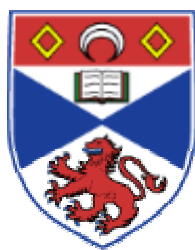


CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS  
WORKING PAPER SERIES



CDMA04/06

## Optimal Simple Rules for the Conduct of Monetary and Fiscal Policy\*

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NOVEMBER 2004

### ABSTRACT

Stabilization policy involves joint monetary and fiscal rules. We develop a model enabling us to characterize systematic simple monetary and fiscal policy over the business cycle. We principally focus on the following question: what are the key properties of the joint simple rule governing the conduct of systematic stabilization policy? We find that conducting stabilization policy incorporates not only a set of monetary policy choices governed by the so-called ‘Taylor principle’ but also fiscal policy that gives considerable force to automatic stabilizers. Recent US and UK monetary and fiscal choices seem broadly consistent with this model. This result is found to be robust to a number of alternate modeling strategies.

**JEL Classification:** E21; E32; E52; E63.

**Keywords:** Optimal simple rules, monetary and fiscal policy, finite lives.

\*We are grateful for comments from seminar participants and colleagues at Cambridge, Cardiff, Edinburgh, Glasgow, Manchester, Milan-Bicocca, Newcastle, Nottingham and St. Andrews Universities. Seminar participants at the Bundesbank, ECB, the Federal Reserve Board and the Reserve Bank of India also provided useful comments. All remaining errors are our own.

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# 1 Introduction

In this paper we characterize stabilization policy as involving systematic monetary and fiscal rules. It is clear that macroeconomics has recently revived a keen interest in the analysis of simple interest rate rules for the conduct of monetary policy. At an empirical level, such rules have been shown to provide plausible characterizations of actual monetary policy across a number of countries (Taylor, 1999). Also, in theoretical models with nominal rigidities and imperfect competition such rules can contribute in important ways to ensuring macroeconomic stability (Woodford, 2003). But much influential analysis of optimal policy has proceeded without considering the fiscal arm of macroeconomic policy.<sup>1</sup> In this paper, we therefore explore macroeconomic policy within the context of joint plans for monetary and fiscal policy. Specifically, we examine the characteristics of the optimal simple rules for the interest rate and the fiscal surplus as they would be set by one representative policymaker.<sup>2</sup>

It is clear that monetary policy has been emphasized as the senior partner in the search for optimal stabilization rules. In particular, there is now a vast literature on desirable simple rules for the conduct of monetary policy. The ‘Taylor Rule’ is an important example of such a rule:

$$i_t = i_t^* + \phi_\pi^m(\pi_t - \pi^*) + \phi_y^m(y_t - y_t^*). \quad (1)$$

Here  $i_t$  denotes the period nominal interest rate, which is the instrument of monetary policy,  $i_t^*$  is a measure of the equilibrium nominal interest rate and comprises the natural real rate and the inflation target,  $(\pi_t - \pi^*)$  measures the deviation of current inflation,  $\pi_t$ , from target,  $\pi^*$ , and  $(y_t - y_t^*)$  is the deviation of output,  $y_t$ , from target,  $y_t^*$ . In addition, it will generally be the case that  $\phi_\pi^m \geq 1$ , and  $0 \leq \phi_y^m < 1$ . The first parametric restriction is known as the Taylor principle (Woodford, 2003), and the

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<sup>1</sup>A good point of departure is the Monetary Policy Rule Homepage set up by John Taylor, <http://www.stanford.edu/~johntayl/PolRulLink.htm>, which lists a large number of papers for which the effects of fiscal policy are implicit or ignored. Taylor (2001) himself provides an exception to this tendency to ignore fiscal policy.

<sup>2</sup>This perspective is obviously simplified, but may not be wildly at odds with reality. In the UK, for example, a senior Finance ministry official observes the deliberations of the Bank of England’s Monetary Policy Committee. The Committee is itself kept closely informed on fiscal plans. Indeed both fiscal and monetary policy are set according to plans to which policymakers have openly committed themselves; a Golden Rule for fiscal expenditure and an Inflation Target for monetary policy—where both rules were devised by the current Chancellor of the Exchequer (Finance Minister).

second would typically see  $\phi_y^m$  somewhat closer to zero than unity.<sup>3</sup>

However, there have been few attempts to analyze analogous rules jointly for both monetary and fiscal policy. Of course, it would hardly be accurate to suggest that the interaction of monetary and fiscal policy has not received much attention. On the one hand, there is a large literature following the concerns raised in Sargent and Wallace (1981). That literature focuses on the results of game theoretic interactions that may arise if monetary and fiscal authorities play a ‘game of chicken’ with the public finances. More recently, there has been the debate surrounding fiscal theories of the price-level.<sup>4</sup> Finally, there is also a growing literature pursuing the Ramsey-inspired approach to monetary and fiscal policy (see, for example, Khan, King and Wolman, 2000).

On the other hand, however, the role that the automatic fiscal stabilizers – the systematic component of fiscal policy – might play in smoothing the business cycle has received significantly less attention from quantitative theorists. The lack of attention is surprising partly because there is a clear need to gauge optimal policy here. On the one hand, there would seem to be widespread agreement that the automatic stabilizers provide a useful role in smoothing the effects of the business cycle, e.g., Eichenbaum (1997). In addition, there is an on-going debate on the need for policy initiatives that would seek to limit fluctuations in the per period fiscal deficit. For example, in the USA discussions continue about a Balanced Budget Amendment and in Europe the Pact for Stability and Growth is a clear attempt to impose some form of upper bound on fiscal deficits.<sup>5</sup> So, if the automatic stabilizers provide a quantitatively significant channel for stabilizing aggregate demand,<sup>6</sup> the key question is: ‘How much variation in the deficit is desirable?’.

Taylor (1997) has recently suggested that in the US the cyclical component of fiscal policy can be well captured by a simple rule analogous to his rule for monetary policy:

$$s_t = s^* + \phi_y^f(y_t - y_t^*). \quad (2)$$

Here  $s_t$  denotes the surplus-GDP ratio,  $s^*$  is a measure of the trend level of that magnitude, and  $\phi_y^f = 0.5$ . Taylor (*op. cit.*) argues that permitting this systematic component of fiscal policy to operate

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<sup>3</sup>Indeed some economists have argued that  $\phi_y^m$  should be identically zero at all dates and in all states (e.g., Christiano and Gust, 1999).

<sup>4</sup>See, for example, Woodford (1997) for a view on the resulting implications for nominal determinacy.

<sup>5</sup>Eichengreen and Wyplosz (1998) is an interesting discussion of these issues.

<sup>6</sup>And there is evidence that this is indeed the case, see Auerbach and Feenberg (2000).

stabilizes the economic cycle, and that discretionary changes in fiscal policy *over and above* these automatic responses are, in general, undesirable.<sup>7</sup>

In our model monetary policy operates principally via the standard channel of setting short-term real interest rates and thereby tilting the planned path of agents' consumption. Fiscal policy operates via two channels: (i) wealth effects from outstanding government debt, and (ii) the aggregate effect of government expenditure. We shall assume that all government expenditure is financed out of lump-sum taxation. This is partly for simplification, but also because the stabilizers themselves are often lump sum in nature. Note that Ricardian Equivalence fails in our model because we employ finite lives with a fixed probability of death and this allows us to assess the impact of the wealth effect of outstanding bonds.

In this paper we analyze rules of the form of (1) and (2) *jointly*. To do so, we shall construct a model in which both monetary and fiscal policy have leverage over aggregate demand and ask what are the joint optimal rules for stabilizing the economy. In particular, we shall adopt a criterion function for the policymaker of the form,  $L = L[(\pi_t - \pi^*), (y_t - y_t^*), \dots]$ , and investigate the constellation of parameters in rules analogous to (1) and (2) such that  $(\phi_\pi^m, \phi_y^m, \phi_y^f) = \arg \min L = L[(\pi_t - \pi^*), (y_t - y_t^*), \dots]$ .<sup>8</sup>

The paper is laid out as follows. In the next section we set out the model and discuss the construction of our aggregate equations. In section 3 we establish our simple rules for the conduct of monetary and fiscal policy. The rule for fiscal policy enables the government to issue debt, raising some modelling issues pertaining to the class of fiscal regime under investigation, and hence equilibrium selection. In section 4 we briefly discuss the algorithm for solving the model for the optimized coefficients on our monetary-fiscal rules.<sup>9</sup> Section 5 illustrates the results of our simulations for alternative versions of our simple rules and considers what happens if one arm of policy behaves in a non-optimal manner, as well as investigating the quantitative significance of the wealth effect associated with government debt. Section 6 offers some conclusions and suggests some directions for future research.

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<sup>7</sup>We note that in the presence of price rigidities, there will be relative price distortions and sluggish adjustment of output towards its flexible price equilibrium. Fiscal policy is thus motivated as an additional tool to facilitate macroeconomic adjustment.

<sup>8</sup>Our solution follows the work of Soderlind (1999) and earlier work by Backus and Driffill (1986). For another recent example see Alexandre, Driffill and Spagnolo (2002).

<sup>9</sup>An appendix, available on the authors' webpages, provides more details.

