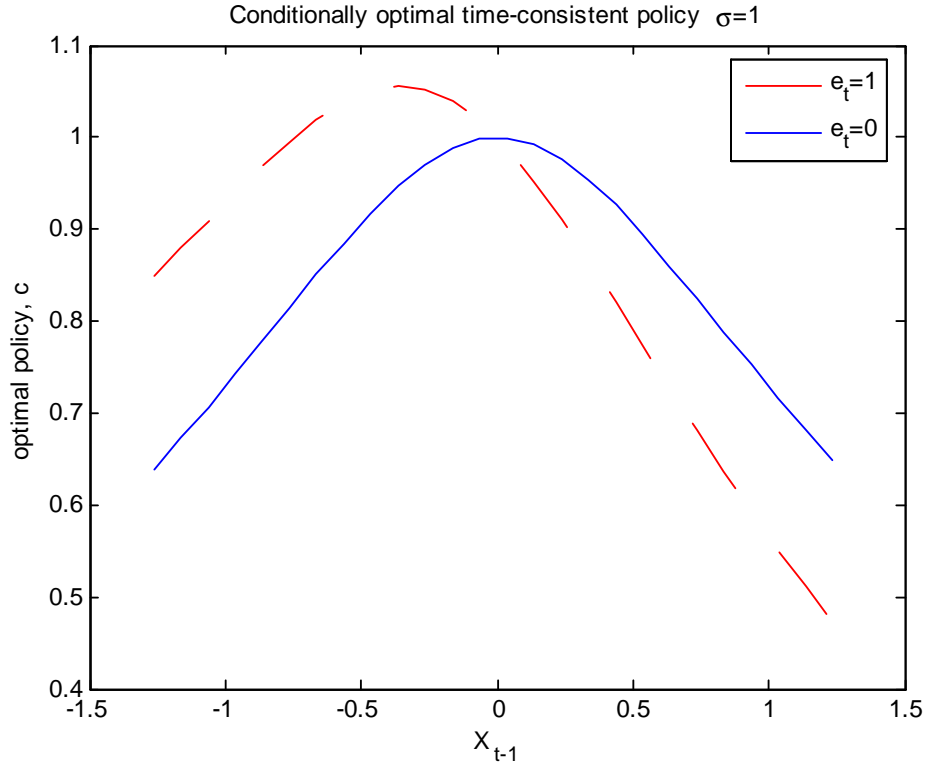
Figure 3.1: Conditionally Optimal Parameter policy, c, for time consitent policy
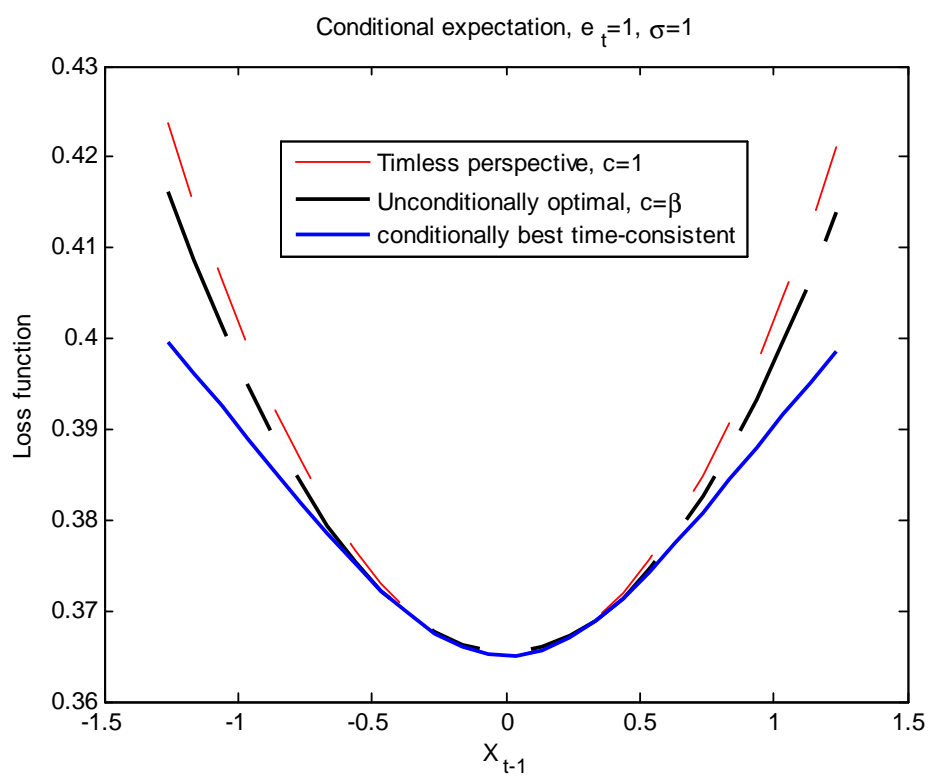
Figure 3.2: Conditional loss function for different policies

