CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS
WORKING PAPER SERIES

CDMA04/04

Tax and Irreversible Investment*

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NOVEMBER 14, 2004

ABSTRACT

We examine the impact of tax policy uncertainty on the irreversible investment decisions of a monopolistically competitive firm. We consider the impact of tax policy in terms of the investment tax credit (ITC) as well as the stochastic tax wedge which determines the after-tax costs of investing. We show that (i) temporary tax incentives have a greater stimulative impact on investment; (ii) greater policy volatility results in greater variability of investment; (iii) a stochastically larger future tax policy lowers current investment; and (iv) greater variability of tax policy around a constant mean leads to lower investment. We examine changes in the average levels of investment and its variability over short- and long-run horizons. We consider a general 3-state process, a log-normal process, and a Poisson process. We carry out an extensive sensitivity analysis that allows us to examine the impact of changes in real interest rates, and changes in the elasticity of demand facing firms.

JEL Classification: E22; E62.
Keywords: Irreversible investment, tax policy, changes in persistence, increases in risk.

* An earlier version of this working paper was presented at the Money, Macro and Finance Group, London, March 2000, and at seminars at Edinburgh, Keele, Manchester and Warwick.

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1 Introduction

An examination of US tax policy shows that it has been volatile at time. Cummins, Hassett and Hubbard (1994) document changes in postwar US tax policy, and note that there have been 13 important changes in the corporate tax code from 1962 to 1988. The post-1987 period has been a relatively stable period where U.S. tax policy is concerned. However, when President George W. Bush became President in 2001 he introduced tax reforms which were adopted after some modifications by the US Congress. A new round of tax changes to stimulate investment and to counter the slowdown in the US and world economies following the terrorist attacks of September 2001 and the uncertainty created by the Iraqi War have also recently been approved by the U.S. Congress. Thus, in spite of long periods of stability, uncertainty and volatility with respect to tax policy still exist.

There are numerous tax provisions which affect corporate investment, three of the most noteworthy being the statutory corporate profits tax rate, depreciation allowance schedules, and the investment tax credit (ITC). A current reduction in the corporate tax rate increases after-tax cash flows but it does not affect the decision to invest directly. By contrast, a current ITC alters the relative price of new investment. Furthermore, the impact of a tax change differs depending on whether it is expected to be permanent versus temporary.[1]

Beginning with the pioneering work of Hall and Jorgensen (1967), the impact of alternative tax policies on investment behavior has been the topic of numerous studies. Hassett and Hubbard (2002) provide an extensive and illuminating survey of taxation and investment.[2] One of the early theoretical contributions is by Sandmo (1974). Abel (1979,1980,1982), Summers (1981), Auerbach (1989),

[1]A temporary ITC is typically expected to have a larger impact than a permanent ITC because it induces an intertemporal allocation of investment. Since a temporary ITC may not be in place at a future date, firms will undertake investments that they would otherwise not have done to take advantage of the temporary tax incentive. See Congressional Budget Office Report (2002) for a recent discussion about the stimulative impacts of alternative tax changes.

[2]See also Demers, Demers, and Atug, section 4.5.

Auerbach and Hines (1987, 1988), and Alvarez, Kannianen, and Sodersten ((1998) analyze changes in corporate tax rates and the ITC using a cost-of-adjustment model based on Q-theory. On the empirical front, Auerbach and Hines (1991,1992) and Cummins, Hassett, and Hubbard (1994,1996) provide empirical evidence regarding the impact of tax policy and tax reform in cost-of-adjustment models with Q-theory. Judd (1987) and Jorgenson and Yun (1986,1990,1991), among others, analyze the impact of tax reform on long-run capital accumulation in perfect foresight general equilibrium frameworks. Their analyses allow for an evaluation of the welfare effects from tax distortions.

A recent article in *Foreign Affairs* (2003) argues that one of the most important obstacles to private investment in the Russian energy sector is the lack of certainty regarding tax and regulatory laws, One way of reducing such uncertainty is through PSA's or "production sharing agreements," which "lock in tax regimes, clarify resource ownership, and guarantee payments in fungible exportable assets (such as oil) that are not so vulnerable to exchange rates." The article also notes the irreversibility underlying such investments: "No PSA really provides an enclave of stability - investors know that they are always vulnerable to 'renegotiation' where the law is weak and once their investments are entrenched."

There are few papers that have investigated the impact of tax policy uncertainty on investment decisions. Aizenman and Marion (1993) consider the impact of randomness in the corporate tax rate on irreversible investment in a general equilibrium model with an AK technology while Altug, Demers, and Demers (2002) consider the impact of greater volatility in the corporate tax rate for a risk neutral monopolistically competitive firm. In both cases, greater volatility in the corporate tax rate has no impact on investment unless it is persistent. The reason is that changes in the current corporate tax rate merely alter the firm's after-tax cash flows when they are serially independent. However, when they are positively serially correlated, there is an information effect from the change in the current tax rate. Consequently, a more variable corporate tax rate will lower current investment by increasing the option value of waiting.[3]

[3]These results hold when there are no depreciation allowances. When there are depreciation allowances, greater

The argument that a more predictable (i.e. more persistent) and less variable ITC leads to lower variability of investment has been made by general equilibrium models that address the issue of the variability of the corporate tax rate and of the ITC. Bizer and Judd (1989) consider a general equilibrium stochastic growth framework with uncertain taxation and which ignores any costs of adjustment. They demonstrate that randomness in the capital income tax raises revenues at a small welfare cost relative to a stable rate whereas a random ITC generates substantial fluctuations in investment at a large welfare cost.[4] Chang (1995) uses a Kydland-Prescott (1982) time-to-build real business cycle framework to investigate the impact of exogenously specified distortionary corporate and personal income taxes on the variability of investment in the presence of productivity shocks. Chang's analysis emphasizes that the variability in exogenous tax policy can have a substantial impact on the variability of investment.

There are also a number of papers that have investigated the impact of nonlinearities in the tax code on investment expenditures. MacKie-Mason (1990) considers tax nonlinearities such as depreciation allowances under two rules for corporate income taxation: full loss refunds and no loss refunds when firms face output price uncertainty. He finds that raising depreciation allowances may deter investment when the interaction of uncertainty and tax nonlinearities are taken into account. Penninga (2000) shows that an investment tax credit together with a tax on future profits reduces the trigger value of investment, and stimulates investment at zero expected cost in an irreversible investment model under uncertainty. By contrast, he shows that financing the costs of accelerated depreciation with a tax on profits will not raise investment at zero expected cost. Faig and Shum (1999) use a model of a firm with financing constraints to analyze the impact of a corporate tax on a firm's irreversible investment decisions taking into account the asymmetry in the corporate income tax code which treats capital gains and losses differently.

volatility in future corporate tax rates will lower investment even when they are IID because there is now a cost effect.

[4] They further argue that it is not useful to summarize the impact of taxes on investment by an "effective tax rate" because it does not recognize the differential effects of uncertainty with respect to the capital income tax and the ITC.

In an influential study, Hassett and Metcalf (1999) study the impact of increases in risk in the investment tax credit when firms must optimally choose the time to undertake an irreversible investment project. When policy uncertainty is in the form of a continuous geometric Brownian motion, they find that a mean-preserving spread increases the median time to investing. By contrast, when the ITC is generated by means of a two-state stationary jump process, they find that an increase in tax risk *increases* investment rather than lowering it.[5] They also consider the impact of counter-cyclical tax policy when the fiscal authority sets the value of the current ITC to vary inversely as a function of the current state of demand. In this case, they show that the impact of an increase in risk on investment is still positive but less than the case with exogenous policy.[6]

In this paper, we examine the impact of changes in tax policy in a framework where investment is irreversible. We consider the impact of changes in the level of the current investment tax credit (ITC) and a stochastic tax wedge as well as of changes in the distribution of their future values (in the sense of stochastic dominance, persistence and mean preserving spreads (MPS).) We examine the response of firms in the short-run and also discuss the long-run implications of our model. Our analysis is based on the irreversible investment model developed in Demers (1985,1991) and Altug, Demers and Demers (1999,2000,2002). It focuses on the profit-maximization problem of a risk neutral monopolistically

[5]The literature on the impact of increases in risk on investment has generated some conflicting results. In a cost-of-adjustment model with a constant returns to scale (CRS) production function and perfect competition, Hartman (1972) showed that greater variability of the output price around a constant mean, that is, a mean-preserving spread, will increase investment. Hartman's conclusion stems from the convexity of the profit function with respect to the output price. See also Pindyck (1982) and Abel (1983,1985). In the absence of CRS, the impact of a MPS depends on the properties of the marginal revenue of capital with respect to the output price.