## 2 The Model

### 2.1 The Representative Agent

The model is constructed around the perpetual youth set up of Blanchard (1985). Following Cardia (1991) and Chadha and Nolan (2003) we convert the model into a discrete-time framework. We assume that there are a large number of identical agents in each cohort, and we assume that the size of each cohort dies away through time. The utility function for a representative agent,  $j$ , is given by,

$$V_0 = \sum_{t=0}^{\infty} \left\{ \left( \frac{1}{1+\delta} \right)^t \left( \frac{1}{1+\lambda} \right)^t E_0 U \left( C_t^j, \frac{M_t^j}{P_t}, L_t^j \right) \right\}. \quad (3)$$

Here  $\delta$  is the subjective discount rate and  $\lambda$  the time-invariant probability of death. This means that the expected remaining lifetime of any agent is equal to  $\lambda^{-1}$ . The utility function is assumed to be concave and separable in its arguments,  $C$ , consumption,  $M/P$ , real money balances, where  $M$  is nominal money balances and  $P$  is the economy-wide price-level and  $L$  is leisure, which is equal to  $1 - N$ . We normalize available time to unity and so  $N$  is interpretable as labor effort.<sup>10</sup> The representative agent maximizes expected utility subject to a sequence of per period flow constraints:

$$(1+\lambda)M_{t-1}^j + (1+\lambda)B_{t-1}^j + W_t N_t^j + \Pi_t - T_t \geq P_t C_t^j + M_t^j + \frac{B_t^j}{1+i_t}, \quad (4)$$

where,  $P_t C_t^j = \int_0^1 p_t(z) c_t^j(z) dz$ , and where (4) holds for all  $t \geq 0$ , and in each state of nature,  $M_{-1}^j$  and  $B_{-1}^j$  given. Let  $z$  index goods in the economy. Then, we have that  $c_t^j(z)$  denotes the representative agent's consumption of good ( $z$ ). In addition,  $W_t$  denotes the nominal wage,  $\Pi_t$  denotes profits remitted from firms where each agent receives a pooled dividend,  $T_t$  denotes lump sum taxes, which are constant across cohorts,  $B_t$  denotes the nominal stock of bonds held over at the end of period  $t-1$  and  $i_t$  is the economy-wide one-period nominal interest rate. The evolution of financial wealth,  $F$ , is given by

$$F_t^j = (1+\lambda)M_{t-1}^j + (1+\lambda)B_{t-1}^j. \quad (5)$$

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<sup>10</sup>We assume log-separability, i.e.  $U(\cdot) \equiv \left[ \log C_t^j + \log \left( \frac{M_t^j}{P_t} \right) + \log(1 - N_t^j) \right]$ , as this facilitates the construction of our aggregate equations.

The sequence of equations (4) together with the transversality condition,  $\lim_{T \rightarrow \infty} \left(\frac{1}{1+\lambda}\right)^T E_0 \prod_{j=0}^{T-1} (1+i_{t+j})^{-1} W_{t+T}^j \rightarrow 0$  help ensure that the agent's optimization problem is well behaved, where this latter condition is weighted by the probability of death. Consumption is defined over the Dixit-Stiglitz aggregator function, where  $\theta$  is the price elasticity of demand for goods:

$$C_t^j \equiv \left[ \int_0^1 c_t^j(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad (6)$$

with the aggregate price-level derived accordingly as:

$$P_t \equiv \left[ \int_0^1 p_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}. \quad (7)$$

## 2.2 Optimality conditions

Let  $\{\mu_t^j\}_{t=0}^\infty$  be a (state-dependent) temporal sequence of Lagrange multipliers. At each date and in each state the following equations, (8)–(11), are amongst requirements for an interior optimum:

$$\left(\frac{1}{1+\lambda}\right)^t \frac{1}{P_t C_t^j} = \mu_t^j; \quad (8)$$

$$\left(\frac{1}{1+\lambda}\right)^t \frac{1}{M_t} + \left(\frac{1}{1+\delta}\right) (1+\lambda) E_t \mu_{t+1}^j = \left(\frac{1}{1+\lambda}\right)^t \mu_t^j; \quad (9)$$

$$\left(\frac{1}{1+\lambda}\right)^t \frac{1}{1-N_t^j} = \mu_t^j W_t; \quad (10)$$

$$\mu_t^j = (1+i_t)(1+\lambda) \left(\frac{1}{1+\delta}\right) E_t \mu_{t+1}^j. \quad (11)$$

Using equations (8) and (11) in (9) uncovers the money demand relation:

$$\frac{M_t^j}{P_t} = C_t^j \left(\frac{1+i_t}{i_t}\right). \quad (12)$$

(8) and (11) imply the consumption Euler equation:

$$E_t P_{t+1} C_{t+1}^j = \frac{1+i_t}{1+\delta} P_t C_t^j. \quad (13)$$

The labor supply function results from (8) and (10):

$$N_t^j = 1 - C_t \left( \frac{W_t}{P_t} \right)^{-1}. \quad (14)$$

Note that the probability of death does not appear in the optimality conditions (12-14). The reason for this is straightforward. Following the assumptions made by Blanchard (1985), insurance companies pay premia to savers in return for their wealth in the event of death. This wealth is then returned lump-sum to those currently alive. Consequently, the market interest rate incorporates this ‘premium’, which just offsets the probability of death uplift in the agent’s discounting of future utility.

## 2.3 The Representative Firm

We assume that there are a large number of infinitely-lived monopolistically competitive firms, who use only labor in the production of their differentiated good.<sup>11</sup> Each period these firms receive a symmetric productivity shock. So for firm  $i$  the production function is given by

$$Y_{i,t} = A_t N_t^\varpi, \quad (15)$$

where  $0 < \varpi < 1$ , and  $A_t$  is the productivity innovation. The firm hires labor in competitive markets and we assume that all agents are equally productive. Consequently, the real wage,  $W_t/P_t$ , is the same across all cohorts, and is given by,

$$\frac{W_t}{P_t} \equiv w_t = \varpi A_t N_{i,t}^{\varpi-1}. \quad (16)$$

We further assume that firms face Calvo-type restrictions in setting prices but that firms meet demand whether or not they have been able to change prices that period. The demand for the  $i^{th}$  product is given by

$$c_t(i) = \left( \frac{p_t(i)}{P_t} \right) Y_t^d, \quad (17)$$

where  $Y_t^d$  denotes aggregate demand, the sum of private expenditure and government expenditure. In the model we develop there will be an effect directly from government expenditure and indirectly via the

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<sup>11</sup>Incorporating economy-wide capital into the current set-up poses few problems. However, we are partly concerned to uncover the quantitative significance of outstanding bonds on private consumption and including a private or public capital stock would make that task somewhat more difficult.

stock of outstanding net bonds. The latter effect, as we demonstrate below, is captured in the measure of private consumption demand. A cost-minimizing firm required to meet current demand will hire labor according to the following optimality condition,

$$w_t = \Lambda_t(\partial Y_{i,t}/\partial N_{i,t}), \quad (18)$$

where  $\Lambda_t$  measures real marginal cost. Total per-period profits then are given by

$$\Pi_t(i) = p_t(i) \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t^d - \Lambda_t \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t^d. \quad (19)$$

As regards price setting behavior we follow Calvo (1983) by assuming that firms who set prices in period  $t$  face a probability,  $\alpha$  ( $0 \leq \alpha < 1$ ) of having to live with the same decision next period. More generally, we assume that a firm which sets its price this period faces the probability  $\alpha^k$  of having to charge the same price in  $k$ -periods time. The optimal price,  $\tilde{p}_t$ , is given by:<sup>12</sup>

$$\tilde{p}_t = \frac{\theta}{(\theta - 1)} \frac{\sum_{k=0}^{\infty} (\alpha\beta')^k E_t(\mu_{t+k}^a P_{t+k}^{\theta} Y_{t+k}^d \Lambda_{t+k})}{\sum_{k=0}^{\infty} (\alpha\beta')^k E_t(\mu_{t+k}^a P_{t+k}^{\theta-1} Y_{t+k}^d)}. \quad (20)$$

Here  $\mu_{t+k}^a$  is aggregate marginal utility, and  $\beta' = \beta/(1 + \lambda)$ . The optimal price is determined by a constant mark up, and the discounted stream of current and expected real marginal cost and demand conditions in which that price may obtain.

## 2.4 Key Aggregate Equations

We turn to the aggregate analogues to equations (12-14). The money demand equation is given by the following expression where  $a$  superscript represents the aggregate equation pertaining to all cohorts at any given time:

$$\left( \frac{M_t}{P_t} \right)^a = C_t^a \left( \frac{1 + i_t}{i_t} \right). \quad (21)$$

Now consider the labor supply function, where the real wage is constant across cohorts. We have:

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<sup>12</sup>The details of this problem are well understood, and we leave them to an appendix, available upon request.



$$N_t^a = 1 - C_t^a \left( \frac{W_t}{P_t} \right)^{-1}. \quad (22)$$

Similarly aggregate consumption is given by

$$E_t P_{t+1} C_{t+1} = (1 + i_t) \beta P_t C_t - \lambda E_t F_{t+1}. \quad (23)$$

This last expression is, of course, identical to (2.11) in the event that  $\lambda = 0$  at each date and in each state of nature. Note that in the standard representative agent framework the Euler equation captures *aggregate* consumption dynamics by construction. In other words, if we use the standard Euler equation and the present value budget constraint, in the infinite-lived representative agent set up, we find that aggregating across all consumers one ends up back at the simple Euler equation as an equation that also represents aggregate consumption-savings behavior. That equivalence between individual and aggregate behavior *vis a vis* consumption does not obtain in our set up. That is, we assume that on death financial wealth is transferred via ‘insurance companies’ as windfall dividends to currently alive agents. This means that consumption by the currently alive cohort is augmented by the annuity value of any expected financial wealth, which means that debt issued by the fiscal authority in this period, as part of expected financial wealth, is not neutral with respect to consumption.<sup>13</sup>

Finally, we turn to aggregate price dynamics. Given our assumption of infinitely lived firms, aggregation here is more straightforward. Dropping the  $i$  index, equation (15) describes aggregate output, (16) describes the marginal product of labor, whilst the evolution of the aggregate price-level is given by a weighted average of this period’s optimal price (common to all firms) and last period’s aggregate price level:

$$P_t = [(1 - \alpha) \tilde{p}_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}]^{1/(1-\theta)}. \quad (24)$$

### 3 Monetary and Fiscal Policy

In this section we consider the operation of monetary and fiscal plans in the US and UK. The presence of price rigidities, see equation (24), ensures sluggish adjustment of output and the price level to their respective flex price levels and appropriate stabilization plans are likely to foster this adjustment. We therefore first consider the stylized facts

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<sup>13</sup>In Chadha and Nolan (2003) a detailed derivation of equation (23) is provided.

on the operation of monetary and fiscal policy in the US and UK and then consider modeling monetary and fiscal policy within the context of our model.