Figure 3.1 shows the policy parameter, c, for the best policy in class (3.7). It is easy to see that the optimal policy parameter, c, is different from one, $c \neq 1$, which corresponds to the timeless perspective policy. Figure 3.2 presents the value of the corresponding conditional loss function. Figures 3.1 and 3.2 demonstrate again that the conditionally optimal policy is not conditionally consistent. At the same time, the timeless perspective policy is not sustainable with respect to the conditional loss function. Therefore, one cannot use the conditional criteria for choosing time-consistent optimal policy.

4. Discussion and conclusion

In this paper we develop a simple and intuitive procedure for uncovering the unconditionally optimal policy that is applicable to a wide variety of examples currently of interest in the literature. UO-policy is time-consistent, conditionally consistent and optimal on the class of rules under consideration. We argued that this perspective on optimal policy formulation is attractive and in this we seem to be going back to a perspective urged by Taylor (1979) and Whiteman (1986) in seminal analyses of policy formulation in linear, rational expectations macroeconomic models. In a particular monetary policy example we showed that UO-policies result, on the average, in higher welfare when one uses the asymptotic loss function as the criterion of policy. However, we also demonstrated that even when time-consistency is not a problem (when the economy, unrealistically, is characterized purely by backward-looking dynamic relations) one may still offer a justification for the UO perspective on policy formulation. In the example in section 3 the key difference between the two classes of policies (conditionally optimal versus unconditionally optimal) showed up in the distribution of initial inflation.

5. Appendix

5.1. An Example With Backward Looking Phillips Curve

This appendix spells out the analysis of the model economy discussed in Section 3.2 of the paper. We recall that the e_t shocks are i.i.d. and distributed as $N(0, \sigma^2)$. Consequent on the recursive structure of the model, we note that at any point in time, t, the state of the economy may be described by the pair (π_{t-1}, e_t).

Therefore, government policy may be written as (5.1)

$$\pi_t = d\pi_{t-1} + \gamma e_t. \tag{5.1}$$

Then, combining (5.1) with (3.2) we receive

$$y_t = (d-1)\pi_{t-1} + (\gamma+1)e_t; \tag{5.2}$$

$$E_t y_{t+1} = (d-1)\pi_t. \tag{5.3}$$

5.1.1. Best unconditional policy

Let us now consider the best unconditional policy (3.5). Combining this with (5.3) we receive

$$(2-d)\pi_t = -y_t. \tag{5.4}$$

Plugging (5.1) and (5.2) into (5.4) we receive (5.5)

$$(2-d)(d\pi_{t-1} + \gamma e_t) = -(d-1)\pi_{t-1} - (\gamma+1)e_t, \tag{5.5}$$

which implies the following restrictions for coefficients α and γ

$$(2-d)d = -(d-1); \tag{5.6}$$

$$(2-d)\gamma + \gamma = -1. \tag{5.7}$$

>From these relations we receive that:

$$d = d_u = \frac{3 - \sqrt{5}}{2};$$

$$\gamma = \gamma_u = -1/(3 - d) = -\frac{2}{3 + \sqrt{5}} = -\frac{3 - \sqrt{5}}{2} = -d_u;$$

$$\pi_t = d_u (\pi_{t-1} - e_t); \tag{5.8}$$

$$y_t = -\left(\frac{d_u - 1}{d_u}\right) \pi_t. \tag{5.9}$$

5.1.2. Best conditional policy

We now consider the best conditional policy, (3.4). Combining this with (5.3) we receive (5.10)

$$\pi_t = -y_t + \beta (d - 1) \pi_t. \tag{5.10}$$

Then, combining (5.1) with (3.2) and (5.10) we receive (5.11)

$$(1 - \beta (d - 1)) (d\pi_{t-1} + \gamma e_t) = -(d - 1) \pi_{t-1} - (\gamma + 1) e_t. \tag{5.11}$$

