CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS
WORKING PAPER SERIES

CDMA05/09

Some Welfare Implications of Optimal Stabilization Policy in an Economy with Capital and Sticky Prices*

Tatiana Damjanovic†
University of St Andrews

Charles Nolan ‡
University of St Andrews

JULY 2005

ABSTRACT

In this paper we review and extend some of the key lessons that seem to be emerging from the Ramsey-inspired theory of dynamic optimal monetary and fiscal policies. We construct measures of the key distortions in our economy; we label these 'dynamic wedges'. Inflation, actual or anticipated, distorts these wedges in the present period, it shrinks the tax base and increases the deadweight loss. We show that, if possible, labour as well as capital ought to be subsidized in steady state. We point to a number of extensions to the Ramsey literature that may help in the formulation of actual policy.

JEL Classification: E31, E61, E62, H21.
Keywords: Optimal taxation, aggregative monetary and fiscal policies.

* This paper was prepared for a special issue of the Scottish Journal of Political Economy on Fiscal and Monetary Interactions. We should like to thank a referee and editor for helpful comments. The usual disclaimer, of course, applies.

† School of Economics and Finance, Castlecliffe, The Scores, St Andrews, Fife KY16 9AL, Scotland, UK. Tel: +44 (0) 1334 462482. E-mail: td21@st-andrews.ac.uk.

‡ School of Economics and Finance, Castlecliffe, The Scores, St Andrews, Fife KY16 9AL, Scotland, UK. Tel: +44 (0) 1334 462425. E-mail: cn14@st-andrews.ac.uk. Web: www.st-andrews.ac.uk/cdma/c.nolan.html.

CASTLECLIFFE, SCHOOL OF ECONOMICS & FINANCE, UNIVERSITY OF ST ANDREWS, KY16 9AL
TEL: +44 (0)1334 462445 FAX: +44 (0)1334 462444 EMAIL: cdma@st-and.ac.uk

www.st-and.ac.uk/cdma

1. Introduction

In this paper we want to offer a selective review, and extension, of some of the key results emerging from the literature on the conduct of optimal monetary and fiscal policy in a closed economy when nominal prices are sticky, taxes are distortionary and the debt instruments available to the government are limited to nominal period debt. Due to space considerations, we shall largely set to one side the issue of commitment. However, by way of indication where, in our view, the literature needs to go, we shall extend the framework normally encountered in the literature in three principle directions. First, in a sticky price environment, we shall model capital accumulation endogenously. Second, we shall incorporate taxation on savings as well as on labour income. Third, we construct what we call 'dynamic wedges'. These are useful summary statistics which reflect how some of the model economy's key distortions evolve through time; it is these distortions that a Ramsey strategy for optimal policy must, in some sense, address. We argue that these wedges imply that there are reasons for believing that price stability is likely to be a key pillar in any optimal monetary-fiscal programme.

2. A brief overview of the literature

For much of the post-war period monetary and fiscal policy were analysed in aggregative macroeconomic models of the IS-LM variety and there was little attempt to understand policy from a welfare maximising perspective; that development had to wait until after the advent of micro-founded general equilibrium macro-models which took root in the 1970s. However, there were some early forays attempting to infer optimal monetary policy by understanding the microeconomic distortions caused by inflation. Three key papers stand out; Bailey (1956), Friedman, (1969) and Phelps (1973). For reasons of space, we shall briefly discuss only the latter two. Friedman argued that if the marginal social cost of supplying money was approximately zero, then to attain an efficient outcome, the private marginal cost of holding money should also be zero. As the private marginal cost is the nominal interest rate, it follows that this should be approximately zero. And since the nominal interest rate is equal to the sum of the real rate and (expected) inflation, inflation should be roughly equal to the negative of the real interest rate: The Friedman Rule concludes that a policy of deflation is optimal from a welfare point of view. Phelps (1973), on the other hand, argues that since governments need to raise revenue to fund their expenditure they should optimally draw on all tax sources. The money base (seigniorage) is one such source and so welfare maximising inflation ought to be positive. Walsh (2003) and Chari and Kehoe (1999) are insightful guides as to how that particular debate played out, but these contributions demonstrate that what distortions exits in the economy will determine what optimal policy should look like. When we come to consider models with sticky prices that issue will prove to be very important.

As regards fiscal policy, Barro (1979) argued that if changing taxes is costly, then taxes (tax rates) should be smoothed through time; sharp swings in tax rates to balance the government's budget each period would be very costly and governments should issue debt in order smooth taxes (i.e., spread the distortion through time).

2.1. Ramsey approach to optimal fiscal and monetary policy

Lucas and Stokey (1983) provided the first attempt to characterise optimal macroeconomic policy in a single coherent framework by applying the Ramsey (1927) principle to optimal dynamic monetary and fiscal policy. They analysed an economy in which unavoidable (stochastic) government expenditure had to be financed out of distortionary taxation. Assuming an environment of complete contingent markets for government debt they came to a number of landmark conclusions. First, tax rates should follow the same stochastic processes as the shocks driving the economy. Second, tax rates should be smoothed over time and across states of nature. Third, state contingent debt would facilitate the smoothing of taxes across states and, under some circumstances, would render optimal policy time consistent. Fourth, the Friedman Rule is optimal; the nominal interest rate should be zero, and period by period agents in the economy should anticipate deflation.

For present purposes, two major extensions to the analysis of Lucas and Stokey (1983) need to be highlighted. First, what happens when debt instruments are limited to one-period nominal debt, a step in the direction of realism? In that case, all else equal, the fiscal authority will be motivated to try to make the nominal period debt stock behave in a manner as close as possible to a full state-contingent stock. For example, when an adverse shock comes along the only way taxes can be smoothed is for some (perhaps all) of the debt stock to be repudiated (inflated away). However the implication of these results, i.e., of smooth tax rates but potentially very volatile inflation, has proven not to be robust to the inclusion of a second distortion, sticky prices, as Siu (2004) and Schmitt-Grohe and Uribe (2004) have recently pointed out. Sticky prices cause there to be a distribution of prices in the economy and standard models imply that actual output is some way below potential output. That distortion is so costly in terms of utility that policymakers have an incentive to stabilise the price-level. This shifts much of the remaining role for policy back onto fiscal policy; tax rates may again become more volatile.

In the next section we develop a sticky-price general equilibrium model with endogenous capital accumulation, incomplete financial markets for government debt, but with a number of different tax rates upon which the government may draw to fund unavoidable expenditure. In the appendix we spell out the full Ramsey problem. However, in the main body of the text we try to keep the discussion as intuitive as possible. Hence, most of our time will be spent discussing the key distortions in our model economy that optimising policies ought to address. We provide a complete analytical solution to the model's endogenous variables in steady state and demonstrate that taxes and monopolistic 'wedges' are the only remaining distortions. However, in the short-run sticky prices are important. We demonstrate this by calculating the dynamic inefficiencies that result from sticky prices interacting with the other distortionary elements in the model economy; we label these 'dynamic wedges'. We try to tease out some implications for the optimal conduct of monetary policy from these dynamic wedges of inefficiency.

3. The Model

There are a large number of identical agents in the economy who evaluate their utility in accordance with the following criterion:

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} U(C_T, M_T/P_T, N(i)_T). \tag{3.1}$$

E_t denotes the expectations operator at time t, β is the discount factor, C is consumption, M is the nominal money stock, P is the price-level and $N(i)$ is the quantity of labour supplied to industry i. Labour is industry specific. For the moment we think of $U(\cdot)$ simply as being concave in its arguments and at least twice differentiable. Consumption is defined over a basket of goods of measure one and indexed by i in the manner of Spence-Dixit-Stiglitz

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \tag{3.2}$$

where the optimal price level is known to be

$$P_t = \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{3.3}$$

The demand for each good is given by

$$y_t^d(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t^d, \tag{3.4}$$

where Y_t^d denotes aggregate demand.

They also face a flow budget constraint,

$$\begin{aligned} \int_0^1 p_t(i)c_t(i)di + M_t + B_t + P_t(K_{t+1} - K_t) \\ = M_{t-1} + \left[1 + i_{t-1}(1 - \tau_t^k)\right] B_{t-1} \\ + P_t(1 - \tau_t^k)(\rho_t - \delta)K_t + W_t(i)N_t(i)(1 - \tau_t^h) + \Pi_t. \end{aligned}$$

As all agents are identical, the only financial assets traded in equilibrium will be those issued by the fiscal authority. Here B_t denotes the nominal value of government bond holdings at the end of date t, $1 + i_t$ denotes the nominal interest rate on this riskless one-period nominal asset, K_t is the capital stock in period t, ρ_t is the rental rate for capital and δ is the depreciation rate of capital. W_t denotes the nominal wage in period t, and Π_t is profits remitted to the individual. We assume the returns on savings are taxed at rate τ^k, while labour income is taxed at rate τ^h.