[6]Another important issue regarding the impact of tax policy on investment behavior has to do with the *time inconsistency* of optimal policy. In a recent analysis, Panteghini and Scarpa (2003) consider this issue in the context of a study of regulatory risk on irreversible investment. Likewise, Erbenova and Vagstad (1999) analyze the interaction of market uncertainty and uncertainty arising from the actions of opportunistic governments which cannot commit to a future course of action on investment behavior.

competitive firm under uncertainty. The industry-specific nature of most investment goods implies that investment decisions are, at least largely, irreversible, and therefore, sensitive to risk and uncertainty. A number of authors have recently emphasized irreversibility and uncertainty as important factors underlying the gradual adjustment of the capital stock; see, for example, Nickell (1977a,1977b, 1978), Demers (1985, 1991), Bean (1989), Bertola (1989), Pindyck (1988), and Bertola and Caballero (1994). The existence of the irreversibility constraint under uncertainty leads to an option value of waiting, or to an endogenous risk premium. Increases in risk increase the option value of waiting and deter investment.

In our framework, a change in the current ITC (or the stochastic tax wedge) alters the relative price of investment goods. We call this the *cost effect* of a change in the ITC (or tax wedge). However, if there is any persistence in the tax process, then there is also an *information effect* from a change in the current ITC (or tax wedge) on investment. The cost effect is unambiguously positive but the nature of the information effect will depend on how persistent the tax policy process is perceived to be. We use this framework to demonstrate the effects of temporary versus permanent tax incentives, and the impact of anticipated tax reforms on investment. For processes that are perceived to be more persistent or permanent, the information effect is negative (e.g., a high ITC today signals a high ITC tomorrow), so that there is a tendency to delay investment. By contrast, for less persistent or temporary tax incentives, since a high ITC today will be followed by a low ITC tomorrow, firms have an incentive to invest more today. Following the earlier analysis of Donaldson and Mehra (1983) and Danthine, Donaldson, and Mehra (1981), we also show that firms' perceptions about the persistence of tax policy can affect the variability of investment expenditures and of the stationary distribution of the capital stock. However, whether variability increases depends on the form of the investment policy function.

We also conduct an extensive analysis of the impact of tax risk on investment behavior. We examine the impact of increasing the variability of tax policy around a constant mean for a general stationary log-normal process and for discrete jump processes. In all cases, we find that an increase in tax risk that is not accompanied by a change in the level of tax incentives reduces investment expenditures and leads to an increase in its variability in the short run and over longer horizons. In what follows, we abstract from general equilibrium considerations. However, we discuss the implications of general equilibrium modelling for our results.[7] For this purpose, we provide a sensitivity analysis regarding changes to the persistence and variability of demand shocks, changes in real interest rates, and changes in the elasticity of demand.

The remainder of this paper is organized as follows. Section 2 presents a discussion of the impact of changes in persistence and increases in risk on the level of investment. Section 3 presents the simulation method while Section 4 presents the simulation results. Section 5 concludes.

2 The Theoretical Framework

Corporate investment is affected by a number of tax provisions. Foremost among these is the corporate income tax rate, depreciation allowance provisions, and the investment tax credit (ITC). In the post World War II period, there have been a number of changes in U.S. corporate tax policy. To describe these changes, we introduce some notation. The firm's after-tax cash flow at t, R_t, is defined as

$$R_t = (1-\tau_t)\Pi(K_t, \alpha_t, A_t, w_t) + \tau_t \sum_{x=1}^{T} D_{x,t-x} p_{t-x}^k I_{t-x} + (1-\gamma_t) p_t^k I_t, \tag{2.1}$$

where $\Pi(K_t, \alpha_t, A_t, w_t)$ is the firm's short-run profit function expressed in terms of the current capital stock K_t, a stochastic demand shift α_t, the level of technology A_t, and a vector of variable input prices w_t. In this expression, τ_t denotes the corporate tax rate at time t, γ_t the investment tax credit (ITC)

[7] Veracierto (2002) and Thomas (2002) find that firm-level investment irreversibility or lumpiness are unimportant at the aggregate level once general equilibrium effects such as endogenous price adjustments are taken into account. By contrast, Caballero (1999) argues that such microeconomic nonlinearities must matter because they are identified at the aggregate level using aggregate data.

at time t as a percentage of the price of the investment good, $D_{x,t-x}$ depreciation allowances per dollar invested for tax purposes for capital equipment of age x on the basis of the tax law effective at time $t-x$, and T the life of the equipment. We let i denote the nominal rate of interest, r denote the real rate of interest, π^e the expected rate of inflation, and define z_t as the present value of tax deductions on new investment, where

$$z_t = \sum_{n=1}^{T} \tau_{t+n} D_{n,t} (1+i)^{-n}, \text{[8]} \tag{2.2}$$

where $i = r + \pi^e$. Letting p_t^k denote the purchase price of investment goods, we can define the stochastic tax wedge or after-tax percentage of p_t^k that the firm pays for investment goods as

$$\widetilde{\zeta}_t \equiv (1 - \widetilde{\gamma}_t - z_t). \tag{2.3}$$

Notice that the corporate tax rate affects the firm's cash flows directly, and it also has an effect through the deductions for depreciations allowances for cash purposes. By contrast, the ITC alters the relative price of new and used investment goods.

In this study, we focus on changes in the level and distribution of the ITC and the stochastic tax wedge. Variation in ζ_t captures variation arising from changes in the corporate tax rate τ_t, and expected inflation π_t^e.[9] An examination the US data shows that variation in the ITC explains 29% of the variation in the stochastic tax wedge for equipment investment over the period 1946-1996, and that the ITC and the present value of depreciation allowances co-vary positively. By contrast, variation in the ITC is much more important for the period 1962-1985, and there is some evidence that the ITC and the present value of depreciation allowances co-vary negatively.[10] Depreciation allowances may be

[8] In our calculations, we assume that $\tau_{t+n} = \tau_t$.

[9] We ignore the direct effects of changes in the corporate tax rate on firms' cash flows. To the extent that the corporate tax rate underwent major changes in a few periods and was relatively stable throughout the period 1952-1986, this assumption is warranted.

[10] The opposite is true for structures investment. Most of the variation in the stochastic tax wedge is due to variation in depreciation allowances, and the covariance between the ITC and depreciation allowances is much lower.

affected by changes in nominal interest rates or expected inflation. In this paper, aside from the induced variation in ζ_t, we do not provide an explicit treatment of changes in nominal interest rates or expected inflation.[11]

2.1 The Firm's Problem

Consider the problem of a monopolistically competitive risk neutral firm which faces uncertainty in its environment arising from shocks to demand and from randomness from tax policy. Versions of this problem are analyzed in detail in Demers, Demers, and Altug (2003). The firm's problem can be decomposed into two parts: first, that of choosing a vector of variable factors of production L_t, and second, of choosing investment subject to an irreversibility constraint. The solution to the first part of the problem delivers a short-run profit function $\Pi(K_t, \alpha_t, A_t, w_t)$ as a function of the current capital stock K_t, a shock to demand α_t, a technology shock A_t, and the vector of variable input prices w_t. We assume that Π is continuous in K_t, α_t, A_t, w_t, increasing in K_t, α_t, and A_t, decreasing in w_t, and strictly concave in K_t.

The focus of our analysis is an examination of changes in tax policy in the presence of demand uncertainty. To allow for an expectations or informational effect, we assume that demand shocks and shocks to tax policy are serially correlated. Specifically, we assume that $\widetilde{\gamma}_t$ and $\widetilde{\alpha}_t$ take values in R^+, and let $G(\gamma_{t+1} \mid \gamma_t)$ and $F(\alpha_{t+1}|\alpha_t)$ denote the (objective) distributions of $\widetilde{\gamma}_{t+1}$ and α_{t+1}, conditional on γ_t and α_t, respectively. Our main analysis treats tax policy as exogenous.[12] To the extent that tax policy has a counter-cyclical component, this can be captured by assuming that α_t and γ_t (or ζ_t) are

[11]The analysis of these issues is of substantial interest. Jorgenson and Yun (1986,1990,1991) examine the tax incentives under alternative tax proposals for different inflation rates. Auerbach and Hines (1988) note that tax rules appear to have been set to "undo" the effects of inflation. Nam and Radelescu (2003) consider tax incentive schemes for seven transition economies, and show that in an inflationary environment generous depreciation provisions do not promote private investment but partly compensate for any additional burdens caused by inflation.