### 3.1 Business Cycle Facts

Table 1 presents some business cycle moments on monetary and fiscal policy over the postwar business cycle in the UK and US comprising the band-pass filtered series for output, the consumer price index and its annual rate of inflation, the policy rate and the fiscal surplus as a percentage of GDP. As standard the price level is countercyclical in both economies but inflation is procyclical with respect to output lags with negative leads for inflation. As far as policy instruments are concerned the business cycle association drawn out from this data is clear: (policy) interest rates and the fiscal surplus are procyclical, although policy rates have negative leads for output. The systematic and positive association of the instruments of stabilization policy, nominal rates and the fiscal surplus, with the business cycle motivate our use of simple rules for understanding monetary and fiscal policy.

### 3.2 Simple Rules

We turn now to consider the aggregate policy rules in place. We shall consider the policymaker as setting the per period interest rate and net taxes—the systematic components of policy – in order to stabilize both output and inflation. That is, we are envisaging policy rules of the following sort:

$$i_t = \phi^m(Y_t, \pi_t, i_{t-1}), \quad (25)$$

and

$$T_t = \phi^f(G_t, T_{t-1}, \gamma B_{t-1}), \quad (26)$$

where  $i_t$  is the nominal interest rate set in period  $t$ ,  $Y_t$  is real aggregate output,  $\pi_t$  is the inflation rate in period  $t$ , and  $T_t$  are the period lump sum taxes. We model monetary policy as control over the short-term (one-period) nominal interest rate, which in the presence of aggregate price stickiness implies some leverage over interest sensitive endogenous variables. In our model that means essentially private consumption, although there are also other effects.<sup>14</sup> While the monetary rule is fairly standard the rule for tax needs some explanation. We take the process for government expenditure as exogenous. We shall assume that the

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<sup>14</sup>For example, the real interest rate influences the net wealth of outstanding bonds. This channel appears to be of second order importance.

fiscal authority sets taxes in response to the level of contemporaneous government expenditure, that seigniorage is returned lump-sum to the private sector, and that taxes respond to the level of debt outstanding at the start of the period. The parameter  $\gamma$  indicates the proportion of debt that is retired each period. This ability of the government to issue debt raises some modeling issues, as the recent debate concerning the fiscal theory of the price-level has emphasized, and we shall briefly discuss these in the next section.

### 3.3 The Government Budget

We shall assume that the government follows a Ricardian fiscal policy. That is, regardless of the sequences of the model's other endogenous variables, the government intends to ensure that its present value budget constraint is met identically at each date and in each state of nature. In our set up the government need not run a balanced budget each period, and hence may be issuing or retiring debt. We constrain fiscal policy however to be working systematically to retire outstanding debt. This raises the issue, therefore, as to whether our debt retirement program is sufficiently robust. In particular, consider the public sector budget constraint at time  $t$ ,

$$\frac{B_t}{(1+i_t)} = B_{t-1} + P_t(G_t - T_t) - (M_t - M_{t-1}). \quad (27)$$

Further, consider the following parameterization of the process governing taxes,

$$T_t = \chi_t G_t - \frac{(M_t - M_{t-1})}{P_t} + \gamma \frac{B_{t-1}}{P_t}. \quad (28)$$

Together these two equations imply that real debt,  $b_t$ , will evolve in the following manner,

$$\frac{b_t}{1+r_t} = (1-\gamma)b_{t-1} + (1-\chi_t)G, \quad (29)$$

where  $r_t$  is the real interest rate. Hence if we define  $(1-\chi_t)G_t$  as the per period deficit,  $D_t$ , then at some point,  $t = T$ , in the future this implies that

$$E_t \frac{b_{t+T}}{\prod_{j=0}^T (1+r_{t+j})} = (1-\gamma)^{T+1} b_{t-1} + E_t \sum_{s=0}^T \prod_{j=0}^{s-1} \left( \frac{1}{1+r_{t+j}} \right) (1-\gamma)^{T-s} D_{t+s}. \quad (30)$$

For policy to be Ricardian we require that  $E_t \frac{b_{t+T}}{\prod_{j=0}^T (1+r_{t+j})}$ , tends to zero as a result of fiscal actions and regardless of the sequence of prices and

interest rates that we see. As  $T \rightarrow \infty$ , the first term on the right hand side does indeed tend to zero. The second magnitude measures the present discounted sequence of net debt issue looking forward from time  $t$ . Even in our simple set-up it is fairly difficult to say anything too precise about the required size of this parameter. Very loosely speaking, for policy to be Ricardian we require that  $\gamma$  eventually becomes and remains sufficiently large across states of nature such that the rate at which debt is retired exceeds the rate at which it grows i.e. the real interest rate.<sup>15</sup>

## 4 Solving for Simple Rules

The model developed in Sections 2 and 3 can be solved for the equilibrium evolution of aggregate wealth, consumption, money holdings, labor supply, inflation, the short term nominal interest rate, the level of taxation, government interest-bearing debt and aggregate output. We outline the solution procedure by first noting that the model requires the following equations, converted into aggregate form as required: (5), (20), (22), (23), (24), (25), (26), (27), together with an equation describing the aggregate economy-wide resource constraint. The feedback coefficients in the policy rules, equations (25) and (26), are left unspecified and we solve for these by adopting a quadratic criterion for the policymaker, following Backus and Driffill (1986), Söderlind (1999), Williams (1999) and Dennis (2001).<sup>16</sup>

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<sup>15</sup>We investigated this issue numerically and found that an annual rate of  $\gamma = 0.06$  is a lower bound on this value in the current set-up. In a deterministic environment, one can make more analytical headway. When this inequality is not satisfied, the model generates multiple solution paths, some of which were consistent with fiscally determined price-levels/inflation rates (as in Woodford, 1997, see also Chadha and Nolan, 2003). We investigate this specific issue analytically in Chadha and Nolan (2004).

<sup>16</sup>We augment the King and Watson (1997) code to perform what is, in effect, a search over the policy parameters such that the policymakers loss function is minimised. Alternative code has been written by Richard Dennis (2001) to solve for optimal simple rules under rational expectations. This latter algorithm, however, requires the  $B$  matrix in (31) to be non-singular. For larger models that is often inconvenient since some manual system reduction is then required. Our code requires neither  $A$  nor  $B$  to be non-singular. The King and Watson (1997) reduction algorithm can deal with singular  $A$  matrices whilst our method of calculating the model's asymptotic variance-covariance matrix does not require the inversion of  $B$  at any step along the way. Dennis' (2001) code however can also be used to solve for the case when precommitment is not feasible. Our code is available on request.

We need to calculate, for a given stochastic structure for the economy's driving processes, the asymptotic variance-covariance matrix for the economy's endogenous variables. We first linearize the model around its non-stochastic steady state. Then we make an initial guess about the optimal policy parameters, given the other parametric assumptions we have made, and verify that the model admits a unique stable rational expectations equilibrium under this parameter constellation. Once we have established the existence of such an equilibrium, we are able to calculate the loss function of the policymaker. We then iterate on this calculation for alternate selections of policy rule parameter values, and compare losses, and continue in this way until a minimum for the loss function is located.

## 4.1 Parameterization of Model

Let us first briefly outline the key baseline parameter values that we adopt for the calibration of the model in Table 2. The model is calibrated at a quarterly frequency using more or less standard parameter values. We assume that  $\lambda$  is determined as a result of the representative agent expecting to live to 70. The discount factor,  $\beta$ , is set at 0.988. As noted above, numerical investigations led us to set the debt retirement rate,  $\gamma$ , to 0.015. We further assume that the steady state consumption:income ratio,  $c/y$ , is equal to 0.6, while the steady state money:wealth ratio,  $m/w$ , was chosen to be 0.1. Roughly speaking the average size of the UK debt-to-GDP ratio over the post-war period has been some 40%. Together with our assumption for  $c/y$ , implies that the steady-state wealth:income ratio for this simple model economy is 0.7. Naturally, that number is required to parameterize our consumption Euler equation.

Specifying the nature of the stochastic forces driving the economy has less precedent in the empirical literature, although important results are emerging (see, for example, Burnside, Eichenbaum and Fisher, 2000, and Finn, 1998). Let  $a_t$ ,  $f_t$ , and  $h_t$  denote the log detrended processes for productivity, fiscal and monetary innovations, respectively. We then assume they can be described adequately for our purposes as follows,

$$\begin{bmatrix} a_t \\ f_t \\ h_t \end{bmatrix} = \begin{bmatrix} \rho_a & 0 & 0 \\ 0 & \rho_f & 0 \\ 0 & 0 & \rho_q \end{bmatrix} \begin{bmatrix} a_{t-1} \\ f_{t-1} \\ h_{t-1} \end{bmatrix} + \begin{bmatrix} x_t \\ g_t \\ q_t \end{bmatrix},$$

where  $x_t$ ,  $g_t$ , and  $q_t$  are the shocks respectively to productivity, fiscal and monetary innovations.