In addition to the standard boundary conditions, the necessary conditions for an optimum include:

$$U_C'(C_t, M_t/P_t, N_t) = P_t \mu_t; \tag{3.5}$$

$$\frac{U_N'(C_t, M_t/P_t, N_t)}{U_C'(C_t, M_t/P_t, N_t)} = -w_t \left(1 - \tau_t^h\right); \tag{3.6}$$

$$\frac{U_m'(C_t, M_t/P_t, N_t)}{U_C'(C_t, M_t/P_t, N_t)} = \frac{i_t(1 - \tau_{t+1}^k)}{\left(1 + i_t(1 - \tau_{t+1}^k)\right)}; \tag{3.7}$$

$$E_t \left\{ \frac{\beta U_C'(C_{t+1}, M_{t+1}/P_{t+1}, N_{t+1})}{U_C'(C_t, M_t/P_t, N_t)} \frac{P_t}{P_{t+1}} \right\} = \frac{1}{1 + i_t(1 - \tau_{t+1}^k)}. \tag{3.8}$$

Equation (3.5) denotes the real marginal utility of income. Equation (3.6) yields the supply of labour given the (post-tax) real wage, w_t. (3.7) is the money demand equation, and (3.8) indicates the growth in consumption over time; the consumption Euler equation. Optimal capital accumulation is described by (3.9) and (3.10),

$$\mu_t P_t = \Psi_t; \tag{3.9}$$

$$\beta E_t \mu_{t+1} P_{t+1} \left[\rho_{t+1} - \left(\rho_{t+1} - \delta \right) \tau_{t+1}^k \right] = \Psi_t - \beta E_t \Psi_{t+1} (1 - \delta). \tag{3.10}$$

Equation (3.9) recognizes the utility foregone from investment at date t. (3.10) captures the dynamic properties of this trade-off, such that higher capital next period, *ceteris paribus*, enables higher consumption next period. The combination of (3.9), (3.10) and the Euler equation, (3.8), yields

$$\beta E_t \frac{C_t}{C_{t+1}} \left[\left(\rho_{t+1} - \delta \right) \left(1 - \tau_{t+1}^k \right) + 1 \right] = 1. \tag{3.11}$$

The complete markets assumption implies the existence of a unique stochastic discount factor,

$$Q_{t,t+1} = \frac{\beta U_C'(C_{t+1}, N_{t+1})}{U_C'(C_t, N_t)} \frac{P_t}{P_{t+1}} \tag{3.12}$$

where

$$E_t \left\{ Q_{t,t+1} \right\} = \frac{1}{1 + i_t(1 - \tau_{t+1}^k)}.$$

We will assume that the individual's utility function is separable and isoelastic

$$U(C, M/P, N(i) = u(C; \xi) + w(M/P; \xi) - v(N(i); \xi).$$

ξ indicates a vector of exogenous shocks. We will also make the following assumptions about the functional form of the utility function:

$$u(C;\xi) = \log(C;\xi); \tag{3.13}$$

$$\upsilon(N(i);\xi) = \frac{1}{1+\upsilon}\left(N(i);\xi\right)^{1+\upsilon}; \tag{3.14}$$

$$w(M/P) = \lambda_m \log\left(\frac{M}{P};\xi\right), \tag{3.15}$$

where $\upsilon, \lambda_m > 0$.

3.1. Representative firm: factor demands

Firms are monopolistic competitors who produce their distinctive goods according to the following constant returns technology:

$$Y_t(i) = F\left(K_t(i), A_t N_t(i)\right) \equiv \left[A_t N_t(i)\right]^{\phi} K_t(i)^{1-\phi}. \tag{3.16}$$

$K_t(i)$ is the real capital stock of the firm in period t, and A_t is a stochastic productivity shock which fluctuates around a deterministic trend growth rate which at time t is denoted by γ^t. The capital stock is not firm specific but is hired on a period by period basis from agents in the economy. As a result, all firms face the same cost of capital. However, labour is industry specific and equilibrium wages depend on relative prices. Therefore marginal costs and capital-labour ratios vary across firms.

As we shall see, there are some rigidities in the setting of prices, but whether or not firms can change prices in a particular period they will meet demand at the posted price. They will aim to meet that demand with the least cost combination of factor inputs. ρ_t denotes the economy-wide rental rate for capital and $w_t(i)$ indicates the industry-wide wage. Consequently, we may write marginal cost,

$mc_t(i)$, as

$$mc_t(i) = \left(\frac{\rho_t}{1-\phi}\right)^{1-\phi}\left(\frac{w_t(i)}{\phi}\right)^{\phi} A_t^{-\phi}. \tag{3.17}$$

It follows that the optimal demand for capital and labour are given by:

$$K_t(i) = \left[A_t \frac{\rho_t}{w_t(i)} \frac{\phi}{1-\phi}\right]^{-\phi} Y_t(i); \tag{3.18}$$

$$N_t(i) = \left[A_t^{-\frac{\phi}{1-\phi}} \frac{\rho_t}{w_t(i)} \frac{\phi}{1-\phi}\right]^{1-\phi} Y_t(i). \tag{3.19}$$

3.2. Representative industry: equilibrium wage and marginal cost

We are now able to solve for the equilibrium wage in a representative industry. We equate labour demand and supply and using our expression for the demand for a particular good i, we receive (3.20)

$$w_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{\frac{-\theta v}{1+(1-\phi)v}} \left[\left(\left[\rho_t \frac{\phi}{1-\phi}\right]^{1-\phi} A_t^{-\phi} Y_t\right)^{v} C_t \frac{1}{1-\tau^h}\right]^{\frac{1}{1+(1-\phi)v}}. \tag{3.20}$$

We can then substitute (3.20) into (3.17) and get

$$mc_t(i) = \phi \left(\frac{p_t(i)}{P_t}\right)^{\frac{-\theta v \phi}{1+(1-\phi)v}} Y_t^{\frac{\phi v}{1+(1-\phi)v}} \left(\frac{C_t}{1-\tau^h}\right)^{\frac{\phi}{1+(1-\phi)v}} \left(\left[\frac{\rho_t \phi}{1-\phi}\right]^{1-\phi} A_t^{-\phi}\right)^{\frac{1+v}{1+(1-\phi)v}}.$$

These are useful formulations as they relate our variables of interest, here wages and marginal cost, to aggregate or economy-wide variables and terms in what we may usefully think of as price dispersion. We shall return to this below.

3.3. Representative firm: price setting

Price dispersion will be crucial to the aggregate dynamic outcomes of our model economy. And although in any particular industry marginal cost is constant across firms facing low demand or high demand states of the world, at the economy-wide level the capital stock is given and short-run variations in output will be closely correlated, ceteris paribus, with labour supply and the equilibrium real wage. We adopt Yun's (1996) variant of the Calvo (1983) set-up. Each period firms adjust their prices for steady state inflation. In addition, a measure, $1-\alpha$, of firms are allowed to adjust prices in a more sophisticated way. Those firms choose the nominal price which maximises their expected profits given that they may have to charge the same price, adjusted for inflation, in $k-$periods time with probability α^k.

In (3.20) we see that equilibrium wages are a function of a price dispersion term. However, we are assuming that firms are cost-takers and that they do not anticipate the change in wages in reaction to their price setting decision. The price setting problem can then be characterized as follows:

$$\max E_t \sum_{k=0}^{\infty} \Omega_{p,t+k} \left(\left(\frac{p_t(i)}{P_t} \right)^{-\theta+1} - \left(\frac{p_t(i)}{P_t} \right)_{t+k}^{-\theta} mc_{t+k}(i) \frac{P_{t+k}}{P_t \pi^k} \right),$$

where

$$\Omega_{p,t+k} = \pi^k \alpha^k \beta^k U_C'(C_{t+k}, N_{t+k}) Y_{t+k} \left(\frac{P_t}{P_{t+k}} \pi^k \right)^{-\theta+1}.$$

The optimal real relative price can be shown to be

$$\frac{p_t'(i)}{P_t} = \left(\frac{\chi_t}{F_t} \right)^{\frac{1+(1-\phi)v}{1+v+(\theta-1)\phi v}},$$

where

$$F_t = E_t \sum_{k=0}^{\infty} (\alpha\beta\pi)^k \widetilde{f}_{t+k} \left(\frac{P_{t+k}}{P_t} \pi^{-k} \right)^{\theta-1}; \tag{3.21}$$

$$\chi_t = E_t \sum_{k=0}^{\infty} (\alpha\beta\pi)^k \widetilde{x}_{t+k} \left(\frac{P_{t+k}}{P_t} \pi^{-k} \right)^{\theta+\frac{\theta v \phi}{1+(1-\phi)v}}; \tag{3.22}$$

and

$$\widetilde{f}_t = \frac{Y_t}{C_t}(\theta - 1); \tag{3.23}$$

$$\widetilde{x}_t = \frac{1}{\phi}\theta\left(A_t^{-\varphi}Y_t\left(\frac{\rho_t\phi}{1-\phi}\right)^{1-\phi}\right)^{\frac{1+v}{1+(1-\phi)v}}\left[\frac{1}{1-\tau_t^h}\right]^{\frac{\phi}{1+(1-\phi)v}} C_t^{-\frac{(1-\phi)(1+v)}{1+(1-\phi)v}}. \tag{3.24}$$

Any producer in industry i given the chance to reprice will chose the price, $p_t'(i)$. The aggregate price-level in our model evolves in the following way

$$P_t = \left[(1-\alpha)\left(p_t'\right)^{1-\theta} + \alpha\left(\pi P_{t-1}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{3.25}$$

where p_t' is the average of new prices set in period t.