[12]Thus, we assume that the demand shock α_t is independent of γ_t or the stochastic tax wedge ζ_t.

correlated.[13]

Using dynamic programming, we can express the firm's problem recursively as

$$V(K_t,\gamma_t) = \max_{I_t} \left\{(1-\tau_t)\,\Pi\,(K_t,\alpha_t,A_t,w_t) - p_t^I I_t + \beta E_t V(K_{t+1},\gamma_{t+1})\right\} \tag{2.4}$$

subject to the law of motion $K_{t+1} = (1-\delta)K_t + I_t$, the irreversibility constraint $I_t \geq 0$, K_t given and where $\beta = (1+r)^{-1}$, $0 < \beta < 1$ is the discount factor and E_t indicates that expectation is taken with respect to $G(\gamma_{t+1}|\gamma_t)$ and $F(\alpha_{t+1}|\alpha_t)$.[14] Let V_K denote the partial derivative of V with respect to K. The first-order necessary and sufficient conditions for the optimization problem at time t are

$$\begin{aligned} -p_t^I + \beta E_t V_K(K_{t+1},\gamma_{t+1}) &= 0 \quad \text{if } I_t^* > 0 \\ &\leq 0 \quad \text{if } I_t^* = 0. \end{aligned} \tag{2.5}$$

Using the envelope theorem, we find the partial derivative of $V(K_{t+1},\gamma_{t+1})$ with respect to K_{t+1} for $t = 1, 2, \ldots$ as

$$\begin{aligned} V_K(K_{t+1},\gamma_{t+1}) = (1-\tau_{t+1})\,\Pi_K\,(K_{t+1},\alpha_{t+1},w_{t+1}) + \\ (1-\delta)\min\left[p_{t+1}^I, \beta E_{t+1} V_K\left((1-\delta)\,K_{t+1},\gamma_{t+2}\right)\right], \end{aligned} \tag{2.6}$$

where $V_K(K_{t+1},\gamma_{t+1})$ is the shadow value of capital. Assume that an interior solution obtains in period t. After substituting for the shadow price and for β, the first-order condition (2.5) for time t can be re-arranged as

$$\begin{aligned} &(1-\tau_{t+1})\Pi_K\,(K_{t+1},\alpha_{t+1},w_{t+1}) = c_t \\ &+ (1-\delta)\left\{E_t\widetilde{p}_{t+1}^I - E_t \min\left[\widetilde{p}_{t+1}^I, (1+r)^{-1}\, E_{t+1} V_K\left((1-\delta)\,K_{t+1},\gamma_{t+1}\right)\right]\right\}, \end{aligned} \tag{2.7}$$

where Π_K is the partial derivative of Π with respect to K_{t+1}, and where $c_t = p_t^I\,(r+\delta) - (1-\delta)\,(E_t\widetilde{p}_{t+1}^I - p_t^I)$ is the firm's cost of capital in the sense of Jorgenson (1967). If investment were reversible, equation

[13] A more general specification might be to assume that there are feedback effects from aggregate investment to the processes determining tax policy.

[14] For simplicity we have suppressed A_t, w_t and α_t as arguments of V.

(2.7) would reduce to $(1-\tau_{t+1})\Pi_K(K_{t+1}, \alpha_t, w_{t+1}) = c_t$, which is the optimality equation for a firm purchasing or selling capital services one period in advance before the uncertainty about ITC is resolved. Note that the choice of investment is no longer a dynamic problem when investment is reversible. The second term on the right-hand side of equation (2.7) is a risk premium that the firm requires for the loss of flexibility that it incurs since it cannot disinvest. It represents a positive marginal adjustment cost arising endogenously from the irreversibility of investment.[15]

Many studies have found implausibly large adjustment costs in models with constant adjustment costs (see, for example, Summers 1981.) Other authors have uncovered evidence suggesting that adjustment costs may be varying over time (see, for example, Oliner, Rudebusch, and Sichel (1996), Demers, Demers and Schaller (1994) and Suzuki and Ogawa (1994)). In a cost of adjustment framework, Abel and Blanchard (1986) relate investment to the determinants of Q, and find that their model leaves unexplained a large, serially correlated component. It has been suggested that compositional effects in investment (especially the shift towards investment in computers) can induce such serial correlation. Tevlin and Whelan (2003) provide a recent analysis in this regard. In particular, they show that in the presence of such serial correlation, it may be difficult to identify the effects of the cost of capital on investment.

In our framework, the tax policy variables affect current investment directly through the current costs of investing, and also through the expected value of the future marginal value of capital. Anticipations of future tax reforms or any changes in the distribution of tax variables will affect investment by altering

[15] An alternative way of interpreting the endogenous risk premium is in terms of an option value of waiting. See Dixit and Pindyck (1994). Such an option value of waiting also arises in an irreversible investment model with learning, and varies in time in response to changes in investor's posterior beliefs, which constitute the state of information. See Demers (1985,1991) and Altug, Demers, and Demers (1999,2000). For a recent test of the option value model, see Harchaoui and Lasseree (2001). See also Abel, Dixit, Eberly, and Pindyck ((1996) for a model with partial irreversibility and expandability, and Abel and Eberly (1994) for a model with asymmetric adjustment costs.

the endogenous risk premium or option value of waiting.[16] This framework allows us to examine the impact of anticipated tax reforms on investment. It also allows us to analyze of the impacts of policy persistence and increases in tax risk on both the level and variability of investment expenditures. We discuss these issues from a theoretical perspective in this section, and provide a quantitative analysis in the next.

2.2 Changes in the Current Level of the ITC

In the literature on investment and tax policy, the emphasis has been on the impact of anticipated tax reforms and permanent versus temporary tax policy changes on investment. In an environment under certainty, Abel (1982) shows that a temporary ITC may not have a larger impact than a permanent ITC if there is constant returns to scale in production. Using numerical simulations, Summers (1981) investigates the impact of taxes on investment taking into account firms' financial policy. Auerbach (1989) examines the impact of anticipated tax reforms in a cost-of-adjustment model with a concave production function. Anticipated changes affect the firm's investment and Q separately. Auerbach and Hines (1988) and Alvarez, Kannianen, and Sodersten extend this literature by allowing for uncertainty about the timing of tax reforms. Auerbach and Hines (1988) argue that investors form expectations of tax reforms under uncertainty. Even in the absence of any policy actions, the *ex ante* value of depreciation allowances change yearly due to changes in expected inflation and real interest rates. Furthermore, even though tax parameters change infrequently, there is typically extended policy debates regarding proposed tax changes.

The existing literature has typically emphasized the difference between temporary versus permanent anticipated changes in the ITC. Yet if investors do not know the future value of the ITC with certainty,

[16] Unlike the analysis of tax policy in cost-of-adjustment models with non-time-varying adjustment costs, our framework allows the endogenous risk premium or option value to vary in response to any variables that firms use to forecast the future marginal value of capital.

then one way of capturing the differences between temporary versus permanent changes in tax variables is to examine their *persistence*. First consider the effect of a current change in the ITC when it is serially independent. In this case, the distribution function is simply $G(\gamma_{t+1})$, so that the expectation of V_K is a constant.[17] If an interior solution exists at time t, we obtain from the first-order condition

$$\frac{\partial I_t}{\partial \gamma_t} = \frac{p_t^k}{-\beta \int V_{KK} dG(\gamma_{t+1})} > 0. \tag{2.8}$$

Thus a higher current ITC has a cost-reducing effect and stimulates investment when the ITC is serially independent.[18]

When there is serial dependence, however, a change in the ITC will also change the distribution function $G(\gamma_{t+1} \mid \gamma_t)$ and the expectation of the future marginal value of capital (V_K). We define a positively (negatively) correlated process as one where the current ITC signals a greater (lower) likelihood of a high ITC in the next period. Assuming an interior solution, and integrating the numerator by parts we have

$$\frac{\partial I_t}{\partial \gamma_t} = \frac{p_t^k}{-\beta \int V_{KK} dG(\gamma_{t+1}|\gamma_t)} - \frac{\beta \int V_{K\gamma} G_{\gamma_t}(\gamma_{t+1}|\gamma_t) d\gamma_{t+1}}{-\beta \int V_{KK} dG(\gamma_{t+1}|\gamma_t)} \tag{2.9}$$

In addition to a cost-reducing term, we now also have an additional term representing the information effect. The sign of the information effect depends on the impact of a change in γ_t on $G(\gamma_{t+1}|\gamma_t)$, and on whether V_K is increasing or decreasing in γ_{t+1}.

The first question can be answered by considering shifts in the distribution of the ITC in the sense of first-order stochastic dominance (FSD).[19] When $\tilde{\gamma}_{t+1}$ is positively serially correlated, $G(\gamma_{t+1} \mid \gamma_t')$ dominates $G(\gamma_{t+1} \mid \gamma_t)$ by FSD for all $\gamma' \geq \gamma$ and for all t. That is, $G_{\gamma_t}(\gamma_{t+1} \mid \gamma_t) \leq 0$ for all γ_{t+1} with a strict inequality for some value of γ_{t+1}. This condition can be interpreted to mean that the future

[17] In what follows, we ignore the impact of demand uncertainty, and write the expectation of the future value function only as a function of the conditional distribution of the tax policy variables.

[18] In the case of a corner solution at time t, an increase in γ_t will lead to a lower incidence of a binding constraint.

[19] A cumulative distribution function $H(\gamma)$ is larger than $G(\gamma)$ in the sense of FSD if and only if $H(\gamma) \leq G(\gamma)$ *for all* γ, with a strict inequality holding for some γ. See Hadar and Russell (1971).

resembles the present. In the case of negative serial correlation, for all $\gamma_t' \geq \gamma_t$, $G_{\gamma_t}(\gamma_{t+1} \mid \gamma_t)$ dominates $G_{\gamma_t}(\gamma_{t+1} \mid \gamma_t')$ by FSD, i.e., $G_{\gamma_t} \geq 0$.