We adopted an agnostic strategy for setting the covariation structure of the forcing variables. First we estimated Solow residuals, Taylor Rules

and Fiscal Rule equations on US and UK data and found little difference in the standard errors of the respective equations.<sup>17</sup> Similarly Cardia (1991) found that the standard deviation of shocks to the monetary and fiscal processes were of similar magnitude in the US data, whilst in the German data the standard deviations of fiscal and productivity shocks were of a similar size. As a point of reference, a standard parameterization in the one-sector RBC literature is that the standard deviation of a quarterly productivity innovation is about 0.007–0.008. From our data the standard error of the Taylor Rules are 0.01–0.011 and the Fiscal Rules are 0.006–0.0018. In practice, therefore we decided simply to set  $\sigma_a = \sigma_f = \sigma_q = 0.01$ .<sup>18</sup>

In terms of the persistence parameters we chose the following:  $\rho_a = 0.9$ ,  $\rho_f = 0.9$ , and  $\rho_q = 0$ . Again, the literature on RBC models leads us to expect persistence in productivity innovations. Similarly, the literature on fiscal shocks also seems to indicate that these are persistent, as do our empirical estimates of these rules.<sup>19</sup> Again, we experimented with differing degrees of persistence and our findings seem largely robust to quite large variations in these parameters. Our assumption that monetary shocks are not persistent reflects a common modelling choice that monetary shocks persist largely due to interest rate smoothing, the observed tendency for lagged interest rates to predict current interest rates. The parameter constellation and covariation structure of the forcing processes are used in conjunction with the model developed to: (i) assess the responses of the aggregate economy to shocks in the forcing variables, and (ii) establish the optimal values of the choice variables in the stabilization policy rules.

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<sup>17</sup>The US dataset runs from 1955:1 to 2000:4: we use the Federal Funds rate as the policy instrument in the Taylor Rule; annual inflation is measured as the four quarter percentage change in the All-Items CPI; GDP in 1995 constant prices is detrended by a quadratic time trend and the Federal Government surplus or deficit is given as a proportion of GDP. For the UK: we use the base rate as the policy instrument; annual inflation is measured as the four quarter percentage change in the RPI; GDP in 1995 constant prices is detrended by a quadratic time trend and the Public Sector Cash Requirement is given as a proportion of GDP, after being seasonally adjusted by X12.

<sup>18</sup>We experimented by re-ordering the shocks, making in turn monetary and then fiscal shocks more volatile. This had very little effect on our results. These results are available on request.

<sup>19</sup>We find 0.9 for the persistence in the US and 0.76 for the UK in our dataset.

## 4.2 Solution Method

A linearized version of the model developed in sections 2 and 3 can be represented in compact form with all variables in percentage deviation from the steady state as

$$AE_t y_{t+1} = By_t + Cx_t \quad \forall t \geq 0, \quad (31)$$

where  $y_t$  is a vector of endogenous variables comprising both predetermined and non-predetermined variables including policy rules for the nominal interest rate and taxes.  $x_t$  is a vector of exogenous variables and  $A, B$  and  $C$  are matrices of fixed, time-invariant coefficients.  $E_t$  is the expectations operator conditional on information available at time  $t$ . King and Watson (1997) demonstrate that if a solution to (31) exists and is unique then we may write such a solution in state-space form as follows:

$$\begin{aligned} y_t &= \Pi s_t, \\ s_t &= Ms_{t-1} + Ge_t, \end{aligned} \quad (32)$$

where the  $s_t$  matrix includes the state variables of the model,  $e_t$  is a vector of shocks to the state variables and  $\Pi, M$  and  $G$  are coefficient matrices. The  $y_t$  matrix has also been augmented to include the model's exogenous state variables. We can use equations (32) to calculate the asymptotic variance-covariance matrix for the model's endogenous variables,<sup>20</sup> which we minimize subject to the policymaker's period loss function,  $L$ , is given by

$$L = \alpha_1 \Sigma_\pi + \alpha_2 \Sigma_y + \alpha_3 \Sigma_i, \quad (33)$$

where  $\Sigma_x$  denotes the asymptotic variance of the annualized value of  $x$ .<sup>21</sup>

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<sup>20</sup> Let  $npd$  denote the number of predetermined variables,  $nx$  the number of exogenous state variables and let  $nnpd$  denote the number of non-predetermined variables. The dimensions of our system are as follows:  $y_t$  is  $[(nnpd + npd + nx) \times 1]$ ,  $s_t$  is  $[(npd + nx) \times 1]$ ,  $\Pi$  is  $[(nnpd + npd + nx) \times (npd + nx)]$ ,  $M$  is  $[(npd + nx) \times (npd + nx)]$ , and  $G$  is  $[(npd + nx) \times (npd + nx)]$ . The interested reader should consult Hansen and Sargent (1998) and our technical annex for further details.

<sup>21</sup> Following Alexandre, Driffill and Spagano (2002), Söderlind (1999) and Williams (1999) we assume that  $\alpha_1 = \alpha_2 = 1$ , and  $\alpha_3 = 0.25$ . In sensitivity analysis, we have relaxed these relative weights in the loss function and examined the resulting loss functions and our conclusions seem robust to this.

## 5 Results

We know from a vast amount of work that has taken place in recent years that a Taylor rule does well in the type of model we have developed above. Although we are not aware of an estimated Blanchard-Yaari model for which the performance of a Taylor rule has been evaluated, our aggregate equations are for the most part similar to those of the representative agent, and so we presume that something like a Taylor rule remains a desirable rule for monetary policy. As important, we also know that something like a Taylor rule tracks real-world policy rates fairly closely, so that if our model is to track data on monetary policy choices it needs to generate a Taylor-like rule. What we also ask of our model, in terms of equation (26), are the optimal weights in the fiscal rule?

Recall from the literature on Taylor rules (see, for example, Woodford, 2003) that a weight of greater than unity on contemporaneous inflation and a weight close to zero on contemporaneous output has desirable stabilizing properties. Our results for optimality of this simple Taylor rule are given in Table 3.

The numbers reported in this table correspond to the optimized coefficients associated with the arguments (indicated to the left) in the reaction functions. In all of the simulation results reported we have constrained fiscal policy not to react to the inflation deviation from target. We see, then, that the Taylor principle is respected in our optimized rule, yielding an impact coefficient on inflation deviations of 1.15, and an eventual (long-run) feedback on inflation of just over 1.5.<sup>22</sup> Note also that, at the optimum, there is a weak contemporaneous feedback from output (0.06). Interest rates are autocorrelated, although to a degree that appears somewhat less than one might expect. Turning to the deficit rule, we see that the deficit reacts robustly to the output gap. The surplus:GDP ratio responds contemporaneously to the output gap with a coefficient of 0.75 (somewhat higher than Taylor's recommendation), and is more highly autocorrelated than the nominal interest rate.

Figures 1-2 illustrate the fit of the optimized rules (solid lines) with data from the US and the UK (dotted lines). The familiar charts of the Taylor Rule tracking actual policy rates in the upper panels of Figures 1 and 2 are now augmented by fiscal rules. The simulated optimal simple rules provide a reasonable visual fit to the data. We are reluctant to claim too much for these simulated rules, as our model is clearly very parsimonious. For example, the simple rules result in fairly robust fiscal

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<sup>22</sup>In fact, Taylor (1993) himself suggested a value of 1.5.



responses which are apparent towards the start of the sample period. In a sense, however, these two sets of charts make clear what we think is a key message of this paper: good monetary policy is mutually predicated on good fiscal policy. Our set up allows us to be clear, within the confines of our model, what ‘good’ fiscal policy looks like and as the monetary rule responds to inflation it would seem so should the fiscal rule act systematically to output i.e., there would seem to be a clear role for automatic stabilizers.

## 5.1 Impulse Responses

Before assessing a number of different policy scenarios, we discuss the impulse responses of output, interest rates, the fiscal balance and inflation to 1% shocks in each of the forcing variables given the optimized coefficients reported in Table 3. From the plots of these responses, a picture emerges of monetary and fiscal policy working as a complementary sequence of choices.

### 5.1.1 Productivity shock

Naturally, output (not shown) responds positively and with a high degree of persistence to a productivity shock. Figure 3 shows inflation mirroring the response, as falling marginal costs put downward pressure on firms’ prices, by falling below baseline for 7 quarters. The optimal policy response involves the nominal interest rate falling below base, while the fiscal surplus rises. Nominal interest rates are cut in order to stabilize falling inflation and lump-sum taxation tempers aggregate demand.

### 5.1.2 Monetary shock

Figure 4 outlines the response of the economy to innovations in the economy’s interest rate rule. Inflation responds quickly to the monetary shock, falling by just under 0.5% in the first period and returns more than half way to base by period 2. Output remains below its steady-state level for some 10 quarters, although it is within 0.1% of base after only 3 quarters. Output falls because a monetary policy shock increases real rates. The dotted line show the more volatile responses of inflation and output when the weight on inflation is somewhat less than optimal at 1.

### 5.1.3 Fiscal shock

Figure 5 depict the response of the deficit, inflation and output to fiscal policy shocks. The increase in output and inflation caused by the impact

on aggregate demand leads to a persistent but small rise in nominal interest rates (not shown). A rise in government expenditure on final goods results in a rise in labor supply which boosts aggregate output (despite pushing down on aggregate consumption) via the wealth effect of government debt. The dotted line corresponds to less active fiscal policy and we observe less movement in the fiscal deficit in response to a fiscal shock.

#### 5.1.4 Persistence of Shocks

Figure 6 illustrates the result of varying the persistence of the monetary and fiscal shocks for the optimal weight in the simple policy rules.<sup>23</sup> We find that increasing the persistence on fiscal shocks from 0-0.8 tends to require more aggressive monetary and fiscal policy and that increasing the persistence of monetary shocks tends to reduce the required weight on the policy rules. We leave further analysis of this interaction between monetary and fiscal policy to future work but suggest that the qualitative results are retained where monetary policy targets inflation and to a lesser degree output and where fiscal policy places a relatively strong weight on output.