3.4. Fiscal Authorities

The government purchases goods and raises revenue through taxes on capital and labour income and from seigniorage. We assume that the government can borrow by issuing a one period risk-free nominal bond. Later on, when we come to characterise the Ramsey policies in steady state it will be interesting to assume some ability to levy lump-sum taxation, but we leave that issue aside for the moment. The nominal value of government debt evolves according to the law of motion

$$B_t = (1 + i_{t-1}\left(1 - \tau_t^k\right))B_{t-1} - S_t - (M_t - M_{t-1}) \tag{3.26}$$

where B_t denotes the end of period liabilities of the government, i_t is the one-period risk free interest rate, S_t is the budget surplus

$$S_t = \int W_t(i)N_t(i)\tau_t^h + \tau_t^k(\rho_t - \delta)P_t(i)K_t(i) - G_tP_t,$$

where G_t denotes real government expenditure. We rule out equilibria related to the fiscal theory of the price level by requiring that the expected path of government surpluses must satisfy an intertemporal solvency condition, by design, for all feasible paths for the model's endogenous variables. Thus, we have that,

$$\left(1+i_{t-1}\left(1-\tau_t^k\right)\right)B_{t-1}+M_{t-1}=E_t\sum_{k=0}^{\infty}Q_{t,t+k}\left[S_{t+k}+\frac{i_{t+k}\left(1-\tau_{t+k}^k\right)}{\left(1+i_{t+1}\left(1-\tau_{t+1}^k\right)\right)}M_{t+k}\right], \tag{3.27}$$

where

$$E_tQ_{t,t+k}=E_t\frac{\beta U_C'(C_{t+k},N_{t+k})}{U_C'(C_t,N_t)}\frac{P_t}{P_{t+k}}=E_t\frac{\beta\mu_{t+k}}{\mu_t}=E_t\prod_{j=0}^{k}\frac{1}{1+i_{t+j}(1-\tau_{t+j}^k)}.$$

The present value budget constraint demonstrates one fundamental way that monetary and fiscal policy are linked; to the extent that (real) monetary growth is higher, taxes may be lower. However, in most advanced economies seigniorage is a relatively minor revenue source for government[1]. That said, once we shift our focus from monetary policy qua monetary growth, to an interest rate path there are important issues to be faced, and there may, in a certain sense, be constraints on the feasible level of the nominal interest rate (See Chadha and Nolan, 2004b).

Finally, there is an economy-wide resource constraint such that total output is equal to the sum of (private plus government) consumption and investment,

$$Y_t=C_t+G_t+I_t. \tag{3.28}$$

4. Characterising the steady-state

In this section we characterise fully the steady state of the model and examine which distortions exist in steady state. First, we make our economy stationary

[1] More fundamentally, it may play no role in an optimal funding strategy for unavoidable government spending for the reasons we briefly surveyed at the beginning of this paper. See our results below regarding the optimality of the Friedman Rule.

by stripping out the effects of trend nominal growth and productivity growth. We assume that productivity grows at a constant rate γ, $A_t = A\gamma^t$. We also assume that government expenditure is a fixed proportion of output. As shown in King, Plosser and Rebelo (2002) the form of household utility we have adopted implies that real consumption, investment, capital stock, output and government expenditure grow at the same rate: $C_t = C\gamma^t$, $I_t = I\gamma^t, K_t = K\gamma^t, Y_t = Y\gamma^t, G_t = G\gamma^t$. Labour supply will be constant in steady state, $N_t = N$, which implies that the real wage grows with productivity, $w_t = w\gamma^t$. Furthermore, we assume that the tax code remains unchanged in steady state. Inflation is constant in steady state and $P_{s+1} = \pi^s P$. With our assumptions the Euler equation (3.8) solves for the equilibrium interest rate

$$i = \left(\frac{\pi\gamma}{\beta} - 1\right)/(1 - \tau^k),$$

while the money demand equation (3.7) gives us the rate of growth of nominal money,

$$\frac{M_{s+1}}{M_s} = \gamma\pi.$$

The latter implies that $M_{s+1} = M\omega^s$, where $\omega = \gamma\pi$. All nominal variables, including debt, B, and the budget surplus, S, will grow at the same rate as nominal money. In steady state, marginal cost and the return on capital will be constant.

Equation (3.5) implies that μ_t should grow with the inverse of nominal money $\mu_{s+1} = \frac{1}{\omega}\mu_s$, while the Lagrange multiplier Ψ_s grows at the same rate as $\mu_s P_s; \Psi_{s+1} = \frac{1}{\gamma}\Psi_s$, which is implied by equation (3.9). We adopt the convenient normalizations that, in steady state, $A = 1$, $P = 1$.

4.0.1. Evolution of Capital

Equation (3.11) yields, in steady state:

$$\rho = \frac{\gamma - \beta}{\beta(1 - \tau^k)} + \delta.$$

The Euler equation solves for the interest rate in steady state

$$1 + i(1 - \tau_t^k) = \frac{\omega}{\beta}. \tag{4.1}$$

In addition, we note that in a stationary steady state of our model we have that:

$$\gamma K = (1 - \delta)K + I \Rightarrow \frac{I}{K} = \delta + \gamma - 1. \tag{4.2}$$

4.0.2. Price setting

From (3.25) we can conclude that all firms will set the same price in steady state, $p/P = 1$ (the steady-state Phillips curve is vertical). As a consequence, a steady state relation between output and consumption may be derived as follows. Recall that pricing dynamics are given by

$$\frac{p_t'}{P_t} = \left(\frac{\chi_t}{F_t}\right)^{\frac{1+(1-\phi)v}{1+v+(\theta-1)v\phi}}. \tag{4.3}$$

Combine this expression with equations (3.21) to (3.24) and note that in steady state we must have that $x = f$. As a result, we find

$$Y^v = C\left(1 - \tau^h\right)\left(\frac{\theta - 1}{\theta}\phi\right)^{\frac{1+(1-\phi)v}{\phi}} \left(\frac{\rho\phi}{1 - \phi}\right)^{-\frac{(1+v)(1-\phi)}{\phi}}. \tag{4.4}$$

This expression will be useful below when we come to calculate closed-form expressions for wages, output and consumption.

4.0.3. Steady state wage

The steady state real wage can be found from (3.20)

$$w = \left(\left[\rho \frac{\phi}{1-\phi} \right]^{(1-\phi)v} \frac{Y^v C}{1-\tau^h} \right)^{\frac{1}{1+(1-\phi)v}}. \tag{4.5}$$

Combining (4.4) and (4.5) we solve for steady state wage (4.6)

$$w = \left(\frac{\theta - 1}{\theta} \phi \right)^{1/\phi} \left(\frac{\rho\phi}{1-\phi} \right)^{-\frac{1-\phi}{\phi}}. \tag{4.6}$$

4.0.4. Steady state marginal cost

It is straightforward to show that steady state marginal cost is given by

$$mc = \frac{w^\phi}{\phi} \left[\frac{\rho\phi}{1-\phi} \right]^{1-\phi} = \frac{\theta - 1}{\theta}, \tag{4.7}$$

which is smaller than unity due to market power.

4.0.5. Solvency constraint

Finally, recall the government's flow budget constraint,

$$B_{t+1} = (1 + i_t \left(1 - \tau^k_{t+1}\right))B_t - S_{t+1} - (M_{t+1} - M_t). \tag{4.8}$$

Nominal debt grows at the same rate as money, so it follows from (3.26) that the steady state surplus may be written as

$$S = \frac{1-\beta}{\beta} B + M \frac{1-\omega}{\omega} \tag{4.9}$$

where B is detrended nominal debt, M is detrended nominal money, and S is the detrended primary surplus. This equation simply says that the primary surplus has to be sufficient to roll over the debt stock, although a higher level of seigniorage will reduce the need for factor taxation (for given G).

4.0.6. Budget surplus

We may calculate the steady state debt to GDP ratio in the following way. Note that we may write the surplus as

$$\begin{aligned} S+G &= wN\tau^h + \tau^k(\rho-\delta)K \\ &= Y\left(\tau^h\frac{\theta-1}{\theta}\phi + \tau^k(\rho-\delta)\frac{\theta-1}{\theta}\left(\frac{1-\phi}{\rho}\right)\right). \end{aligned}$$

However, using our expression for the steady-state surplus and noticing that government expenditure is a fixed proportion of output in steady state, it follows that

$$\frac{1-\beta}{\beta}B + M\frac{1-\omega}{\omega} = (\Omega_{sy}-g)\,Y,$$

where

$$\Omega_{sy} = \tau^h\frac{\theta-1}{\theta}\phi + \tau^k(\rho-\delta)\frac{\theta-1}{\theta}\left(\frac{1-\phi}{\rho}\right).$$

Using the money demand equation, equilibrium money M can the be expressed as a function of consumption

$$M = \lambda_m C\frac{\omega}{\omega-\beta}. \tag{4.10}$$

Finally, we recover,

$$\frac{1-\beta}{\beta}B + \frac{\lambda_m(1-\omega)}{\omega-\beta}C = (\Omega_{sy}-g)\,Y. \tag{4.11}$$

Relation (4.11) defines the closed-form for the steady state level of the ratio of debt to GDP, once we have expressions for output and consumption.