Turning to the sign of $V_{K\gamma}$, when $\widetilde{\gamma}_{t+1}$ is positively serially correlated, the marginal value of capital V_K is decreasing in γ_{t+1} (see Lemma 1). This occurs because capital can be bought more cheaply in the future and because a higher ITC at $t+1$ signals an even higher one in the future.[20] Both of these effects reduce the marginal value of additional capital to be carried into $t+1$. When $\widetilde{\gamma}_{t+1}$ is negatively serially correlated, a higher ITC at $t+1$ signals a lower one in the future, so that the two effects work in opposite direction, rendering the sign of $V_{K\gamma}$ indeterminate.

In the positively correlated case (since $V_{K\gamma} < 0$ and $G_{\gamma_t} \leq 0$), the information effect in (2.9) depresses current investment by inducing firms to wait. With a positive cost effect and a negative information effect, the sign of $\partial I_t / \partial \gamma_t$ becomes indeterminate and must be determined quantitatively. In the negatively correlated case, $G_{\gamma_t} \geq 0$. Assuming that $V_{K\gamma} < 0$, the information effect in (2.9) will be positive. Thus, when the ITC follows a negatively correlated process, both the cost effect and the information effect are conducive to stimulating current investment, so that a higher ITC today has an unambiguously positive effect on current investment ($\partial I_t / \partial \gamma_t > 0$).

We can use this framework to examine the response of firms to temporary versus permanent changes in tax policy and to analyze the impact of anticipated tax reforms. Suppose firms expect a change in tax policy that is likely to eliminate tax incentives that are in place today. Our results for the negatively correlated case implies we would expect a spurt of activity as firms strive to take advantage of tax incentives that they know will not be available in the future. The higher the negative correlation, the more certainty that firms attach to a decline in the ITC, conditional on its being high today, and the greater the expected investment response. We can also examine the behavior of firms in the immediate aftermath of a reform.[21] If firms know with certainty the date at which tax incentives are

[20] The first effect operates when there is an interior solution and the second when there is a corner solution.

[21] Cummins, Hassett, and Hubbard (1994) suggest that firms may be aware of future tax policy changes because they

likely to be removed, then we would expect to observe a decline in their investment activity in the period immediately following a reform. In this case, firms reduce their invest both because current tax incentives are low, and also because they expect them to persist with high probability. We could also model uncertainty with respect to the *timing* of a tax reform by assuming that firms attach a positive probability to a shift to a new tax regime with a different distribution for the tax policy variables.[22]

2.3 FSD Shifts in the Distribution of the ITC

Suppose next that there is a change in government where the new administration is favorable to increasing the ITC. Such a situation can be characterized as a FSD shift in the distribution of the ITC, such that all the moments of the distribution are now larger. In other words, the firm now expects a higher ITC on average, but is also more uncertain of its true value.[23]

Formally, consider two distributions $\widehat{G}$ and G such that $\widehat{G}(\gamma_{t+1} \mid \gamma_t)$ dominates $G(\gamma_{t+1} \mid \gamma_t)$ by FSD. For a given observed value of γ_t, $\widetilde{\gamma}_{t+1}$ is stochastically larger under $\widehat{G}$ than under G. Now, as can be easily shown, $E_{\widehat{G}} V_K(K_{t+1}, \gamma_{t+1}) > (<) E_G V_K(K_{t+1}, \gamma_{t+1})$ as $V_{K\gamma} > (<) 0$ [24] where the subscript on the expectation operator denotes the relevant distribution function. Hence, denoting the optimal investment under G and $\widehat{G}$ by I_G^* and $I_{\widehat{G}}^*$, from the first-order condition (2.5), optimal investment is higher (lower) under $\hat{G}$ relative to G ($I_{\widehat{G}}^* > (<) I_G^*$) depending on whether $V_{K\gamma}(K_{t+1}, \gamma_{t+1}) > (<) 0$.

The probability of a higher future ITC increases with a shift in the distribution from G to $\widehat{G}$. With positive serial correlation, since the marginal value of capital, V_K, is unambiguously decreasing in

follow the legislative debate in newspapers and the media.

[22] Altug, Demers, and Demers (2000) propose such a regime shift model with time-varying transition probabilities.

[23] One might argue that investment opportunities in transition countries have undergone such shifts, as regimes that are more favorable to market forces have taken power. However, at the same time as they have increased the expected net return to investing (perhaps through tax incentives or other reductions in the after-tax costs of investing), they have also made it more uncertain.

[24] Using integration by parts, $E_{\widehat{G}} V_K(K_{t+1}, \gamma_{t+1}) - E_G V_K(K_{t+1}, \gamma_{t+1}) \equiv \int V_K \left(d\widehat{G} - dG \right) = \int V_{K\gamma}(G - \widehat{G}) d\gamma > (<) 0$ as $V_{K\gamma}(K_{t+1}, \gamma_{t+1}) > (<) 0$

γ_{t+1} $(i.e., V_{K\gamma} < 0)$, the firm delays investment and reduces current investment expenditures under $\widehat{G}$. The same is true for the case of serial independence (since $V_{K\gamma} < 0$) and for negative serial correlation with $V_{K\gamma} < 0$. When the ITC is negatively correlated and $V_{K\gamma} > 0$ investment rises under $\widehat{G}$. We summarize this discussion by the following proposition.

Proposition 1 *Let $\widehat{G}(\gamma_{t+1} \mid \gamma_t)$ dominate $G(\gamma_{t+1} \mid \gamma_t)$ by FSD.*

(a) If $\widetilde{\gamma}_{t+1}$ is positively serially correlated, or is serially independent, then I_t^* is lower under $\widehat{G}$.

(b) If $\widetilde{\gamma}_{t+1}$ is a negatively serially correlated, then I_t^* is higher (lower) under $\widehat{G}$ if $V_{K\gamma} > (<) 0$.

2.4 Changes in Persistence

Persistence in policy (or the lack of it) affects the way decision-makers evaluate their future prospects. In this section, we examine the impact of changes in policy persistence on the variability of investment expenditures. Our analysis draws on Donaldson and Mehra (1983) and Danthine, Donaldson, and Mehra (1981), who examine changes in persistence in a stochastic growth framework.

Suppose that tax policy leads to a distribution for the ITC that is more persistent than the original one. Assume that $\gamma_t \in [\gamma^L, \gamma^H]$, and let (G^2) denote the distribution that is more persistent relative to the original distribution (G^1).[25] Suppose that $\tilde{\gamma}_t$ follows a positively serially correlated process (or a negatively correlated process where $V_{K\gamma} < 0$). Let $\gamma_t = \gamma^L$. Then the distribution function exhibiting greater persistence (G^2) attaches greater probability to lower realizations of $\widetilde{\gamma}_{t+1}$ than G^1. Hence, $G^1(\gamma_{t+1}|\gamma^L)$ dominates $G^2(\gamma_{t+1}|\gamma^L)$ by FSD. From the first-order condition in equation (2.5), we find that investment falls under G^1.[26] Thus, $I(K_t, \gamma^L, G^1) < I(K_t, \gamma^L, G^2)$.

[25]For discrete distributions, this can be expressed as $Pr^2(\gamma_{t+1} = \gamma' | \gamma_t = \gamma') > Pr^1(\gamma_{t+1} = \gamma' | \gamma_t = \gamma')$ and $Pr^2(\gamma_{t+1} = \gamma'' | \gamma_t = \gamma') < Pr^1(\gamma_{t+1} = \gamma'' | \gamma_t = \gamma') \quad \forall \gamma'' \neq \gamma'$. For continuous distributions, this means that G^2 attaches higher probability mass to a neighborhood of the current observation than G^1 does. Thus, the probability that "the future resembles the present" is higher under G^2 than G^1. See Donaldson and Mehra (1983, p. 308.)

[26]This follows from the fact that $\beta E_{G^1} \left[V_K(K_{t+1}, \gamma_{t+1}) | \gamma^L\right] \leq \beta E_{G^2} \left[V_K(K_{t+1}, \gamma_{t+1}) | \gamma^L\right] \leq (1 - \gamma^L - z_t) p_t^k$.

Similarly, for $\gamma_t = \gamma^H$, the distribution G^2 attaches greater probability to high realizations of $\widetilde{\gamma}_{t+1}$ than G^1 and hence, dominates the latter by FSD. Therefore, by the first-order condition in equation (2.5), investment falls under G^2, that is, $I(K_t, \gamma^H, G^1) > I(K_t, \gamma^H, G^2)$.[27] Since the policy function $I(K_t, \gamma_t, G^1)$ is continuous in γ_t, this implies that there exists some $\hat{\gamma}$ such that $I(K_t, \hat{\gamma}, G^1) = I(K_t, \hat{\gamma}, G^2)$, and

$$\begin{aligned} I(K_t, \gamma, G^1) &< I(K_t, \gamma, G^2) \quad \gamma^L \leq \gamma < \hat{\gamma} \\ I(K_t, \gamma, G^1) &> I(K_t, \gamma, G^2) \quad \hat{\gamma} < \gamma \leq \gamma^H. \end{aligned}$$

The impact of changes in persistence on variability depends on the sign of the optimal policy function with respect to the current ITC. For the positively correlated case, the sign of $\partial I_t / \partial \gamma_t$ depends on whether or not the positive cost-reducing effect of a higher current ITC overcomes the negative information effect. For the negatively serially correlated case, the impact of changes in persistence in the distribution of the ITC depends on the sign of the marginal value of capital with respect to γ, $V_{K\gamma}$. Depending on which of these effects prevails, we can have a variety of outcomes. We summarize this in terms of the following proposition.