## 5.2 Policy Experiments

In this subsection we discuss a number of policy experiments that flow naturally from the optimal simple rules we have already characterized. However, here we exogenously restrict the policy parameter space, which allows us to explore further the interactions between joint (constrained) optimal plans for monetary and fiscal policy.

### 5.2.1 $\phi_y^f = 0$

Let us suppose that fiscal policy, in the first instance, is constrained not to respond to output at all. Table 4 shows the resulting implications for the monetary rule.

Here we have constrained fiscal policy merely to follow an autocorrelated process. When fiscal policy does not react to output,  $\phi_y^f = 0$ , we find that the long run coefficient on inflation in the interest rate rule moves sharply up at the optimum. The long-run coefficient on inflation,  $\phi_\pi^m$  is found to be over 5, regardless of the extent of interest rate smoothing (which we have constrained to take the values in Table 3). We note that should the policy maker wish to limit her *dynamic* response to inflation in the region of 1 – 1.5, then in the presence of a passive fiscal stance, significant interest rate smoothing should be employed i.e.

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<sup>23</sup>We are grateful to a referee for suggesting this exercise.

the lagged coefficient on interest rates should be above 0.5. In short, a passive fiscal policy shifts the burden of stabilization entirely onto monetary policy, causing that arm of policy to be somewhat more active than the Taylor principle might suggest.

### 5.2.2 $\phi_y^f \leq 0$

Table 5 considers further implications for the monetary policy rule when fiscal policy is set procyclically rather than counter-cyclically. The first point to note is that for high long run negative feedback from the output gap, i.e.,  $\phi_y^f > 0.8$ , the system quickly becomes unstable. But if we allow for small negative feedback we find that the weights on both output and, in particular, inflation rise considerably in order to attain an optimum. This result implies that, to some limited extent, monetary policy can substitute for poorly designed fiscal policy by becoming significantly more aggressive.

### 5.2.3 $\phi_\pi^m \rightarrow 1$

Table 6 considers the implications for fiscal policy as the long run coefficient on inflation in the interest rate rule tends to one.<sup>24</sup> The first two sets of results (columns 2-5) show that if the IRR maintains a long run coefficient on inflation of at least one,  $\phi_\pi^m \geq 1$ , optimal fiscal policy will alter little. The next three sets of results vary the long run response of the IRR to inflation, from  $\phi_\pi^m = 1.8, 1.1$  and  $1.0$ . We see that as the IRR response to inflation tends towards one, and given that the output response is small, the optimal FR responds to output more powerfully. This means that fiscal policy can, to some extent, act as a substitute for weak IRR responses to inflation but the policy losses at each of these optima suggest that this strategy is somewhat unattractive.

### 5.2.4 High inflation aversion

Table 7 shows the implications for fiscal policy from a monetary policy-maker who places a high weight on inflation stabilization. The Table shows that active monetary policy in this case engenders similarly active fiscal policy in order to reach the optimum. This means that aggressive monetary policy will be complemented by a similar fiscal policy in order to stabilize the economy optimally. The high weight on inflation in the monetary rule results in a volatile real interest rate, in turn generating

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<sup>24</sup>The columns headed IRR refer to Interest Rate Rule and those headed FDR refer to the Fiscal Deficit Rule. For IRR go to the l.h.s. column for a key and for FDR go to the r.h.s. for a key.

a volatile path for consumption and hence output. Consequently this elicits a robust response from fiscal policy.

### 5.2.5 Wealth Effect of Bonds

The wealth effect impacting on aggregate consumption dynamics in equation (2.21) is filtered by a parameter that represents the steady-state ratio of wealth to consumption,  $\frac{w}{c}$ , multiplied by the probability of death,  $\lambda$ . We note that if  $\lambda = 0$  that bonds are not net wealth. For  $\lambda \neq 0$ , and  $b_{t-1} > 0$ , outstanding bonds will tend to boost aggregate demand. In this section we therefore examine the impact on the optimal rule of allowing the probability of death to increase. In the following tables  $\lambda = 0.003$  implies expected remaining life is 70 years,  $\lambda = 0.005$  implies a horizon of 50 years and  $\lambda = 0.007$  implies a 35 year horizon.

Table 8 shows that halving life expectancy makes very little difference to the optimal weights. Recall that Ricardian Equivalence fails in this set-up solely because of the overlapping generations structure of the model, since not all of the currently alive can expect to live sufficiently long to face the obligations of current debt creation. Figure 7 illustrates these results in more detail. We solve for the optimal long run weight on output as we vary both the ratio of wealth to consumption and life expectancy and find little change in the optimal long run parameter. However, Figure 8 shows that there is some response in the optimal weight as the relative weight on fiscal shocks to productivity shocks increases.<sup>25</sup>

The simulations in this section have shown that (i) joint monetary-fiscal plans may have a number of attractive properties for systematic stabilization policy; (ii) passive fiscal policy, at the optimum, may provide a rationale for aggressive monetary policy; (iii) near breaching of the Taylor principle, in terms of the unit response of interest rates to inflation, will stimulate active fiscal policy at an optimum; (iv) that aggressive monetary policy will again engender active fiscal policy; and (v) optimal fiscal policy seems most sensitive to a greater degree of stochastic variation in aggregate spending rather than parameters in the aggregate consumption dynamics. In each case understanding optimal monetary policy requires a characterization of the concomitant fiscal policy choices, where output stabilization seems to lie very much within the correct scope of fiscal rather than monetary policy.

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<sup>25</sup>We are grateful to a referee for suggesting this illustration to us.

## 6 Conclusions

We develop, calibrate and simulate a micro-founded model in which prices are rigid and firms produce differentiated goods. Fiscal policy acts to influence aggregate demand by influencing current expenditure by creating a wealth effect. Monetary policy regulates inflation by setting the price of intertemporal trade. At the policy optimum, under commitment, we find a clear case for the policymaker (when pursuing a simple rule) observing the Taylor principle in conjunction with letting automatic stabilizers induce a non-zero fiscal balance through the economic cycle.

We show that neglect of the role of automatic stabilizers in designing optimal policy will have implications for optimal monetary policy. For example, we show that ‘passive’ fiscal policy necessitates large (i.e.  $\phi_\pi^m > 5$ ) long run responses to inflation in the interest rate rule for the previous optimum to be reached. Also we indicate how aggressive monetary policy may result in an aggressive set of fiscal plans. The recent literature on monetary policy rules has perhaps implicitly assumed the ‘correct’ fiscal policy will be followed. In this paper we have developed a model that allows us to say what form that fiscal policy takes. Our brief comparison of actual and ‘optimal’ policy suggests some insight has been gained.

Future work may seek to extend the fiscal environment in three directions. First we may consider incorporating distortionary taxation. Recent work by Auerbach and Feenberg (2000) suggests that this will be an important channel through which the systematic effects of fiscal policy operate. That will also constitute an additional and likely more important channel through which Ricardian Equivalence will fail. Second, incorporating ‘useful’ government expenditure, perhaps via influencing the marginal return to private investment, and a distinction between government employment and government purchases may prove valuable. Finn (1998) demonstrates this latter distinction is of crucial importance, as they have offsetting impacts on output. Third, we might well consider to what extent the current stabilization debate in advanced economies can best be framed with reference to the commitment-type solutions studied here. It is worth considering more rigorously the implications of non-collusive behavior between the monetary and fiscal policy makers, especially within a monetary union.

Finally, we noted at the start of this paper that Taylor Rules for monetary policy were unlikely to describe literally what monetary policymakers actually do. Similarly, the simple fiscal rules that we have constructed, although providing a valuable calculation of the magnitude of deficits and surpluses that one might want to occur in actual economies, tell us little about the micro-structure of the tax

and benefits system. Nevertheless, it would appear that the design of optimal stabilization policy requires consideration of both monetary and aggregate fiscal plans.

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**Table 1a: US Business Cycle Dynamics**

	$\sigma_i$	$\frac{\sigma_i}{\sigma_y}$	$y_{t-4}$	$y_{t-3}$	$y_{t-2}$	$y_{t-1}$	$y_t$	$y_{t+1}$	$y_{t+2}$	$y_{t+3}$	$y_{t+4}$
$y_t$	1.563	1.0	0.140	0.415	0.696	0.912	-	-	-	-	-
$p_t$	1.339	0.857	0.089	-0.067	-0.238	-0.408	-0.548	-0.647	-0.683	-0.663	-0.600
$\pi_t$	1.397	0.894	0.671	0.614	0.488	0.304	0.086	-0.131	-0.313	-0.440	-0.508
$i_t$	1.500	0.960	0.435	0.554	0.631	0.627	0.513	0.308	0.039	-0.235	-0.464
$s_t$	0.772	0.494	0.367	0.563	0.713	0.776	0.724	0.541	0.285	0.018	-0.205

**Table 1b: UK Business Cycle Dynamics**

	$\sigma_i$	$\frac{\sigma_i}{\sigma_y}$	$y_{t-4}$	$y_{t-3}$	$y_{t-2}$	$y_{t-1}$	$y_t$	$y_{t+1}$	$y_{t+2}$	$y_{t+3}$	$y_{t+4}$
$y_t$	1.458	1.0	0.255	0.508	0.751	0.932	-	-	-	-	-
$p_t$	2.126	1.458	-0.128	-0.265	-0.400	-0.522	-0.615	-0.652	-0.627	-0.541	-0.409
$\pi_t$	2.504	1.71	0.453	0.357	0.206	0.008	-0.206	-0.391	-0.518	-0.570	-0.548
$i_t$	1.496	1.026	0.605	0.624	0.591	0.487	0.310	0.085	-0.155	-0.372	-0.536
$s_t$	1.503	1.031	0.033	0.122	0.181	0.204	0.201	0.190	0.187	0.195	0.207

Notes: (i) all data are from 1955:1 to 2001:4; (ii) we show the results for Band-Pass filtered series with a 12-quarter moving average window; (iii) Hodrick-Prescott filtered series are available on request; (iv) column 2 is the standard deviation of the filtered series,  $i$ ; (v) column 3 is the standard deviation scaled by output; (vi) we examine the correlation with output at leads and lags; (vii) the first row gives the autocorrelation function of output.