4.0.7. Output, consumption, capital and labour

Using the economy-wide resource constraint, the capital accumulation equation (4.2) and the demand for capital (3.18) we find that

$$\Omega_{cgy}Y = C, \tag{4.12}$$

where $\Omega_{cgy} = 1 - (\delta + \gamma - 1)\,\frac{\theta-1}{\theta}\frac{1-\phi}{\rho} - g$. Combining (4.12) and (4.4) allows us to solve for optimal output as a function of parameters and tax rates:

$$Y = \left(\frac{\theta-1}{\theta}\phi\right)^{\frac{1+(1-\phi)v}{\phi(v+1)}} \left(\frac{\rho\phi}{1-\phi}\right)^{-\frac{(1+v)(1-\phi)}{\phi(v+1)}} \left(1-\tau^h\right)^{\frac{1}{1+v}} \Omega_{cgy}^{-\frac{1}{1+v}}; \tag{4.13}$$

$$C = \left(\frac{\theta-1}{\theta}\phi\right)^{\frac{1+(1-\phi)v}{\phi(v+1)}} \left(\frac{\rho\phi}{1-\phi}\right)^{-\frac{(1+v)(1-\phi)}{\phi(v+1)}} \left(1-\tau^h\right)^{\frac{1}{1+v}} \Omega_{cgy}^{\frac{v}{1+v}}, \tag{4.14}$$

where we recall that $\rho = (\gamma - \beta) / \left(\beta(1-\tau^k)\right) + \delta$. Finally, it is straightforward to show that

$$K = \left(\frac{\theta-1}{\theta}\phi\right)^{\frac{1+v-\phi}{\phi(v+1)}} \left(\frac{\rho\phi}{1-\phi}\right)^{-\frac{1}{\phi}} \left(1-\tau^h\right)^{\frac{1}{1+v}} \Omega_{cgy}^{-\frac{1}{1+v}}; \tag{4.15}$$

$$N = \left(\frac{\theta-1}{\theta}\phi\right)^{\frac{-\phi}{\phi(v+1)}} \left(1-\tau^h\right)^{\frac{1}{1+v}} \Omega_{cgy}^{-\frac{1}{1+v}}. \tag{4.16}$$

5. The steady state wedges

In our model economy there are three basic distortions (apart from the cost of holding money). First, producers have monopolistic power. Second, there is (flat-rate) taxation of factor income. Third, there are nominal rigidities in the pricing of final output. The first two distortions are present not only on the dynamic path of the economy through time, but also in steady state. The final distortion is present only in the dynamics. We focus here on the steady-state distortions to labour supply and savings. We then go on to look at the impact of sticky prices on the dynamics of the economy.

5.1. The steady state labour wedge

The steady state labour wedge is the ratio of the marginal rate of substitution between consumption and leisure to the marginal product of labour. It is derived using (3.5) and (3.6), from the representative agent's optimality conditions. Formally the wedge is defined by (5.1):

$$\textit{labour wedge} = -\frac{U'_N}{U'_C} / \frac{\partial F(K,N)}{\partial N}. \tag{5.1}$$

We recall the labour supply equation (3.6) and apply the definition of the wedge using our particular production function:

$$\textit{labour wedge} = w\left(1-\tau^h\right)/\frac{w}{mc} = \left(1-\tau^h\right)mc = \left(1-\tau^h\right)\frac{\theta-1}{\theta}.$$

In an economy where the labour supply decision is not distorted the wedge will equal unity. This would be the case if there were no labour taxes ($\tau^h = 0$) *and* consumption goods were perfect substitutes ($\theta \to \infty$). The deviation of the wedge from unity is a measure of economic inefficiency. When taxes are positive, the labour wedge is always smaller than unity; therefore an increase in the wedge corresponds to greater efficiency. The labour wedge grows with competition and declines with tax. In the case of perfect competition it is clear that the efficient labour tax rate is zero. When competition is not perfect the government should subsidise labour ($\tau^h < 0$) in steady state. This seems clear enough from the expression above. However, we demonstrate that result formally in the appendix. It is an analagous result to that of Judd (2002) for the case of capital taxation (see below).

5.2. The steady state investment wedge

For the case of savings and investment, we can examine prices and quantities to see how our model economy deviates from an efficient outcome. First, we look at the behaviour of the rental rate on capital as that makes clear the effect of taxation. Then we examine quantities, the ratio of capital to labour will reflect the monopolistic structure of the final goods producers. Let us use the following definition of the investment wedge,

$$investment\ wedge = \frac{\beta u'(C_{t+1})}{u'(C_t)} / \int \frac{\partial \left[F(K_i, N_i) + K_i(1-\delta)\right]}{\partial K_i} di \tag{5.2}$$

The investment wedge can be thought of as the ratio between the marginal gain to consumers from savings and the marginal return of the real sector from investment. In the nondistorted steady state it should be equal to unity, which we can demonstrate using the Euler equations for savings, investment, (3.11), and the fact that in the non-distorted economy $\frac{\partial F(K_i,N_i)}{\partial K_i} = \widetilde{\rho}$, where

$$\widetilde{\rho} = \frac{\gamma}{\beta} - (1-\delta).$$

Equation (4.1) implies that $\frac{\beta u'(C_{t+1})}{u'(C_t)} = \frac{\beta}{\gamma}$ in steady state, while the factor demand equation implies that $\frac{\partial F(K,N)}{\partial K} = \frac{\rho}{mc}$ and therefore

$$investment\ wedge = \frac{\gamma}{\beta} / \left(\frac{\rho}{mc} + 1 - \delta \right). \tag{5.3}$$

ρ is the return on capital in a distorted steady state, $\rho = \frac{\gamma - \beta}{\beta(1-\tau_t^k)} + \delta$, and this clearly increases as the capital tax rate rises away from zero. When this tax is positive $\frac{\rho}{mc} > \widetilde{\rho}$, and the investment wedge is smaller than unity. Therefore, an increase in the investment wedge implies higher efficiency.

The investment wedge is falling in the market power of firms $\frac{1}{mc}$ and in taxes; in other words, the 'distance' between the efficient economy and the distorted economy increases as taxes rise and as the extent of monopoly power increases. In steady state we see that $\frac{\beta u'(C_{t+1})}{u'(C_t)} = \frac{\beta}{\gamma}$ does not depend on the capital tax rate. However, the capital tax rate makes firms worse off due to the increasing cost of capital. It follows that the efficient wedge is achieved when

$$\frac{\rho}{mc} = \widetilde{\rho}.$$

To see the impact of deviations from perfect competition we note that the steady-state capital labour ratio is given by[2]

$$\frac{K}{N} = \left(1 - \frac{1}{\theta}\right)^{\frac{1}{\phi}} \left(\frac{\rho}{1-\phi}\right)^{-\frac{1}{\phi}}.$$

The distortion to the capital-labour ratio, consequent on the monopolistic power of final producers, distorts investment behaviour in addition to taxation.

In the appendix we solve for the full dynamic Ramsey plan focussing, in particular, on the steady state. Here we provide a brief intuitive discussion of our results. First, we note that if we have access to lump sum taxation, then steady state capital taxation may be negative, as Judd (2002) found. However, we also find that labour taxes in steady state are also liable to be negative in that case. The reason for this is that the Ramsey planner is attempting to correct the distortions in the economy and a key distortion is that output in this economy is too low as a result of the monopoly distortion. Access to lump sum taxation thus frees the fiscal authorities to address that distortion using the other taxation levers. Of course, in the absence of such fiscal flexibility these results will not go through. This result is also sensitive to issues such as the extent of depreciation allowances. Before a clear conclusion emerges on optimal steady state taxes, further investigation is clearly needed.

Finally, we note that the Friedman rule is optimal, and a policy of mild deflation is prescribed (again, we prove this in the appendix). However, that conclusion may be tenuous in the sense that it relies, in part, on the assumption that all prices are equal in steady state. If there remained a distribution of prices,

[2] Of course ρ in this expression will also reflect the impact of taxation. However, if taxation were zero or even negative, such that the rate of return was at its undistorted level, then the impact of imperfect competition would still be present. The fact, however, that capital taxtation need not be zero in steady state, and may optimally be positive, serves to underline something that we emphasise later, namely that the distortions in the economy will interact in ways that will often reinforce one another.

then a policy of mild inflation may yield output gains (as in Nicolae and Nolan, forthcoming) that would have to be balanced against the costs of holding money balances.