Proposition 2 *(i) Suppose γ_t is positively correlated and $\partial I_t / \partial \gamma_t > 0$, or γ_t is negatively correlated and $V_{K\gamma} < 0$. Then lower persistence in the tax credit induces greater variability of investment, and thus, of the stationary distribution for the capital stock, that is, $I(K_t, \gamma^H, G^1) \geq I(K_t, \gamma^H, G^2) \geq I(K_t, \gamma^L, G^2) \geq I(K_t, \gamma^L, G^1)$.*

(ii) Alternatively, suppose γ_t is positively correlated and $\partial I_t / \partial \gamma_t < 0$, or γ_t is negatively correlated and $V_{K\gamma} > 0$. Then greater persistence in the tax credit induces greater variability of investment, and thus, of the stationary distribution for the capital stock, that is, $I(K_t, \gamma^L, G^2) \geq I(K_t, \gamma^L, G^1) \geq I(K_t, \gamma^H, G^1) \geq I(K_t, \gamma^H, G^2)$.

[27] As before, this follows from $\beta E_{G^2} \left[V_K(K_{t+1}, \gamma_{t+1}) | \gamma^H \right] \leq \beta E_{G^1} \left[V_K(K_{t+1}, \gamma_{t+1}) | \gamma^H \right] \leq (1 - \gamma^H - z_t) p_t^k$.

Thus, depending on the impact of a change in the current ITC on the current costs of investing or on the marginal value of capital, "informational changes" arising from the distribution of the ITC can increase or decrease the variability of investment and of the stationary distribution of the capital stock.

2.5 Increases in Risk

Now we ask what would happen to investment behavior if the fiscal authority were to randomize the ITC around some known mean value. It will be convenient to consider the tax wedge $\widetilde{\zeta}_{t+1} \equiv (1 - \widetilde{\gamma}_{t+1} - z)$ instead of the ITC.

We begin by defining a mean-preserving spread (MPS) in the distribution function of $\widetilde{\zeta}_{t+1}$, $F(\zeta)$.[28]

Definition 3 *A cdf $\widehat{F}(\zeta)$ is a MPS of another cdf $F(\zeta)$ if and only if $\int_{\zeta^L}^{y} \left[F(\zeta)d\zeta - \widehat{F}(\zeta)\right] d\zeta \leq 0$ for all y, with strict inequality for some ζ; $= 0$ for $y = \zeta^H$.*[29]

Our first result concerns the case where the tax wedge is independently distributed.

Proposition 4 *If $\widetilde{\zeta}_{t+1}$ is independently distributed, a MPS unambiguously reduces investment.*

When $\widetilde{\zeta}_{t+1}$ is independently distributed, Lemma 2 shows that the shadow price of capital V_K is concave in ζ. Consequently, analogously as in the proof of Proposition 1, a mean-preserving increase in the distribution of the stochastic tax wedge depresses the expected future marginal value of capital, and by the first-order condition (2.5), reduces current investment.

Turning to the case where the tax wedge is positively serially correlated, there is an information effect from a stochastic change in the tax wedge, an effect that is absent from the serially independent case. In particular, a MPS in the distribution of $\widetilde{\zeta}_{t+1}$ conditional on ζ_t randomizes not only $\widetilde{\zeta}_{t+1}$,

[28] Note that, as can be easily shown, a MPS in $G(\gamma_{t+1}|\gamma_t)$ is equivalent to a MPS in $F(\zeta_{t+1}|\zeta_t)$.

[29] A MPS is a special case of a second-order stochastic dominance ranking (SSD), in that the mean of the two distributions are equal. In the case of SSD, the integral condition is the same but the equality of means is not required. See Hadar and Russell (1971).

but also the *future* tax wedge, ζ_{t+2} through its impact on $F(\zeta_{t+2}|\zeta_{t+1})$. A sufficient condition for investment to fall at date t is that the tax wedge at time $t+2$, $\widetilde{\zeta}_{t+2}$, be *a stochastically increasing and concave function (SICV)* of the previous period's tax wedge, ζ_{t+1}.[30] If ζ_{t+2} is any increasing and concave function of ζ_{t+1}, then $\widetilde{\zeta}_{t+2}$ will be a SICV function of ζ_{t+1}.[31]

Proposition 5 *Let the tax wedge be positively serially correlated. If $\widetilde{\zeta}_{t+2}$ is a stochastically increasing and concave function of $\tilde{\zeta}_{t+1}$, a MPS in $F(\widetilde{\zeta}_{t+1}|\zeta_t)$ reduces I_t^*.*[32]

To gain some intuition about the notion of stochastic concavity, we note that when this condition holds, if the policy-maker could randomize the tax wedge ζ_{t+1} at time $t+1$, the distribution of $\widetilde{\zeta}_{t+2}$ conditional on ζ_{t+1} would "worsen" in the sense of second order stochastic dominance (SSD) and hence, the conditional mean of $\widetilde{\zeta}_{t+2}$ would fall, in other words, the firm would expect a lower tax wedge at time $t+2$.[33]

Firms facing irreversible investment decisions and uncertain tax policy have the option to decide when and how much to invest. They respond to the greater riskiness by reducing and postponing investment at time t because the value of waiting (or the endogenous risk premium) has risen. When the tax wedge is stochastically increasing and concave, a MPS in the conditional distribution of $\widetilde{\zeta}_{t+1}$, $F(\zeta_{t+1}|\zeta_t)$ (which is equivalent to a MPS in the conditional distribution of the tax wedge at $t+1$) leads the firm to lower investment at t and $t+1$ in the expectation that since the conditional mean of the

[30]The process $\widetilde{\zeta}_{t+2}$ is a stochastically increasing and concave (SICV) function of ζ_{t+1} if and only if $\int_{\zeta^L}^{x} F(\zeta_{t+2}|\zeta_{t+1})d\zeta_{t+2}$ is decreasing and convex in ζ_{t+1} for all x. See Shaked and Shanthikumar (1993), Chapter 6, Theorem 6.A.6 (a), p. 173.

[31]More generally, Jewitt (1988) shows that the stochastic concavity of ζ_{t+2} will be ensured if its conditional density function, $f(\zeta_{t+2}|\zeta_{t+1})$, is totally positive. (See Karlin, 1968).

[32]Proof of Proposition 5 follows analogously as in the proof of Proposition 1, applying Lemma 3.

[33]The notion of stochastic concavity appears in several areas of economics, particularly in the literature on principal-agent problems. Thus, for example, Jewitt (1988) derives sufficient conditions to ensure the validity of the first-order approach to the principal-agent problem. One of the conditions he imposes is the same as our sufficient condition.

tax wedge at $t+2$ is lower, investment at $t+2$ will be stimulated.

As Hasset and Metcalf (1999) note, an increase in the variance of the tax credit creates an implicit "subsidy" for firms because they can always wait to invest in the high tax credit (or low capital cost) state. Our framework also displays a "subsidy" effect of a MPS on investment. If as a result of the MPS, the stochastic tax wedge facing firms is more spread out and riskier in the future, investment tends to fall today. However, there is also an *intertemporal substitution effect* of a MPS for the case of a serially correlated tax wedge process in that the MPS randomizes the tax wedge in the next period. The randomization of tax policy in one period (i.e. in period $t+1$) leads to a reduction in investment at t and and $t+1$, and an increase at $t+2$. Whether a net quantitative reduction in investment occurs can only be determined numerically.

Finally, we examine mean-preserving increases in risk in the negatively correlated case.

Proposition 6 *Let the tax wedge be negatively serially correlated. Assume* $V_{K\zeta} > 0$. *If* $F_{\zeta_t\zeta_t}(\zeta_{t+1}|\zeta_t) > (<)0$ *investment falls (rises) with a MPS in* $F(\zeta_{t+1}|\zeta_t)$.[34]

In this case, provided that the sign of $V_{K\zeta}$ can be determined numerically, we can determine the impact of a MPS by imposing a stronger condition than in the positively correlated case, namely, the convexity or concavity of the distribution function $F(\zeta_{t+1}|\zeta_t)$ in ζ_t.[35] If, as ζ_t increases, the probability of a lower cost tomorrow (due to a higher ITC) increases more and more rapidly (i.e $F_{\zeta_t\zeta_t} > 0$), a condition which we could term optimism, a MPS in the distribution of $\widetilde{\zeta}_{t+1}$ lowers current investment.

[34]Proof of Proposition 6 follow analogously as in the proof of Proposition 1, applying Lemma 4.

[35]The assumption of convexity of the distribution function is also used in the principal-agent literature. See Mirrlees (1975) and Jewitt (1988).