**Table 2: Calibration parameters for quarterly model**

Symbol	Value	Description
$\lambda$	0.00357	Expected life remaining: 70 years
$r$	0.0125	Real interest rate
$\beta$	0.988	Subjective discount factor
$\delta$	0.01325	Subjective discount rate
$\gamma$	0.015	Rate of debt retirement
$\frac{c}{y}$	0.6	Steady-state consumption-output ratio
$\frac{m}{w}$	0.1	Steady-state money-wealth ratio
$\kappa$	0.5	Phillips curve slope
$\frac{w}{c}$	0.7	Steady-state wealth-consumption ratio

**Table 3: The Optimal Simple Rules**

	Interest rate rule		Fiscal rule	
$\pi_t - \pi^*$	1.15		$\pi_t - \pi^*$	0
$y_t - y_t^*$	0.06		$y_t - y_t^*$	1.75
$R_{t-1}$	0.25		$D_{t-1}$	0.57

**Table 4: Interest Rate Rule Implications of Fiscal Deficit Smoothing**

	Interest rate rule					Fiscal rule	
$\pi_t - \pi^*$	4.83	3.55	1.54	1.01	$\pi_t - \pi^*$	-	
$y_t - y_t^*$	0.80	0.55	0.19	0.19	$y_t - y_t^*$	-	
$R_{t-1}$	0.25	0.3	0.5	0.8	$D_{t-1}$	0.9	

**Table 5: Interest Rate Rule Implications of Procyclical Deficit Smoothing**

	Interest rate rule				Fiscal rule	
$\pi_t - \pi^*$	6.17	10.46	19.35	49.36	$\pi_t - \pi^*$	-
$y_t - y_t^*$	1.13	2.08	4.07	10.77	$y_t - y_t^*$	$\phi_y^f$
$R_{t-1}$	0.25	0.25	0.25	0.25	$D_{t-1}$	0.8
Fiscal Rule $\phi_y^f$	-0.05	-0.10	-0.15	-0.20		

**Table 6: Fiscal Policy Implications of Threshold Taylor Observance**

	IRR	FR	IRR	FR	IRR	FR	IRR	FR	IRR	FR
$\pi_t - \pi^*$	0.5	-	0.5	-	0.9	-	0.9	-	0.8	-
$y_t - y_t^*$	0.5	1.69	0.1	1.72	0.1	1.68	0.1	2.19	0.1	2.27
$\rho$	0.5	0.68	0.5	0.65	0.5	0.61	0.2	0.44	0.2	0.47

**Table 7: Fiscal Policy Implications of the High Inflation Aversion**

	IRR	FR	IRR	FR	IRR	FR
$\pi_t - \pi^*$	4.5	-	3.5	-	2.0	-
$y_t - y_t^*$	0.1	3.06	0.1	3.03	0.1	2.57
$\rho$	0.2	0.44	0.2	0.47	0.2	0.52

**Table 8: Optimal Simple Rules for alternative  $\lambda$**

	Interest rate rule				Fiscal rule		
$\lambda$	0.003	0.005	0.007		0.003	0.005	0.007
$\pi_t - \pi^*$	1.1502	1.1496	1.1487	$\pi_t - \pi^*$	0	0	0
$y_t - y_t^*$	0.0604	0.0592	0.0574	$y_t - y_t^*$	1.7524	1.7524	1.7524
$R_{t-1}$	0.2495	0.2489	0.2481	$D_{t-1}$	0.5726	0.5730	0.5734

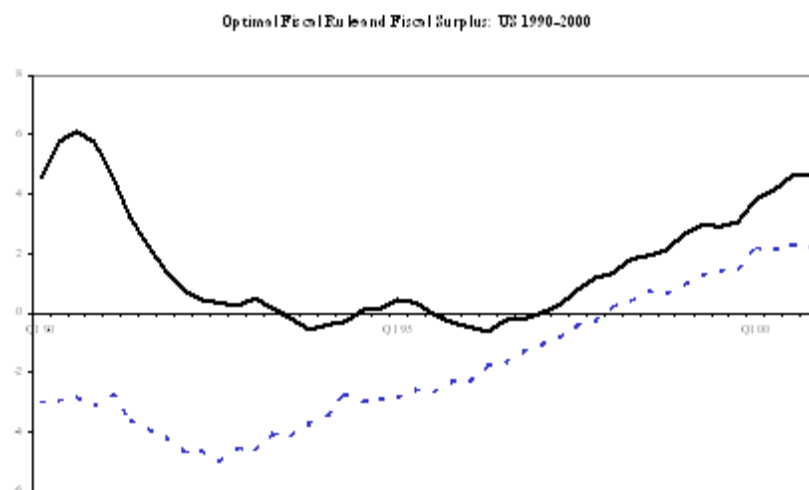
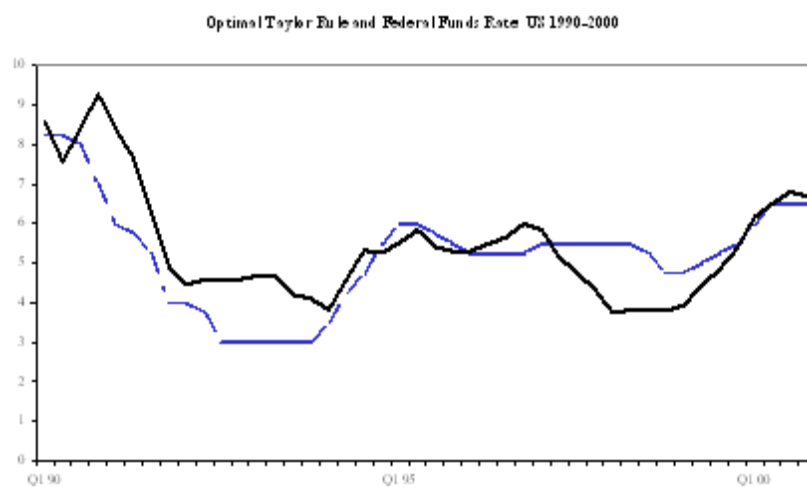