6. Price rigidity and the dynamic wedges

For the formulation of rules governing the conduct of aggregate monetary and fiscal policies in response to shocks, be they one-off or of a more systematic nature (as in the demand and supply shocks that are thought to drive fluctuations in activity at the business cycle frequencies), we need to understand how these wedges evolve through time. The Ramsey problem set out in the appendix helps us address these issues also, in principle at least. However, the Ramsey solution can be difficult to characterise analytically. So in this section we focus on a more limited set of issues: Is monetary policy still likely optimally to be directed at price stability in the model we have developed? To cast some light on that issue, therefore, we derive expressions for the aggregate wedges as they evolve through time; what we might label 'dynamic wedges'. It becomes apparent that the distortions underlying these wedges interact in a potentially reinforcing way. For example, we shall see that price instability interacts with the tax on labour in a way that increases that distortion and pushes our model economy further from the efficient outcome. In constructing these dynamic wedges we find it useful to derive a measure of aggregate (or average) economy-wide marginal cost (which is proportional to the economy-wide capital stock and labour input). This is a little time consuming and so most of the calculations are relegated to an appendix. However, to give a flavour of the nature of the aggregation issues we face we outline the calculation of the aggregate capital stock.

$$\begin{aligned}
K_t \quad : \quad &= \int_0^1 K_t(i)di = \int_0^1 \left[A\frac{\rho_t}{w_t(i)}\frac{\phi}{1-\phi}\right]^{-\phi} Y_t(i)di = \\
&= \left[A\rho_t\frac{\phi}{1-\phi}\right]^{-\phi} Y_t \int_0^1 \left[w_t(i)^{\phi}\left(\frac{p_t(i)}{P_t}\right)^{-\theta}\right] di = \\
&= \left[A\frac{\rho_t\phi}{1-\phi}\right]^{-\phi}\left[\left(\left[\frac{\rho_t\phi}{1-\phi}\right]^{1-\phi} A_t^{-\phi}Y_t\right)^{v}\frac{C_t}{1-\tau^h}\right]^{\frac{\phi}{1+(1-\phi)v}} Y_t \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{\frac{-\theta v\phi}{1+(1-\phi)v}-\theta} di = \\
&= \left(A_t^{-\phi}Y_t\right)^{\frac{1+v}{1+(1-\phi)v}}\left[\rho_t\frac{\phi}{1-\phi}\left(\frac{1-\tau_t^h}{C_t}\right)\right]^{\frac{-\phi}{1+(1-\phi)v}} \Delta_t\left\langle -\Lambda_{50}\right\rangle . \qquad (6.1)
\end{aligned}$$

Here Δ_t is a measure of price dispersion

$$\Delta_t\left\langle -\Lambda_{50}\right\rangle = \int \left(\frac{p_t(i)}{P_t}\right)^{-\Lambda_{50}} di, \qquad (6.2)$$

and we define

$$\Lambda_{50} := \frac{\theta\,(1+v)}{1+(1-\phi)\,v}.$$

The first line of this expression uses (3.18), while the second line incorporates the Dixit-Stiglitz demand function. The third line incorporates (3.20) and the fourth line simply gathers terms. We see immediately that if prices were equal the price dispersion term would equal unity and we would have an expression for the natural level of capital[3].

6.1. The dynamic labour wedge

As we noted, the steady state labour wedge is defined as the ratio of the marginal rate of substitution between consumption and leisure and the marginal product

[3] In that case, however, consumption, output and the return on capital would also be at their natural levels. Our point is simply that, absent the price dispersion term, the expression in the text for the aggregate stock has the same functional form as an economy with no price rigidity.

of labour. The dynamic analogue of this is simply,

$$labour\ wedge_t(i) = w_t(i)\left(1-\tau^h\right)/\frac{\partial F(K_t(i),N_t(i))}{\partial N_t(i)} = \left(1-\tau_t^h\right)mc_t(i),$$

which we may rewrite as

$$labour\ wedge_t(i) = \left(1-\tau_t^h\right)mc_t(i)\left(\frac{p(i)}{P_t}\right)^{\frac{-\theta v\phi}{1+v(1-\phi)}}. \tag{6.3}$$

If we calculate the average labour wedge across industries then we can show that

$$labour\ wedge = \left(1-\tau_t^h\right)mc_t^*\Delta_t\left\langle\frac{-\theta v\phi}{1+v(1-\phi)}\right\rangle.$$

We notice that $\Delta\langle x\rangle \simeq 1+\frac{1}{2}\frac{x}{1-\theta}\left(\frac{x-1+\theta}{1-\theta}\right)var_i\left(\left[\frac{p(i)}{P_t}\right]^{1-\theta}\right)$, and we assume that our parameters are chosen in such way that inequality (6.4) holds, which is true when $v<1/(1-2\phi)$, as would normally be the case for most standard parametrizations of this model,

$$\frac{-\theta v\phi}{1+v(1-\phi)}+\theta-1 = \frac{\theta\left(1+v-2v\phi\right)+1+v(1-\phi)}{1+v(1-\phi)} > 0. \tag{6.4}$$

Then $\Delta\langle x\rangle \simeq 1-\vartheta var_i\left([p(i)/P_t]^{1-\theta}\right)$ where $\vartheta>0$, and so

$$labour\ wedge \simeq \left(1-\tau_t^h\right)mc_t^*\left[1-\vartheta var_i\left([p(i)/P_t]^{1-\theta}\right)\right]. \tag{6.5}$$

This wedge is clearly likely to be time-varying, but is it dynamic? The answer is: Yes. As we argue below, the variance term is a function of expected future inflation. That is because price setters take into account future variables when they set prices. We see that when labour taxes are set to zero and price dispersion is also zero, then the wedge is simply equal to our measure of average marginal cost, mc_t^*. However when both labour taxes are non-zero and the distribution of prices is non-degenerate, the 'distance' between our model economy and the efficient level of activity (i.e., the outcome of a perfectly competitive, flexible-price economy with no distortionary taxation) opens up. Moreover, the tax distortion and the price distortion clearly interact; a given wedge associated with some tax rate may be exacerbated by a degree of price dispersion, in the event that other instruments are unavailable to smooth the excess burden of taxation (such as state-contingent debt).

6.2. The dynamic investment wedge

Following a similar approach as for labour, we may write the dynamic aggregate wedge for savings/investment as

$$investment\ wedge = E_t\left[\frac{\pi_{t+1}}{1+i_{t+1}(1-\tau^k_{t+1})}\right] / \int\left((1-\phi)\frac{Y_t(i)}{K_t(i)}+1-\delta\right)di.$$

We recall that $Y_t(i) = Y_t\left[\frac{p(i)}{P_t}\right]^{-\theta}$, and $K_t(i) \equiv K^*_t\left(\frac{p_t(i)}{P_t}\right)^{\frac{-\theta v\phi}{1+(1-\phi)v}-\theta}$. Therefore $\frac{Y_t(i)}{K_t(i)} = \frac{Y_t}{K^*_t}\left(\frac{p_t(i)}{P_t}\right)^{\frac{\theta v\phi}{1+(1-\phi)v}}$, which implies

$$\int\left((1-\phi)\frac{Y_t(i)}{K_t(i)}+1-\delta\right)di = \frac{Y_t}{K^*_t}\Delta_t\left\langle\frac{\theta v\phi}{1+v(1-\phi)}\right\rangle+1-\delta,$$

where K^*_t is the level of capital that would be employed should all firms charge equal prices. One can show that $\Delta\left\langle\frac{\theta v\phi}{1+v(1-\phi)}\right\rangle \simeq 1 + \frac{1}{2}\frac{1}{(\theta-1)^2}\frac{\theta v\phi}{1+v(1-\phi)}\left(\frac{\theta v\phi}{1+v(1-\phi)}+\theta-1\right)var_i\left([p(i)/P_t]^{1-\theta}\right)$. Finally, we may write the investment wedge as

$$investment\ wedge = E_t\left[\frac{\pi_{t+1}}{1+i_{t+1}(1-\tau^k_{t+1})}\right] / \left(\frac{\rho_t}{mc^*_t}\left(1+\vartheta_k(\theta)\,var_i\left\{[p(i)/P_t]^{1-\theta}\right\}\right)+1-\delta\right), \tag{6.6}$$

where we defined

$$\vartheta_k(\theta) = \frac{1}{2}\frac{\theta^2}{(\theta-1)^2}\frac{v\phi}{1+v(1-\phi)}\left(\frac{v\phi}{1+v(1-\phi)}+1-\frac{1}{\theta}\right).$$

It is straightforward to show that $\frac{\partial\vartheta_k(\theta)}{\partial\theta} < 0$ which, ceteris paribus, implies that the investment wedge increases with the degree of competition (i.e., moves us closer to the efficient outcome). However, the degree of competition also impacts on marginal cost, as we saw when we calculated the steady state wedge where it showed up the marginal cost term. The overall impact on the wedge from competition is beneficial. On the other hand, an increase in the rate of capital taxation reduces investment and, other things constant, increases the return on capital, $\partial\rho_t/\partial\tau_t^k > 0$; the investment wedge is therefore declining in the capital tax. And as we found before, the level of distortion may be increasing in the degree of price dispersion in the economy. However, future expected inflation and taxation are also now important. For example, ceteris paribus, a rise in expected inflation will decrease the size of the investment wedge (moving it farther from the efficient outcome). The dominant channel appears to be that higher future inflation brings forward consumption lowering current savings. In addition, higher expected inflation will also widen current period price dispersion as producers who get a chance to change prices this period internalize that into current period pricing decisions. This anticipation of future events also impacts on the labour market wedge, making that wedge, in effect, also 'forward-looking'. In Damjanovic and Nolan (2005b) we investigate that particular effect in a simpler set-up where we can quantify the impact of inflation on taxes for a given a level of government expenditure. We also look at recent OECD data on the relationship between inflation and taxes.