3 Simulation Procedure

3.1 Parameterizing the Model

We now make specific assumptions about the production function and the inverse demand function. We assume a constant returns to scale Cobb-Douglas production as

$$Y_t = A_t K_t^{\eta} L_t^{1-\eta}, \tag{3.1}$$

where A_t is a shock to technology, and $0 < \eta < 1$. Let p_t denote the output price. We assume that the firm faces a constant elasticity demand function. The inverse market demand function is given by

$$p_t = (\alpha_t)^{-1/\varepsilon} (Y_t)^{1/\varepsilon}, \tag{3.2}$$

where $\varepsilon < -1$ is the price elasticity of demand, and α_t is a variable representing the state of demand. Under these assumptions, the firm's short-run profit function (which has been optimized over the choice of the variable factors) is given by

$$\Pi\left(K_t, \alpha_t, w_t\right) \equiv \upsilon_t K_t^{\mu} \alpha_t^{1-\mu}$$

where $a = (1-\eta)(1+\varepsilon)$ and $b = 1 - \eta(1+\varepsilon)$. Then $\upsilon_t = A_t^{\mu/\eta} w_t^{a/b} [N^{-a/b} - N^{-\varepsilon/b}] > 0$, $N = (1+\varepsilon)\varepsilon^{-1}(1-\eta) < 1$, and $0 < \mu = -\eta(1+\varepsilon)/(1-\eta(1+\varepsilon)) < 1$.

The values of the elasticity of demand ε and the elasticity of output with respect to capital η are determined using the implications of the firm's profit maximization problem under imperfect competition. In a static version of the firm's problem, the markup of price over marginal cost is defined as $\text{MRKP} = p_t/\text{MC}_t$, where MC_t is the marginal cost of producing an additional unit of output. Under profit maximization, marginal cost equals marginal revenue. Making use of this fact and the form of the inverse demand function in equation (3.2), the markup can be expressed as $\text{MRKP} = \varepsilon/(1+\varepsilon)$. The value of ε is determined using the markup estimates reported by Morrison (1992) and Roeger

(1995). Morrison's (1992) estimates imply an average markup of 1.162 for U.S. manufacturing over the period 1960-1985 while Roeger's (1995) estimates imply an average markup of 1.6054 using U.S. 2-digit manufacturing industries for the period 1953-1984. This implies that $-7.17 \leq \varepsilon \leq -2.65$. We use a simple average of Morrison's and Roeger's estimates as the baseline value of ε, namely, $\varepsilon = -3.607$, and also consider two other values as $\varepsilon = -5$ and $\varepsilon = -2.65$,

The firm's profit maximization problem also implies that the elasticity of output with respect to capital η can be measured by the share of capital in total factor costs, c_K, which, in turns, equals $c_K = (\text{MRKP}/\phi)s_K$, where s_K is the revenue share of capital, ϕ is the degree of returns to scale, and MRKP is the markup. Under constant returns to scale, $\phi = 1$. The direct measurement of the revenue share of capital s_K depends on the definition of capital stock and of national income, with estimates ranging from 0.25 to 0.43 (see Cooley, 1995, chapters 1 and 6). Since capital in our model corresponds more closely to a narrower definition of capital such as the sum of producers' durable equipment and structures, we set $s_K = 0.25$ to obtain the value of η used in our study.

We set the baseline value of the discount factor β equal to $\beta = 0.947$, which implies an annual real interest rate of $r = 5.6\%$. This is typically the value that is used in calibrated real business cycle models based on the steady state properties of the model. (See Cooley (1994), page 21.) The historical data implies that the observed real interest rate is much lower. From 1926 to 1997, the 20-year Treasury *real* bond return was 2.4% with a standard deviation of 10.5%. Alternatively, from 1946 to 1997, the 20-year Treasury real return was 1.6% with a standard deviation of 11.8%. This is just a re-statement of the equity premium puzzle of Mehra and Prescott (1984). (See Weil (1989).) Thus, for sake of comparison, we also assume that $\beta = 0.9766$, which implies a real interest rate of 2.4%.

In the simulations, we analyze the impact of tax policy uncertainty in the presence of demand uncertainty, holding constant real wages, the technology shock, and the real price of capital. We make use of manufacturing data for 1987 to obtain estimates of these quantities to apply our experiments for the 1986 pre- and post-reform period. In the simulations, we set $A = 6$, $p^k = 1.18$, and $w = 8$.[36]

We infer a process for the demand shock process using the form of the inverse demand function in (3.2). This approach is similar to that in Caballero and Engel (1996), who estimate the marginal profitability of capital using 2-digit and 4-digit industry data, *conditional on assumptions about market structure and the production technology.* Measuring Y_t as real GDP for the manufacturing sector (in billions of 1996 dollars), and p_t as the GDP deflator for manufacturing for the period 1987-2000, the standard deviation of $\ln(\alpha_t)$ is 0.2363. Allowing for a deterministic trend, the demand shock can be represented by a stationary $AR(1)$ process as

$$\underset{}{\ln(\alpha_t)} = \underset{(0.806)}{-0.7295} + \underset{(0.110)}{0.6439}\ln(\alpha_{t-1}) + \underset{(0.012)}{0.0120}t + \epsilon_t$$

Notice that the trend term is not significant. In the absence of a trend, we have

$$\ln(\alpha_t) = \underset{(0.017)}{0.0868} + \underset{(0.060)}{0.7936}\ln(\alpha_{t-1}) + \epsilon_t,$$

with standard errors in parentheses.[37] In our simulations we assume that $\ln(\alpha_t)$ follows a stationary $AR(1)$ process with a mean of zero, autoregressive parameter $\rho = 0.75$, and unconditional standard deviation $\sigma_\alpha = 0.2$. (The implied conditional standard deviation is $\sigma_\epsilon = 0.1323$.) We also consider a more persistent process for $\ln(\alpha_t)$ by assuming that $\rho = 0.85$, and one which satisfies a MPS by

[36] Let Y_t denotes real output in manufacturing (Source: BEA Industry Accounts Data, *Gross Domestic Product by Industry*, K_t the real value (or the ratio of the current-cost valuation to the investment deflator) of net capital stock of private assets in manufacturing (Source: BEA Fixed Asset Tables, Table 3.1ES *Current-Cost Net Stock of Private Fixed Assets*), and L_t hours worked for full-time and part-time employees. The technology shock A_t is measured as the Solow residual for 1987 as $\ln(A_t) = \ln(Y_t) - \eta\ln(K_t) - (1-\eta)\ln(L_t) = 2.14$ or $A_t = 8.53$. The real price of capital is defined as the ratio of the capital stock deflator to the GDP deflator, which equals 1.14 for 1987. The value of the real wage $w = 12.65$ is calculated as the ratio of the average hourly earnings of production workers in manufacturing industries to the GDP deflator for 1987. (Source: BLS, *National Employment, Hours and Earnings.)*

[37] The above measure of $\ln(\alpha_t)$ is obtained by scaling the firm's revenue function by 1000.

assuming that $\sigma_\epsilon = 0.1871$, holding constant the mean of α_t.

3.2 Parameterizing the Processes for Tax Policy

Table 1 shows the corporate tax rate, depreciation allowances for tax purposes and the ITC for 27 classes of equipment, 23 classes of business structures, and residential structures. King and Fullerton (1984) and Jorgenson and Sullivan (1981) have studied the impact of changes in the cost of capital on investment. More recently, Jorgenson and Yun (1991) provide a comprehensive discussion of changes in U.S. tax policy and the impact of tax reform on investment. The data in Table 1 constitute the basis of the results reported in Jorgenson and Yun (1991).[38] The ITC was first introduced in 1962, and it underwent several changes in subsequent periods. Depreciation allowances can take the form of accelerating depreciation when depreciation allowed for tax purposes is greater than economic depreciation or expensing.[39] There were numerous changes to the U.S. tax code associated the capital consumption allowances.[40] As a major piece of postwar tax legislation, the Tax Reform Act of 1986 eliminated the ITC, modified the ACRS by lengthening capital cost recovery periods, and lowered the federal corporate tax rate to 34 percent. The Tax Reform Act of 1986 also eliminated the differential tax treatment of alternative assets.[41]

Figure 1 graphs the behavior of the stochastic tax wedge ζ_t for the period 1946 to 1996. An increase in ζ_t signifies an increase in the after-tax cost of investing while a decrease signifies a decline. Significant declines in the after-tax cost of investing occurred after the 1953 and 1962 tax reforms. By contrast, the Tax Reform Act of 1986 led to a significant increase in the after-tax cost of investing for both

[38] These data were kindly provided to us by Professor Dale Jorgenson in private correspondence.

[39] See Auerbach and Jorgenson (1980) for a discussion of expensing.

[40] Liberalizations occurred in 1954, in 1962, and in 1981 (when the Accelerated Cost Recovery System (ACRS) was introduced).

[41] Concurrently with changes in the U.S. tax code, there were tax reforms in Canada, Japan, and the U.K. which had similar effects. See Daly, Mercier, and Schweitzer (1988) and the references therein for a fuller discussion.