Fig. 1 - US Rules in Practice and Theory

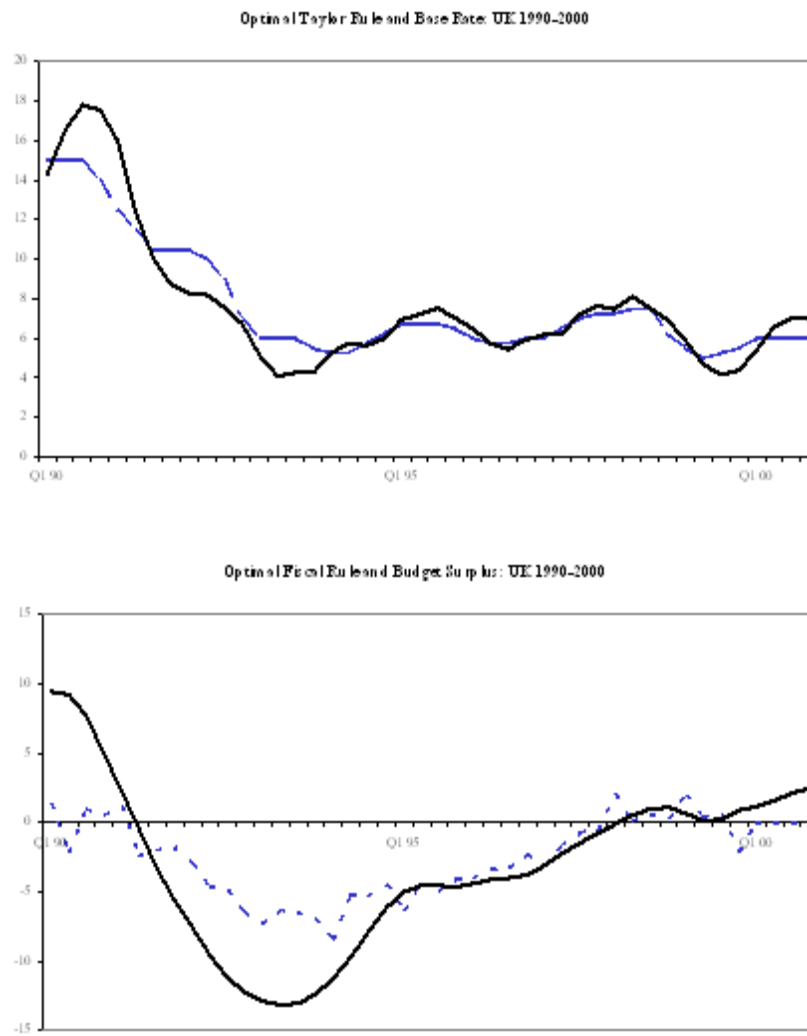


Fig. 2 - UK Rules in Practice and Theory

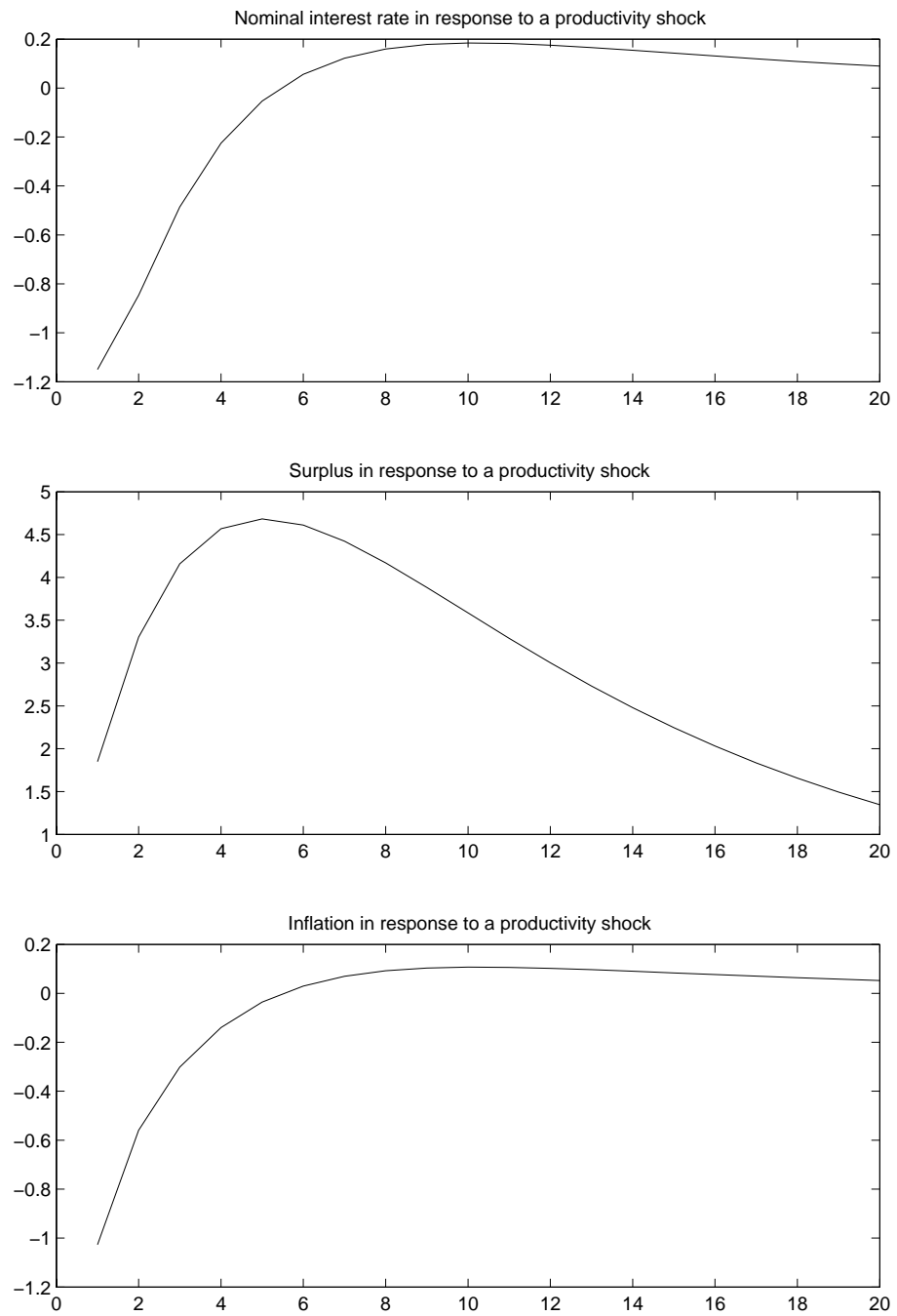
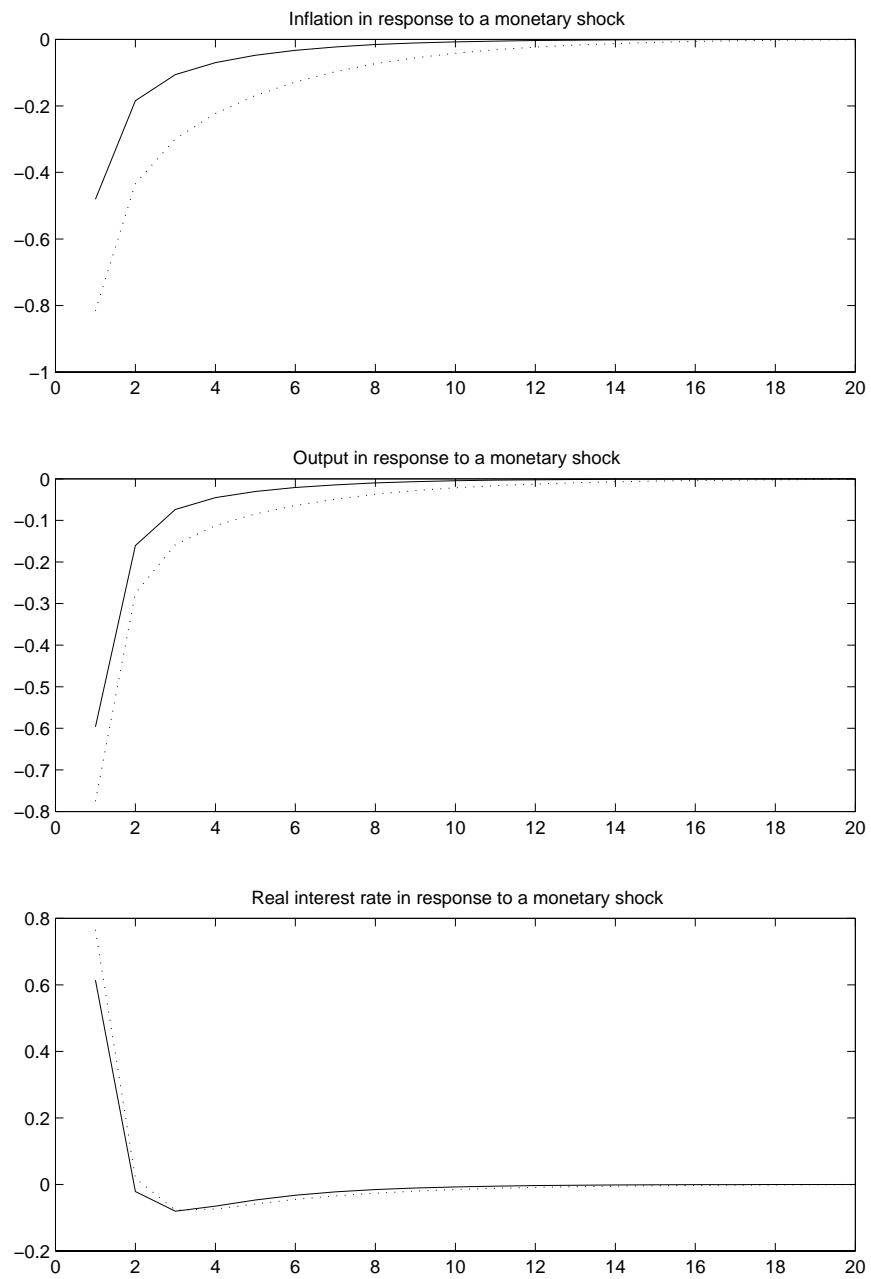


Figure 3: Key responses to a productivity shock



Note: dotted line indicates setting long run inflation weight in monetary policy as 1 instead of 1.5 of optimum