We may glean a number of things from the analysis of these dynamic wedges. First, the addition of capital into this model is unlikely to overturn the broad case for price stability.[4] As we mentioned earlier, when prices are sticky, there

[4] We are deliberately leaving to one side the Friedman Rule. In calibrated models with more complex features, its prescription of deflation is often overturned. Alternatively, one may want to think of a cashless economy in which case it is redundant, and the dynamic wedges we are

is a costly misallocation of resources in the economy. The temptation to exploit inflation to emulate state-contingent government pay-outs (and hence smooth the excess burden of taxation) needs to be tempered by the realisation that such a policy will widen the dispersion of prices and move, ceteris paribus, the economy further from the efficient outcome. In our model such a policy will in general result in further distortions to the labour supply and investment decisions. An intertemporal dimension is particularly important in the case of the investment wedge. Any attempt to use surprise inflation in this economy to inflate away government debt is liable not only to increase current price dispersion but may also increase future expected inflation. In addition, a policy of systematically exploiting inflation in this way can interact with taxes further worsening the situation. Second, even when volatility in capital taxes is, other things constant, a desirable policy (as it may be, given that capital is a state variable and in inelastic supply from the date t perspective), a policy of inflation cannot help us smooth one distortion through time in this economy without having an impact on other distortions. In other words, inflation cannot easily be used to massage one distortion without having likely adverse implications for the other distortions. Finally, we note that the distortions captured in our dynamic wedges can be shown to imply that above some critical value for inflation, further inflation unambigously increases the distance between the distorted economy and the efficient outturn. This is a somewhat subtle issue which we explore further in Damjanovic and Nolan (2005a).

---

considering are the key distortions. Finally, in practice deflation rarely seems to appeal to central bankers as a part of an optimal monetary policy.

7. Conclusions and directions for future research

In this paper we have reviewed some aspects of the literature on optimal monetary and fiscal policy. We have extended this discussion into an environment where price are sticky and capital is endogenous. We clarified the distortions that existed in that model economy, both in steady state and dynamically. We concluded that when government debt is not state-contingent and prices are sticky then an efficient monetary policy is unlikely to seek to use inflation to ameliorate the remaining distortions in the economy. Incorporating capital into this picture, we conjectured, seems to bolster further the case for price stability.

There are a number of avenues of investigation that need to be pursued. The arguments in this paper, with respect to the dynamic properties of monetary policy, were somewhat loose and intuitive. A more formal characterization of the dynamic Ramsey programme for both aggregate monetary and fiscal policy is required, incorporating a richer array of fiscal instruments than is present hitherto in the literature. That work is in progress (Damjanovic and Nolan, 2005a,b).

Second, the (largely separate) literatures on optimal monetary and fiscal policy have found many additional distortions to be empirically important. For example, in the monetary literature sticky wages, habits in consumption, rule of thumb behaviour, learning, to name but a few, have been mentioned by a number of economists as important components of realistic macroeconomic models. The fiscal literature has also noted the importance of some factors: The nature of government spending (on final consumption or for employment purposes), the degree of monopoly power, the size of depreciation allowance, the potential importance of the automatic stabilisers, have all been highlighted. Understanding how these factors interact and influence (jointly) optimal monetary and fiscal policies is likely to yield important insights.

Finally, an important avenue for investigation would seem to be the incorporation of non-Ricardian elements into the model. Again, this seems empirically realistic, although it may complicate considerably the characterization of the Ramsey plans.

References

[1] Bailey, M. J. (1956). The welfare cost of inflationary finance. *Journal of Political Economy*, April pp. 93-110.

[2] Barro R. J. (1979). On the determination of the public debt. *The Journal of Political Economy*, Vol. 87, No. 5, Part 1 October, pp. 940-971.

[3] Calvo, G. (1983). Staggered contracts in a utility-maximizing framework. *Journal of Monetary Economics*, September Vol. 12, 383–98.

[4] Chadha, J. S. and Nolan C. (2003). The interaction of monetary and fiscal policy: solvency and stabilisation issues, in *Essays in Honour of Sir Alan Walters*, Edward Elgar, edited by Kent Matthews and Patrick Minford.

[5] Chadha, J. S. and Nolan, C. (2004a). On the interaction of monetary and fiscal policy, in *Dynamic Macroeconomic Analysis: - Theory and Policy in General Equilibrium* edited by Sumru Altug, Jagjit Chadha and Charles Nolan, Cambridge University Press.

[6] Chadha, J. S. and Nolan, C. (2004b). Interest rate bounds and fiscal policy. *Economics Letters*, Volume 84, Issue 1 , July 2004, Pages 9-15.

[7] Chadha, J. S. and Nolan, C. (2007). Optimal Simple Rules for the Conduct of Monetary and Fiscal Policy, forthcoming, *Journal of Macroeconomics*.

[8] Chari, V. V. and Kehoe, P. (1999). Optimal Fiscal and Monetary Policy, in Handbook of Macroeconomics, Vol.1C, ed. John B. Taylor and Michael Woodford, Elsevier Science, North-Holland, 1999.

[9] Damjanovic, T. and Nolan, C. (2005a). Monetary policy, fiscal policy and the welfare analysis of optimal stabilization policy in an economy with sticky prices and capital. In progress.

[10] Damjanovic, T. and Nolan, C. (2005b). The interaction between fiscal and monetary policies: taxes and inflation. Forthcoming in *The Oxford Review of Economic Policy* (November).

[11] Friedman, M. (1969). *The Optimum Quantity of Money and Other Essays*, University of Chicago Press.

[12] Guo J-T. and Lansing, K.J. (1999). Optimal taxation of capital income with imperfectly competitive product markets. *Journal of Economic Dynamics and Control*, 23 pp. 967-995.

[13] Judd, K. (2002). Capital-Income taxation with imperfect competition. *American Economic Review* 92, May, 417-421.

[14] King, R. G., Plosser, C. I. and Rebelo, S. T. (2002). Production, growth and business cycles: technical appendix. *Computational Economics*, 20, 87-116.

[15] Lucas, R. E., Jr. and Stokey, N. L.(1983). Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics*, 12: 55-93.

[16] Nicolae, A. and Nolan, C. (forthcoming). The impact of imperfect credibility in a transition to price stability. *The Journal of Money, Credit, and Banking.*

[17] Phelps, E. S. (1973). Inflation in the theory of public finance. *Swedish Journal of Economics*, March, pp. 67-82.

[18] Ramsey, F. P. (1927). A contribution to the theory of taxation. *Economic Journal* 37: 47–61.

[19] Schmitt-Grohe, S. and Uribe, M. (2004). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory* 114, February, 198-230.

[20] Siu, H. (2004). Optimal fiscal and monetary policy with sticky prices. *Journal of Monetary Economics* 51(3).

[21] Walsh, C. (2003), *Monetary Theory and Policy*, MIT Second Edition.

[22] Yun, T (1996). Nominal price rigidity, money supply endogeneity and business cycles', *Journal of Monetary Economics* 37: 345-370.

8. Appendix

8.1. The Ramsey problem

In this appendix we set out the Ramsey problem. Our principal aim is methodological; we want to set out the key steps in characterising the Ramsey plan. We then emphasise some central steady state results. We do not pursue to any extent the characterisation of the fully dynamic Ramsey plan (see Damjanovic and Nolan (2005a) for further analysis).

Household utility is described by

$$U_{t0} = \sum_{k=0}^{\infty} \beta^k \left[\log C_{t+k} - \frac{N_{t+k}^{v+1}}{v+1} + \lambda_m \log m_{t+k} \right].$$

We know that money demand is optimally described by

$$m_t = C_t \frac{1 + i_t^k}{i_t^k}. \tag{8.1}$$

So we may write the utility function in the following way

$$U_{t0} = \sum_{k=0}^{\infty} \beta^k \left[(1+\lambda_m) \log C_{t+k} - \frac{N_{t+k}^{v+1}}{v+1} + \lambda_m \left(\log \left(1 + i_{t+k}^k\right) - \log i_{t+k}^k \right) \right].$$

We need to maximize utility subject to the agent's budget constraint, market clearing conditions and the agent's Euler equations.