Figure 1: After-tax Cost of One Dollar of Investment

equipment and non-residential structures investment. The declines in the after-tax cost of investing can be attributed to the increase in the corporate income tax rate up to 1952 and the liberalizations that occurred in depreciation allowances for tax purposes. With respect to ζ_t and z_t, we can identify three regimes. Between 1962 and 1986 there is greater variability and changes in persistence of ζ_t and z_t. Prior to 1962 and after 1986, there is less variability. The ITC was off before 1962 but it was on between 1962 and 1986 (except for 1970 when it was off). After 1986 it was off. Thus, focussing on the period between 1946 to 1996, one could conclude that the ITC is a discrete process. However, between 1962 and 1986, variation in the ITC contributes to the variability of the tax wedge ζ_t.

In this paper, we focus on tax incentives for shorter-lived assets such as producers' durable equipment of various categories. No doubt long-lived investment projects are also affected by changes in the tax code, and react to greater riskiness in the environment or to greater policy variability.[42] However, tax incentives are typically thought to be more important for equipment assets. Thus, we set the depre-

[42]See Altug (1993) and Demers, Demers, and Altug (2003), who analyze the impact of irreversibility and uncertainty in a time-to-build model of investment.

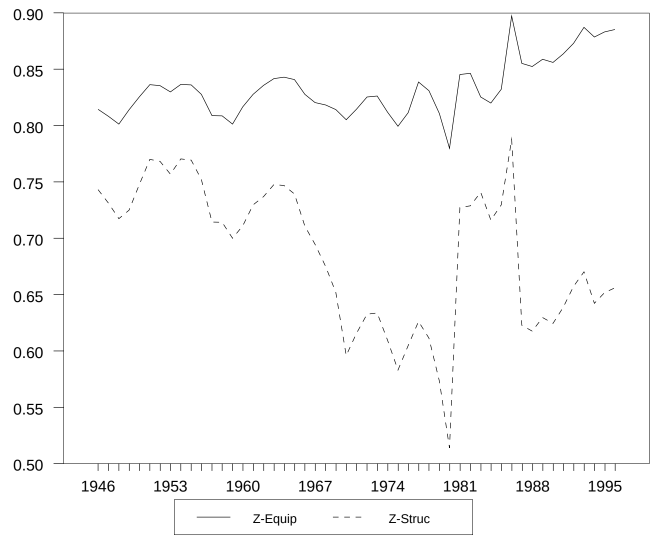

Figure 2: Depreciation Allowances for Equipment and Structures

Figure 3: ITC for Equipment and Structures

ciation rate δ equal to as the average depreciation rate for equipment assets considered by Jorgenson and Yun (1991), Table 3.19, p. 79. The investment tax credit γ_t is measured as the average rates of the ITC, and depreciation allowances z_t as the average rates of depreciation allowances for equipment assets.

In what follows, we consider alternative process for the ITC and the stochastic tax wedge over the period 1946-1996. For the period 1946-1996, the ITC may be reasonably characterized as a three-state process. For the period 1962-1986, one may also consider the ITC to be drawn from a continuous distribution. For the case of a discrete ITC, we set z equal to its sample average over the entire period, that is, $z = 0.40$. When considering a log-normal model for the stochastic tax wedge, we allow for time series variation in γ_t and z_t. Finally, we set the corporate tax rate τ equal to its average value for the sample period 1946-1996, that is, $\tau = 0.4817$.

Table 2 shows the baseline parameter settings used in the simulations.

Table 2: Parameter Values

ϵ	η	β	δ	τ	w	p^k	A	ρ	$\bar{\alpha}$	σ_ϵ
-3.607	0.346	0.947	0.13	0.4817	8	1.18	6	0.75	0	0.1323

3.3 The Optimal Investment Policy Function

We solve for the optimal investment policy function using numerical dynamic programming with value iteration. (See Bertsekas, 1976.) To account for the presence of the irreversibility constraint, we convert the choice problem into one of choosing next period's capital stock K_{t+1} to satisfy the constraint $K_{t+1} \geq (1-\delta)K_t$. Since both K_t and K_{t+1} must belong to the finite set of feasible points denoted $\mathcal{K}$, the elements of the set $\mathcal{K}$ are chosen such that $(1-\delta)K$ also belongs to this set. This is accomplished by choosing the elements of $\mathcal{K}$ as $K_{M-j+1} = (1-\delta)^{j/n}\widetilde{K}$, $j = 1, \ldots, M$, where n is a positive integer.

(See Sargent (1980) for a similar approach.) In the real business cycle literature, it is customary to approximate continuous Markov processes for the exogenous forcing variables with discrete Markov chains.[43] In our simulations we approximate the continuous $AR(1)$ process for the demand shock with a discrete Markov chain following the approach described by Tauchen (1989).

We implement the Tauchen approximation with 100 or 50 discrete values for $\ln(\alpha_t)$, depending on the distribution for the tax policy variables. The parameters of the underlying continuous process are estimated very accurately using the Tauchen approximation. With 100 grid points, the estimated conditional standard deviation and autoregressive parameter are 0.1325 and 0.7495 when the true values are 0.1323 and 0.75.[44] We choose 100 points in the capital stock grid, with $n = 6$ and $\tilde{K} = 3$. This yields the capital stock grid $\mathcal{K} = [0.2945, 2.9312]$, or, in billions of dollars, $[294.5, 2931.2]$. This choice of the capital stock grid ensures that the firm's optimal choice of investment (or next period's capital stock) remains within the same capital stock grid across different experiments. According to this parameterization, the percentage change across consecutive values in the capital stock grid is 2.35%. The convergence criterion for value iteration satisfies $|V^j - V^{j-1}| < 0.001$.[45]

4 Results

4.1 Changes in the Current ITC and Changes in Persistence

In this section, we examine discrete 3-state probability distributions for the investment tax credit γ_t. The frequency distribution for the ITC is plotted in Figure 4 for the period 1946-1996. This figure shows that the assumption of a general 3-state probability distribution for γ_t is not unwarranted. We define the transition probabilities by $p_{ij} = Pr(\gamma_{t+1} = \gamma_j | \gamma_t = \gamma_i)$ for $i, j = 1, 2, 3$, and the matrix of

[43] See Danthine, Donaldson, and Mehra (1989) for a discussion of computational issues in RBC models.

[44] For $\rho = 0.85$, we obtain $\hat{\sigma}_\epsilon = 0.1328$ and $\hat{\rho} = 0.8495$, and for $\sigma_\epsilon = 0.1871$ we obtain $\hat{\sigma}_\epsilon = 0.1874$ and $\hat{\rho} = 0.7495$.

[45] More generally,Danthine, Donaldson, and Mehra (1989) assume that $|V^j - V^{j-1}| < c\|K\|$, where $\|K\| = 0.0025$ is the norm of the capital stock partition and $c = 0.5$.

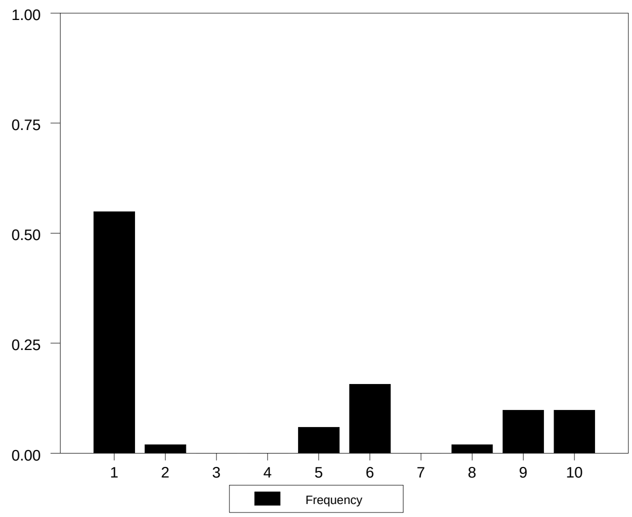

Figure 4: Frequency Distribution for ITC on Equipment

transition probabilities associated with each distribution by Γ_m^{pc}, $m = 1, 2, 3$. For the results in this section, we assume that $\gamma_i \in \{0, 0.05, 0.1\}$. For theoretical completeness, we consider IID, positively correlated (PC), and negatively correlated (NC) distributions. The distributions are ranked according to their persistence.

The solution for the model delivers decision rules that show next period's optimal capital stock as a function of the current capital stock, the current state of demand, and the current ITC. In Table 3, we summarize the response of investment to changes in the ITC in two different ways. First, we consider the **one-step ahead** response of firms, averaged across all possible capital stock states and across all possible states of demand.[46] Using this approach, columns (1) and (2) Table 3 reports the incidence of the binding irreversibility constraint, and the average level of investment for each ITC state, while columns (3) and (4) report the averages of these quantities with respect to the unconditional distribution for γ. To gauge the variability of investment under alternative distributions, column (5) of Table 3 also

[46]The unconditional probability distribution for the state of demand is used to average across the possible states of demand whereas averaging across the possible capital stock states is done by simple averaging.

reports the coefficient of variation (CV) for investment expenditures over different values of γ. Finally, as a way of distinguishing the response of investment to different ITC distributions, we calculate the standard deviations of the reported averages across the possible states of demand. These standard deviations are reported in parentheses.