Figure 4: Key responses to a monetary shock

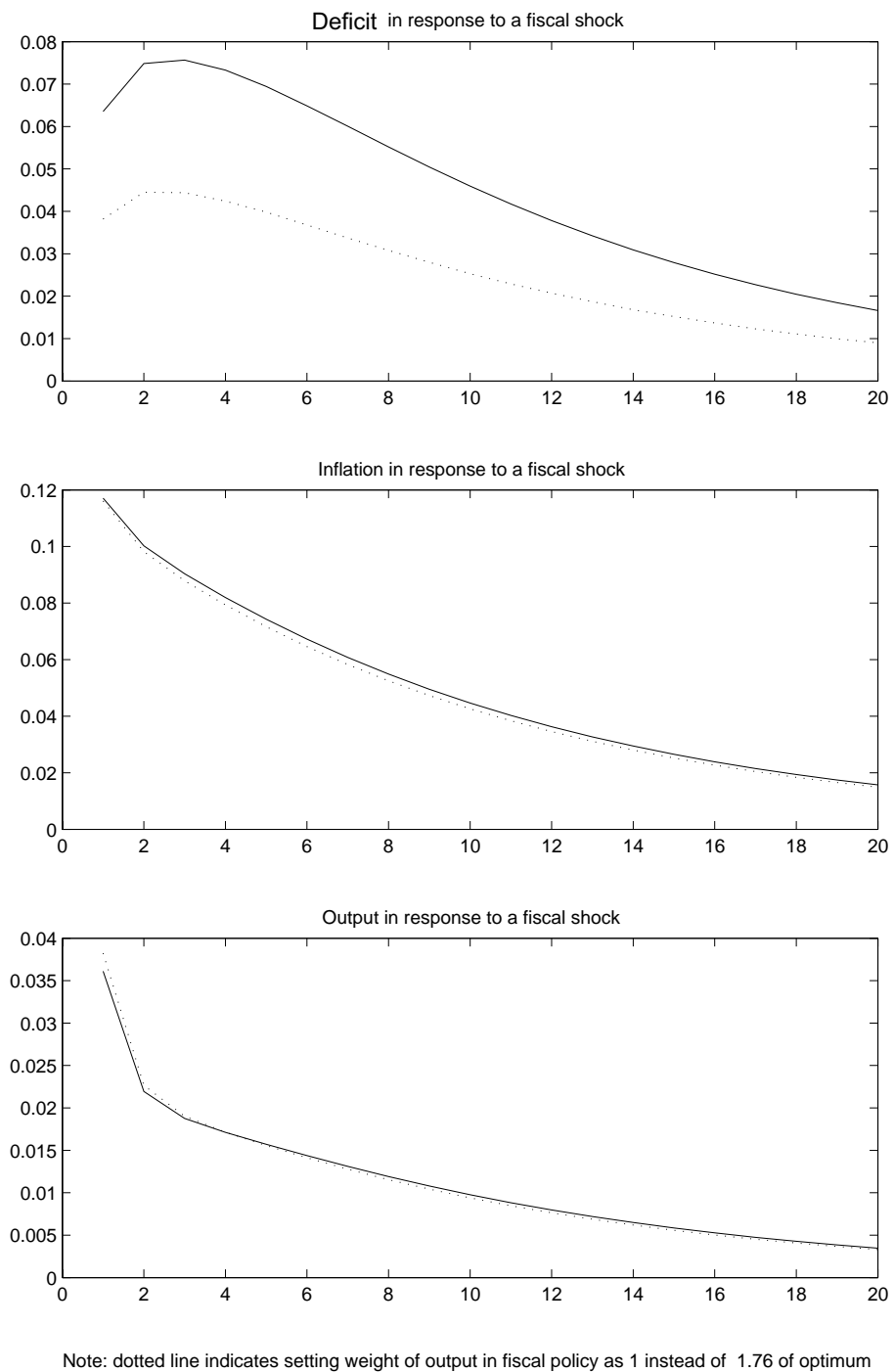


Figure 5: Key responses to a fiscal shock



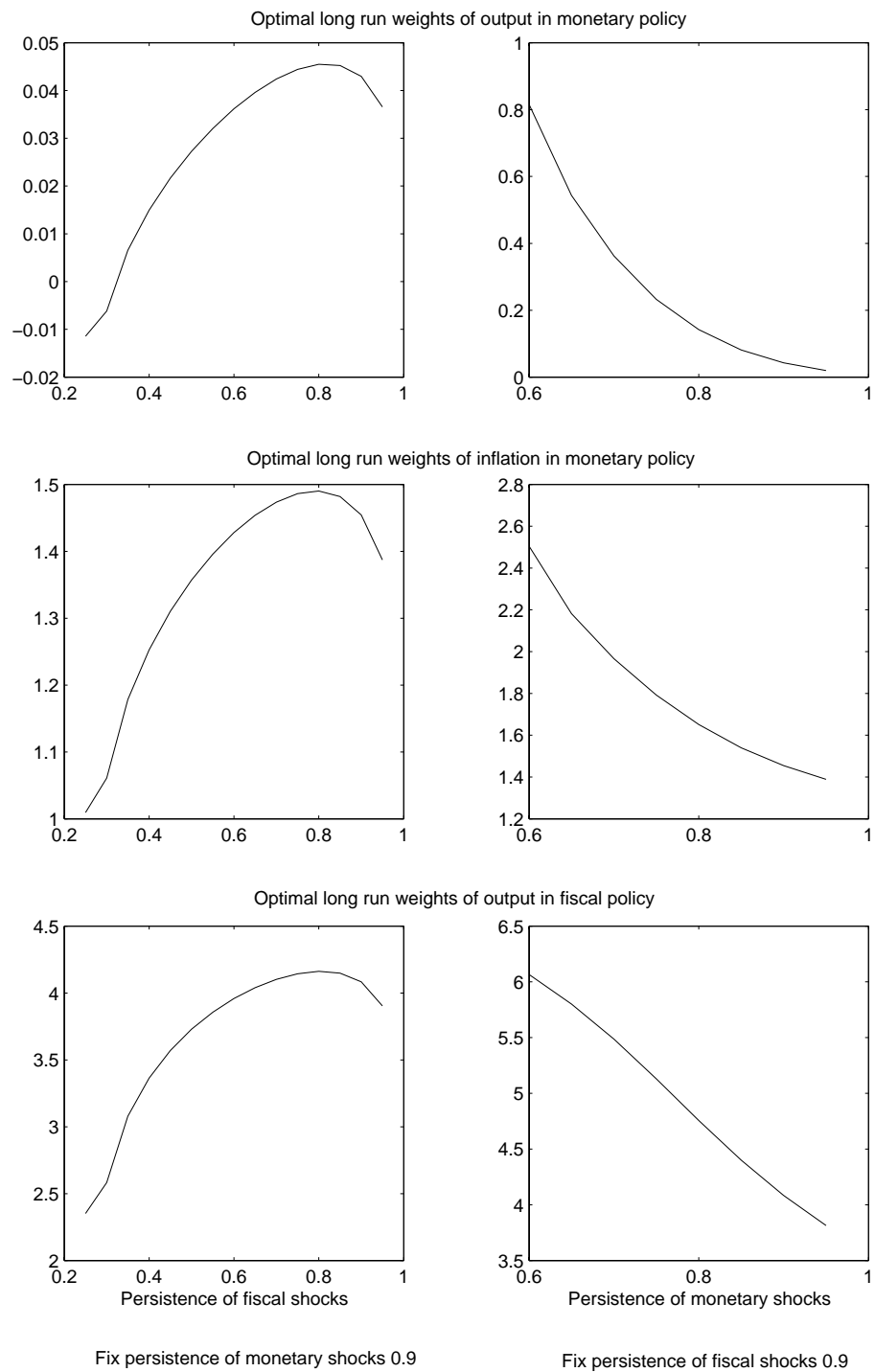


Figure 6: Changing the persistence of shock process

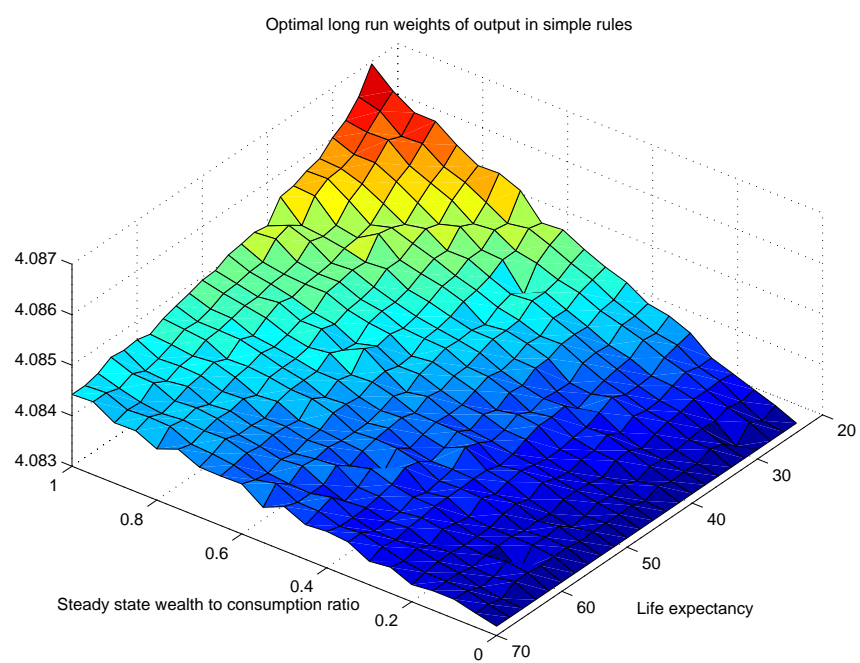


Figure 7: Illustration of wealth effect

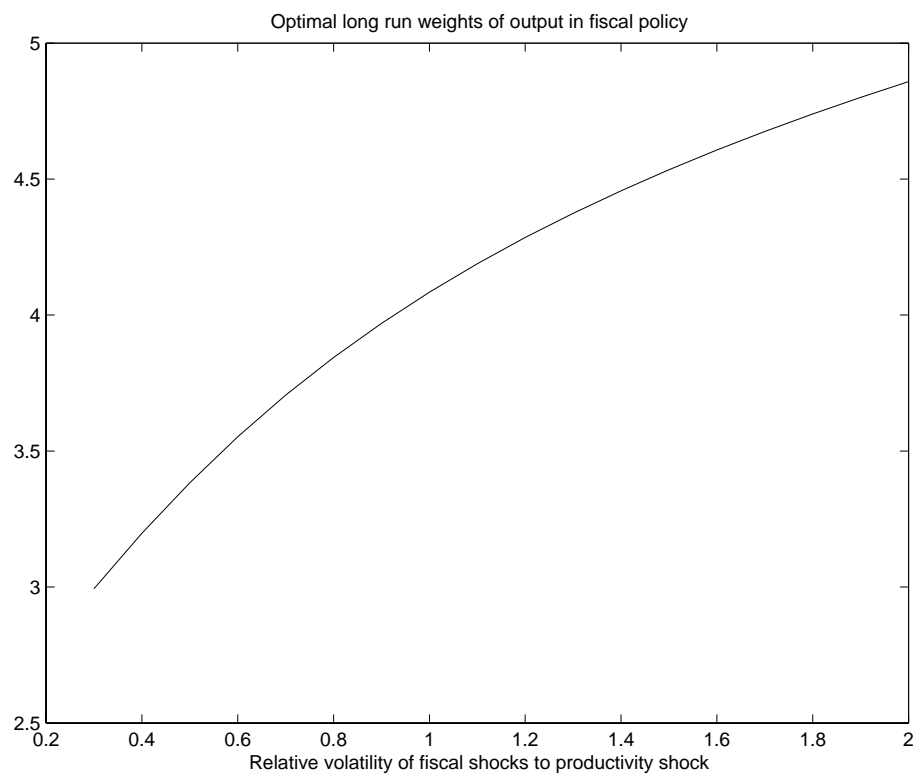


Figure 8: Changing relative volatility of fiscal shocks

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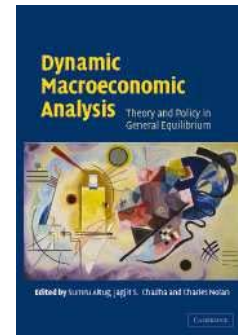
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