We may write the present value budget constraint as

$$W_{t0} = \sum_{k=0}^{\infty} \beta^k \Delta_{t+k} \langle -\Lambda_{50} \rangle \left[-N_{t+k}^{v+1} + 1 - \frac{Y_{t+k}}{C_{t+k}} (1 - mc_{t+k}) \right] - \frac{G_{t+k}}{C_{t+k}} (1 - \Delta_{t+k} \langle -\Lambda_{50} \rangle) + \lambda_m \tag{8.2}$$

where

$$W_t := \left(\left(1 + i_t \left(1 - \tau_t^k\right)\right) B_t + M_t \right) \frac{1}{C_t P_t} + \Delta_t \langle -\Lambda_{50} \rangle \frac{1}{\beta} u_{c,t-1} K_t^*.$$

This expression corresponds to the private agent's analogue to (3.27) in the main text. Market clearing requires that

$$(A_t N_t)^{\phi} K_t^{1-\phi} = C_t + G_t + K_{t+1} - (1-\delta)K_t,$$

so that our present-value constraint becomes

$$W_{t0} = \sum_{k=0}^{\infty} \beta^k \Delta_t \langle -\Lambda_{50} \rangle \left[-N_{t+k}^{v+1} + 1 - \frac{(A_{t+k} N_{t+k})^{\phi} K_{t+k}^{1-\phi}}{C_{t+k}} (1 - mc_{t+k}) \right] - \frac{G_{t+k}}{C_{t+k}} (1 - \Delta_{t+k} \langle -\Lambda_{50} \rangle) + \lambda_m.$$

The consumption Euler equation, (3.8) from the main text, is given by

$$\frac{\beta C_{t+k}}{C_{t+k+1}} \frac{\pi_{t+k}}{\pi_{t+k+1}} = \frac{1}{1 + i_{t+k+1}^k}.$$

Equation (3.25) provides us with the law of motion for inflation and may usefully be rewritten as

$$\Delta_{t+1} \langle -\Lambda_{50} \rangle = (1-\alpha) \left(\frac{1 - \alpha (\pi_{t+1})^{\theta-1}}{(1-\alpha)} \right)^{\frac{\varkappa}{1-\theta}} + \alpha \Delta_t \langle -\Lambda_{50} \rangle \pi_{t+1}^{-\Lambda_{50}}, \tag{8.3}$$

where we define

$$\pi_{t+1} := \frac{P_{t+1}}{P_t \pi}.$$

We also make the standard assumption that the nominal interest rate is positive

$$i_t^k \geqslant 0. \tag{8.4}$$

The problem facing the "planner" is then given by the following Lagrangian function

$$
\begin{aligned}
L &= \sum_{k=0}^{\infty}\beta^k\left[(1+\lambda_m)\log C_{t+k}-\frac{N_{t+k}^{v+1}}{v+1}+\log\left(1+i_{t+k}^k\right)-\log i_{t+k}^k\right]+ \\
& d_1\sum_{k=0}^{\infty}\beta^k\left[\begin{array}{c}\Delta_t\left\langle-\Lambda_{50}\right\rangle\left[-N_t^{v+1}+1-\frac{(A_{t+k}N_{t+k})^{\phi}K_{t+k}^{1-\phi}}{C_{t+k}}\left(1-mc_{t+k}\right)\right] \\ -\frac{G_{t+k}}{C_{t+k}}\left(1-\Delta_{t+k}\left\langle-\Lambda_{50}\right\rangle\right)+\lambda_m\end{array}\right]+ \\
& \sum_{k=0}^{\infty}\beta^k d_{2t+k}\left(\left(A_{t+k}N_{t+k}\right)^{\phi}K_{t+k}^{1-\phi}-C_{t+k}-G_{t+k}-K_{t+k+1}+(1-\delta)K_{t+k}\right)+ \\
& \sum_{k=0}^{\infty}\beta^k d_{3t+k}\left(\frac{\beta C_{t+k}}{C_{t+k+1}}\frac{\pi}{\pi_{t+k+1}}-\frac{1}{1+i_t^k}\right)+ \\
& \sum_{k=0}^{\infty}\beta^k d_{4t+k}\left(\Delta_{t+1+k}\left\langle-\Lambda_{50}\right\rangle-(1-\alpha)\left(\frac{1-\alpha\left(\pi_{t+k+1}\right)^{\theta-1}}{(1-\alpha)}\right)^{\frac{-\Lambda_{50}}{1-\theta}}-\alpha\Delta_{t+k}\left\langle-\Lambda_{50}\right\rangle\pi_{t+k+1}^{-\Lambda_{50}}\right)+ \\
& \sum_{k=0}^{\infty}\beta^k d_{5t+k}i_t^k.
\end{aligned}
$$

We seek the first-order necessary conditions with respect to C, K, N, i^k, the variables which determine steady state policy. We give each of these in turn:

$$
\begin{aligned}
C &: (1+\lambda_m)\frac{1}{C_{t+k}}+d_1\Delta_{t+k}\left\langle-\Lambda_{50}\right\rangle\frac{\left(A_{t+k}N_{t+k}\right)^{\phi}K_{t+k}^{1-\phi}}{C_{t+k}^2}+d_1\frac{G_{t+k}}{C_{t+k}^2}\left(1-\Delta_{t+k}\left\langle-\Lambda_{50}\right\rangle\right)+ \\
& -d_{2t+k}+d_{3t+k}\frac{\beta}{C_{t+k+1}}\frac{\pi}{\pi_{t+k+1}}-d_{3t+k-1}\frac{\beta C_{t+k-1}}{C_{t+k}^2}\frac{\pi}{\pi_{t+k}}; \qquad (8.5)
\end{aligned}
$$

$$
\begin{aligned}
K_{t+k} &: d_1(1-\phi)\Delta_{t+k}\left\langle-\Lambda_{50}\right\rangle\frac{\left(A_{t+k}N_{t+k}\right)^{\phi}K_{t+k}^{-\phi}}{C_{t+k}} \\
& +d_{2t+k}\left((1-\phi)\left(A_{t+k}N_{t+k}\right)^{\phi}K_{t+k}^{-\phi}+(1-\delta)\right)-\frac{1}{\beta}d_{2t+k-1}; \qquad (8.6)
\end{aligned}
$$

$$
\begin{aligned}
N &: -N_{t+k}^v+d_1\Delta_t\left\langle-\Lambda_{50}\right\rangle\left((1+v)N_{t+k}^v\left(\frac{1}{\left(1-\tau_{t+k}^h\right)\phi}-1\right)-\phi A_{t+k}\frac{\left(A_{t+k}N_{t+k}\right)^{\phi-1}K_{t+k}^{1-\phi}}{C_{t+k}}\right) \\
& +\phi d_{2t+k}A_{t+k}\left(A_{t+k}N_{t+k}\right)^{\phi-1}K_{t+k}^{1-\phi}; \qquad (8.7)
\end{aligned}
$$

$$i_{t+1}^k : \lambda_m \left(\frac{1}{1+i_{t+1}^k} - \frac{1}{i_{t+1}^k} \right) + d_{5t+k} = 0 \tag{8.8}$$

Equation (8.8) implies that $d_{5t+k} > 0$ and inequality (8.4) is binding. It follows that the optimal interest rate is zero $i_{t+1}^k = 0$. Hence, as claimed in the main text, the Friedman rule in our model economy is optimal, both in and out of steady state.

We turn to the steady state implications of this system of dynamic equations. (8.5) - (8.7) transform into:

$$Cd_2 = (1+\lambda_m) + d_1 \frac{Y}{C}(1-mc); \tag{8.9}$$

$$\begin{aligned} d_1 (1-\phi) \frac{Y}{K}(1-mc) + d_2 \left((1-\phi) \frac{Y}{K} + (1-\delta) - \frac{\gamma}{\beta} \right) C &= 0; \\ d_1 \frac{\rho}{mc}(1-mc) + \left(\frac{\rho}{mc} - \frac{\gamma}{\beta} + (1-\delta) \right) \left[(1+\lambda_m) + d_1 \frac{Y}{C} \right] &= 0; \qquad (8.10) \\ d_1 \left[\frac{\rho}{mc}(1-mc) + \frac{Y}{C} \left(\frac{\rho}{mc} - \tilde{\rho} \right) \right] &= -\left(\frac{\rho}{mc} - \tilde{\rho} \right) (1+\lambda_m); \end{aligned}$$

$$-\frac{N^{v+1}C}{\phi Y} + d_1 \left(-(1+v) \frac{N^{v+1}C}{Y\phi} - 1 \right) + d_2 C = 0; \tag{8.11}$$

The Lagrange multipliers are detrended in following way: $d_{2t+k} = d_2 \frac{1}{\gamma}^{t+k}$, $d_{3t+k} = d_3$, $d_{4t+k} = d_4$.We also define $\tilde{\rho} = \frac{\gamma}{\beta} - (1-\delta)$ which equals the capital return in steady state with zero capital tax.

When the economy is perfectly competitive, $mc = 1$, and equation (8.10) implies $\rho = \tilde{\rho}$, which corresponds to the optimality of the zero capital tax rate, as found by Judd (1985) and Chamley (1986).

8.2. When lump sum taxation is allowed

In the first-best situation, where government is allow to use a lump-sum tax / subsidy, to find an optimal tax policy we do not need to take into account the solvency constraint. In this case $d_1 = 0$, $Cd_2 = (1 + \lambda_m)$. Equation (8.10) solves for the optimal capital return

$$\rho = \tilde{\rho} mc,$$

which gives us the optimal capital tax policy in a steady state with monopolistic distortion

$$\frac{\gamma - \beta}{\beta(1 - \tau^k)} + \delta = mc\left(\frac{\gamma}{\beta} - 1 + \delta\right);$$
$$\tau^k = (mc - 1)\frac{(\gamma - \beta + \delta\beta)}{mc\,(\gamma - \beta + \delta\beta) - \delta\beta}.$$

The optimal capital tax rate τ^k is negative, and grows in the degree of competitiveness. For a perfectly competitive market structure $mc = 1$ and τ^k is zero. This result appears similar to that of Guo and Lansing (1999).