Just as before, this relations delivers useful information on parameters α and γ

$$(1 - \beta (d - 1)) = -\frac{(d - 1)}{d};$$

$$\gamma (2 - \beta (d - 1)) = -1.$$

We can solve these as follows

$$d = d_c = \frac{2 + \beta - \sqrt{\beta^2 + 4}}{2\beta};$$

$$\gamma_c = -d_c.$$

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And so, the optimal policy then will result in the following dynamic paths:

$$\pi_t = d_c\left(\pi_{t-1} - e_t\right); \tag{5.12}$$

$$y_t = -\left(\frac{d_c - 1}{d_c}\right)\pi_t. \tag{5.13}$$

5.1.3. Unconditional expectation

We use (5.9), (5.13) to express unconditional expectations in term of inflation

$$EL_t = \frac{1}{1-\beta}\left(1 + \left(\frac{d-1}{d}\right)^2\right)E\pi_t^2. \tag{5.14}$$

We can find $E\pi_t^2$ by applying the unconditional operator to the square of (5.8) or (5.12).

$$E\pi_t^2 = d^2 E\pi_t^2 + d^2\sigma^2,$$

which results in (5.15)

$$E\pi_t^2 = \frac{d^2}{1-d^2}\sigma^2. \tag{5.15}$$

>From (5.12) we may easily conclude that the dispersion of π_t is smaller when d is smaller, and that therefore it is smaller when the government employs the unconditionally optimally policy as compared with the conditionally optimal policy.

Plugging (5.15) into (5.14) we receive the final expression for unconditional expectation of the loss criterion:

$$EL_t = \frac{1}{1-\beta}\left(1 + \left(\frac{d-1}{d}\right)^2\right)\frac{d^2}{1-d^2}\sigma^2.$$

It is then easy to show that

$$\frac{\partial EL_t}{\partial d} = \frac{\sigma^2}{1-\beta}\frac{(4d-2)\left(1-d^2\right) + 2d\left(2d^2 - 2d + 1\right)}{\left(1-d^2\right)^2};$$

$$= -\frac{2\sigma^2}{1-\beta}\frac{d^2 - 3d + 1}{\left(1-d^2\right)^2},$$

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which is zero when $d = d_u$

5.1.4. Conditional expectation

Now we can calculate the conditional expectation of the loss function, which can be simplified as (5.16)

$$E_t L_t = \left(1 + \left(\frac{d-1}{d}\right)^2\right) \sum_{j=0}^{\infty} \beta^j E_t \pi_{t+j}^2, \qquad (5.16)$$

where

$$\pi_t = d\left(\pi_{t-1} - e_t\right). \qquad (5.17)$$

Integrating (5.17) forward we receive

$$\pi_{t+k} = d^{k+1} \pi_{t-1} + \sum_{j=o,k} (-d)^{j+1} e_{t+k-j},$$

which implies

$$E_t \pi_{t+k}^2 = d^{2(k+1)} \pi_{t-1}^2 + \sum_{j=o,k} d^{2j} \sigma^2.$$

We may simplify this last expression usefully in the following way:

$$E_t \pi_{t+k}^2 = d^{2(k+1)} \pi_{t-1}^2 + d^2 \sigma^2 \frac{1 - d^{2(k+1)}}{1 - d^2}.$$

Finally, the unconditional expectation can be calculated as follows:

$$\begin{aligned} E_t L_t &= \left(1 + \left(\frac{d-1}{d}\right)^2\right) \sum \beta^k E_t \pi_{t+k}^2; \\ &= \sigma^2 d^2 \frac{1}{1 - d^2} \left(\frac{1}{1 - \beta} - \frac{d^2}{1 - \beta d^2}\right) + \frac{d^2}{1 - \beta d^2} \pi_{t-1}^2. \end{aligned}$$

And finally, it can then be shown that this can be simplified as

$$E_t L_t = \frac{2d^2 - 2d + 1}{1 - \beta d^2} \left(\sigma^2 \frac{1}{(1 - \beta)} + \pi_{t-1}^2\right).$$

It is easy then to show that for *any* exogenously given π_{t-1}, it achieves its minimum at $d = d_c$.

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