The first order condition with respect to labour (8.11) is

$$\frac{N^{v+1}C}{\phi Y} = 1 + \lambda_m. \tag{8.12}$$

We recall that $\frac{N^{v+1}C}{\phi Y} = \frac{wN}{\phi Y}(1 - \tau^h) = mc(1 - \tau^h)$, and so we solve for the optimal labour tax

$$\tau^h = 1 - \frac{(1 + \lambda_m)}{mc}. \tag{8.13}$$

Optimal τ^h is also negative. The government should pay higher subsidies to labour when the economy is less competitive and the larger is λ_m.

8.3. Aggregate relations

8.3.1. Aggregating capital

We can derive the following expression for aggregated capital.

$$
\begin{aligned}
K_t \quad : \quad &= \int_0^1 K_t(i)di = \int_0^1 \left[A\frac{\rho_t}{w_t(i)}\frac{\phi}{1-\phi}\right]^{-\phi} Y_t(i)di \\
&= \left[A\rho_t\frac{\phi}{1-\phi}\right]^{-\phi} Y_t \int_0^1 \left[w_t(i)^{\phi}\left(\frac{p_t(i)}{P_t}\right)^{-\theta}\right] di = \\
&= \left[A\frac{\rho_t\phi}{1-\phi}\right]^{-\phi}\left[\left(\left[\frac{\rho_t\phi}{1-\phi}\right]^{1-\phi} A_t^{-\phi}Y_t\right)^{v}\frac{C_t}{1-\tau^h}\right]^{\frac{\phi}{1+(1-\phi)v}} Y_t\int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{\frac{-\theta v\phi}{1+(1-\phi)v}-\theta} di = \\
&\left(A_t^{-\phi}Y_t\right)^{\frac{1+v}{1+(1-\phi)v}}\left[\rho_t\frac{\phi}{1-\phi}\left(\frac{1-\tau_t^h}{C_t}\right)\right]^{\frac{-\phi}{1+(1-\phi)v}}\Delta_t\left\langle-\Lambda_{50}\right\rangle
\end{aligned}
\tag{8.14}
$$

Where Δ_t is the measure of price dispersion

$$
\Delta_t\left\langle-\Lambda_{50}\right\rangle = \int\left(\frac{p_t(i)}{P_t}\right)^{-\Lambda_{50}} di \tag{8.15}
$$

and we define

$$
\Lambda_{50} := \frac{\theta(1+v)}{1+(1-\phi)v}
$$

We introduce the following notation: K_t^* is the level of capital should all firms charge the same price ($\Delta_t\left\langle-\Lambda_{50}\right\rangle = 1$). Hence

$$
K_t^* := \left(A_t^{-\phi}Y_t\right)^{\frac{1+v}{1+(1-\phi)v}}\left[\rho_t\frac{\phi}{1-\phi}\frac{1-\tau^h}{C_t}\right]^{\frac{-\phi}{1+(1-\phi)v}}. \tag{8.16}
$$

8.3.2. Aggregating labour

$$\begin{aligned}
N_t &= \int_0^1 N_t(i)di = \int_0^1 \left[\frac{\rho_t}{w_t(i)}\frac{\phi}{1-\phi}\right]^{1-\phi} A_t^{-\phi} Y(i)di \\
&= A_t^{-\phi}\left[\rho_t\frac{\phi}{1-\phi}\right]^{(1-\phi)} Y_t J_t^{\frac{-(1-\phi)}{1+(1-\phi)v}} \int_0^1 w_t(i)^{-(1-\phi)} \left(\frac{p_t(i)}{P_t}\right)^{-\theta} di \\
&= A_t^{-\phi}\left[\rho_t\frac{\phi}{1-\phi}\right]^{(1-\phi)} Y_t^{1+v} J_t^{\frac{-(1-\phi)}{1+(1-\phi)v}} \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{\frac{\theta v(1-\phi)}{1+(1-\phi)v}} \left(\frac{p_t(i)}{P_t}\right)^{-\theta} di = \\
& \left(A_t^{-\phi}Y_t\right)^{\frac{1}{1+(1-\phi)v}} \left[\frac{\phi\rho_t}{1-\phi}\right]^{\frac{(1-\phi)}{1+(1-\phi)v}} \left[\frac{C_t}{1-\tau^h}\right]^{\frac{-(1-\phi)}{1+(1-\phi)v}} \Delta_t\left\langle\frac{-\theta}{1+(1-\phi)\,v}\right\rangle
\end{aligned}$$

We define w_t^* in a manner analogous to K_t^* and similarly N_t^*, for the case where $p_t(i) = P_t$. Hence,

$$w_t^* \; : \; = \left[\left(\left[\rho_t\frac{\phi}{1-\phi}\right]^{1-\phi} A_t^{-\phi}Y_t\right)^v \frac{C_t}{1-\tau^h}\right]^{\frac{1}{1+(1-\phi)v}} \tag{8.17}$$

$$N_t^* \; : \; = \left[\left(\left[\rho_t\frac{\phi}{1-\phi}\right]^{1-\phi} A_t^{-\phi}Y_t\right)\left(\frac{C_t}{1-\tau^h}\right)^{-(1-\phi)}\right]^{\frac{1}{1+(1-\phi)v}} \tag{8.18}$$

As a result we get that,

$$N_t = N_t^*\Delta_t\left\langle\frac{-\theta}{1+(1-\phi)\,v}\right\rangle.$$

About the CDMA

The **Centre for Dynamic Macroeconomic Analysis** was established by a direct grant from the University of St Andrews in 2003. The Centre funds PhD students and facilitates a programme of research centred on macroeconomic theory and policy. The Centre has research interests in areas such as: characterising the key stylised facts of the business cycle; constructing theoretical models that can match these business cycles; using theoretical models to understand the normative and positive aspects of the macroeconomic policymakers' stabilisation problem, in both open and closed economies; understanding the conduct of monetary/macroeconomic policy in the UK and other countries; analyzing the impact of globalization and policy reform on the macroeconomy; and analyzing the impact of financial factors on the long-run growth of the UK economy, from both an historical and a theoretical perspective. The Centre also has interests in developing numerical techniques for analyzing dynamic stochastic general equilibrium models. Its affiliated members are Faculty members at St Andrews and elsewhere with interests in the broad area of dynamic macroeconomics. Its international Advisory Board comprises a group of leading macroeconomists and, ex officio, the University's Principal.

RECENT WORKING PAPERS FROM THE CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS

Number	Title	Author(s)
CDMA04/01	Interest Rate Bounds and Fiscal Policy	Jagjit S. Chadha (St Andrews) and Charles Nolan (St Andrews)
CDMA04/02	The Impact of Imperfect Credibility in a Transition to Price Stability	Anamaria Nicolae (Durham) and Charles Nolan (St Andrews)
CDMA04/03	Monetary Policy Rules, Asset Prices and Exchange Rates	Jagjit S. Chadha (St Andrews), Lucio Sarno (Warwick and CEPR) and Giorgio Valente (Warwick)
CDMA04/04	Tax Policy and Irreversible Investment	Sumru Altug (Koç, CEPR and Economic Research Forum), Fanny S. Demers (Carleton and McGill) and Michel Demers (Carleton and McGill)
CDMA04/05	The Role of Preference Shocks and Capital Utilization in the Great Depression	Mark Weder (Humboldt)
CDMA04/06	Optimal Simple Rules for the Conduct of Monetary and Fiscal Policy	Jagjit S. Chadha (St Andrews) and Charles Nolan (St Andrews)
CDMA04/07	Money, Debt and Prices in the UK 1705-1996	Norbert Janssen (Bank of England), Charles Nolan (St Andrews) and Ryland Thomas (Bank of England)
CDMA05/01	Labour Markets and Firm-Specific Capital in New Keynesian General Equilibrium Models	Charles Nolan (St Andrews) and Christoph Thoenissen (St Andrews)
CDMA05/02	The Impact of Simple Fiscal Rules in Growth Models with Public Goods and Congestion	Sugata Ghosh (Cardiff) and Charles Nolan (St Andrews)
CDMA05/03	Inflation Targeting, Committee Decision Making and Uncertainty: The Case of the Bank of England's MPC	Arnab Bhattacharjee (St Andrews) and Sean Holly (Cambridge)

CDMA05/04	How to Compare Taylor and Calvo Contracts: A Comment on Michael Kiley	Huw Dixon (York) and Engin Kara (York)
CDMA05/05	Aggregate Dynamics with Heterogeneous Agents and State-Dependent Pricing	Vladislav Damjanovic (St Andrews) and Charles Nolan (St Andrews)
CDMA05/06	Aggregation and Optimization with State-Dependent Pricing: A Comment	Vladislav Damjanovic (St Andrews) and Charles Nolan (St Andrews)
CDMA05/07	Finance and Growth: A Critical Survey	Alex William Trew (St Andrews)
CDMA05/08	Financial Market Analysis Can Go Mad (in the search for irrational behaviour during the South Sea Bubble)	Gary S. Shea (St Andrews)
CDMA05/09	Some Welfare Implications of Optimal Stabilization Policy in an Economy with Capital and Sticky Prices	Tatiana Damjanovic (St Andrews) and Charles Nolan (St Andrews)

For information or copies of working papers in this series, or to subscribe to email notification, contact:

Alex Trew
Castlecliffe
School of Economics and Finance
University of St Andrews
Fife, UK, KY16 9AL

Email: awt2@st-and.ac.uk; Phone: +44 (0)1334 462445; Fax: +44 (0)1334 462444.