First, note that if the averages in columns (1) and (2) are viewed as the average response of firms with differing capital stocks and differing states of demand, there is a large proportion of firms which do not invest.[47] Recall that the investment response to changes in the ITC includes both a *cost effect* and an *information effect*. The cost effect is positive for all seven of the distributions in Table 3. However, the information effect is negative for the PC distributions, that is, a current high (low) ITC today signals that the ITC will be high (low) tomorrow. Thus, investors do not have an incentive to shift their investment plans to take advantage (or to avoid) a currently high (low) ITC today. By contrast, the sign of the investment response for the NC distributions depends on the sign of $\partial V_K / \partial \gamma$. Table 3 shows that investment is increasing in the ITC for all cases.[48] However, investment increases by a smaller amount for the PC cases relative to the IID or NC cases. Put differently, the results in Table 3 imply that tax incentives that are perceived as being more temporary tend to induce a a spurt of investment activity.

Next, suppose we are interested in predicting the effects of a given tax regime on investment expenditures. For example, we might view the post-1987 period as one in which investors expected the ITC to remain at zero for the foreseeable future. For the ITC distributions Γ_2^{pc} and Γ_3^{pc}, the investment response conditional on $\gamma_t = 0$ is not out of line with actual investment in manufacturing for 1987. According to NIPA data, the real value of investment in private equipment and software was \$359.93 in 1987.[49] In terms of our model, average investment, conditional on investing, is \$252 billion for $\gamma_t = 0$

[47]For all the distributions considered in Table 2, more than 40% of firms do not invest on average.

[48]As a check of the numerical results, we obtained an independent measure of the shadow price of capital V_K by numerically solving the functional equation for V_K. For both the PC and NC cases, we find that V_K is decreasing in γ.

[49]According to BEA Fixed Asset Table 5.4, historical-cost investment in private equipment and software in manufac-

when the ITC is expected to last with probability 0.8 (see Γ_2^{pc}) and \$349 billion when the low ITC state is expected to last with probability 0.99 (or to be permanent). (See Γ_3^{pc}.)[50]

In our model, we can examine the response of firms to the *removal* of an ITC with high probability by considering their investment response under two separate distributions for tax policy. Conditional on being in the high state, if investors expected current tax incentives to be removed with high probability, then their current investment response would be \$448 billion, or conditional on investing, \$654 billion (see Γ_3^{nc}). By contrast, once firms knew that they had transited to a distribution in which the low state was expected to persist with high probability (as implied by Γ_3^{pc}), then their predicted investment response would be \$180 billion, or conditional on investing, \$349 billion.

Auerbach and Hasset (1991) examine investment at the level of industrial industries and assets and reconcile the predicted effects of the Tax Reform Act of 1986 with the pattern of investment in the years 1987-1989. They note that the strong performance of equipment investment reflects compositional changes in the nation's capital stock that is attributable to the growth in computers and communications equipment and instruments, and that reflects a secular shift from structures to equipment investment. Nevertheless, they note that the over-prediction of equipment investment based on non-tax factors is consistent with the 1986 tax policy changes. Cummins and Hassett (1992) find that the investment forecast error following the Tax Reform Act of 1986 is negative, suggesting that investment was on average lower than would have been predicted based on pre-reform information sets. Likewise, Cummins, Hasset, and Hubbard (1994,1996) find significant effects of tax-adjusted Q and the cost of capital in years following major tax reforms such as 1973, 1982, or 1987. These results imply that failure to take

turing is \$674.4 billion dollars in 1996. From NIPA Table 7.6, the chain-type quantity index for 1987 is 53.37, yielding the result in the text.

[50] We calculate the average level of investment, conditional on investing, from the relation $E(I_t) = Pr(I_t > 0)E(I_t | I_t > 0)$, where $Pr(I_t > 0)$ is the proportion of firms that undertake positive levels of investment. The unconditional average for investment, $E(I_t)$, is \$109 billion for Γ_2^{pc}, and \$180 billion for Γ_3^{pc}. From Table 3, we can calculate $P(I_t > 0) = 0.417$ for the first case and 0.502 for the second, yielding the result.

account changes in the distribution of tax policy facing firms would imply that a simple investment equation would tend to over-predict investment.

In the above analysis, we averaged over firms across firms with different capital stocks. However, this may hide substantial variation in the investment response. Firms that have relatively little capital will tend to increase their investment in response to a currently high ITC state.[51] To control for heterogeneity in firms' investment responses as a function of their capital stocks, we simulate the optimal response of firms for a given initial capital stock and for randomly generated sequences of demand shocks and ITC states.[52] Figure 5 plots the long-run average response of firms as a function of time for the different ITC distributions. We find that starting from any initial capital stock, firms tend to reach their long-run desired capital stocks fairly rapidly. For any ITC distribution, firms that have relatively low capital stocks tend to invest more on average, and firms that have relatively high capital stocks tend to invest less on average over the 10-year period. Consequently, in Columns (6) and (7) of Table 3 we report the 10-year average investment response and the coefficient of variation of investment expenditures for the simulated investment series as a function of the median capital stock.

Table 3 shows that firms facing less persistent ITC distribution would tend to have higher investment on average. Moving from Γ_3^{nc} to Γ^{iid} to Γ_3^{pc}, conditional on having the median capital stock, we find that investment falls from \$175 billion to \$167 billion to \$143 billion. In terms of the dynamics of the investment response displayed in Figure 5, firms that face more persistent or positively correlated ITC processes initially undertake a larger investment relative to less persistent or negatively correlated

[51] If the stationary distribution for the capital could be calculated accurately, we could account for this variation by averaging with the stationary probabilities.

[52] We consider initial capital stocks in the set $k_0 \in \{0.514, 0.918, 1.641\}$, and simulated 1000 independent sequences of 10 periods each for the demand shock and the ITC. We simulate their initial values using the relevant unconditional probability distributions, and generate the sequence of (simulated) demand shocks and ITC's using the transition probabilities for α_t and the alternative probability distributions considered for the ITC, respectively.

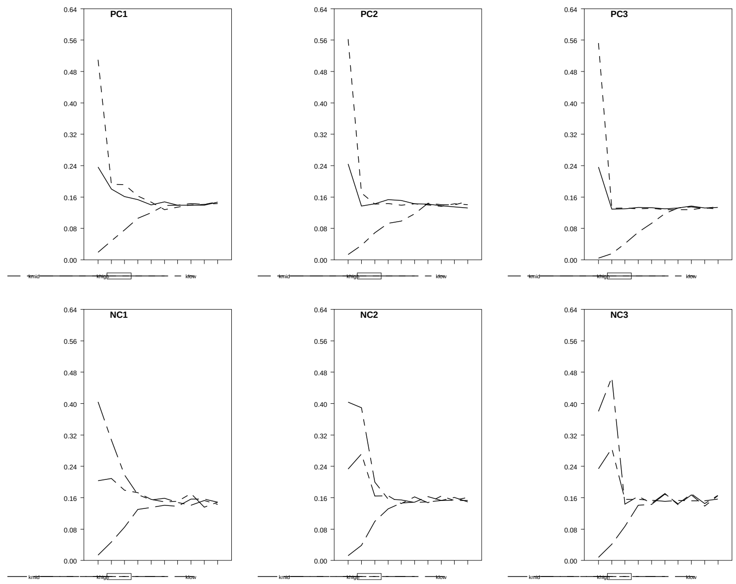

Figure 5: Long-run Response for PC and NC Models

processes.[53] This mitigates their subsequent investment response and leads to a lower long-run average investment. By contrast, firms that face less persistent distributions tend to invest little in the low or medium ITC states in the expectation that the ITC state will be higher next period. Thus, their investment response is spread out over a longer period, leading to higher investment on average.

As we discussed in Section 2, changes in the persistence of the ITC distribution also affect the *variability* of investment expenditures. From this perspective, we note that a decline in persistence unambiguously leads to an increase in variability in the short-run. The coefficients of variation (CV) reported in column (5) can be ranked according to the persistence of the underlying ITC distributions: the less persistent the distribution, the higher the CV. The increased variability in investment occurs at both the extensive and intensive margins, conditional on investing. For the IID, PC, and NC

[53]This fact is also confirmed by the one-step ahead average response of investment across all ITC states reported in column (4) of Table 3. These unconditional averages are computed using the unconditional probabilities for γ_t, and they show that average investment falls from \$283 to \$242 to \$227 billion as we move from Γ_2^{pc} to Γ^{iid} to Γ_2^{nc}.

distributions, we find that firms that face a less persistent process tend to invest relatively less in the low ITC state and relatively more in the high ITC state. As a consequence, the variability of investment expenditures increases in the short-run. The CV's increase from 1.19 to 1.41 to 1.56 as we move from Γ_2^{pc} to Γ^{iid} to Γ_2^{nc}. For the IID and PC cases, we also find less persistent distributions are associated with greater variability of the long-run average response of investment. Moving from Γ^{iid} to Γ_1^{pc} to Γ_3^{pc}, the CV for investment expenditures in the long-run falls from 1.37 to 1.28 to 0.83. Comparing the NC with the PC distributions, we also find that the investment response is more variable under the NC relative to the PC cases.[54]

Chang (1995) uses a Kydland-Prescott (1982) time-to-build real business cycle framework to investigate the impact of tax policy uncertainty on the variability of investment in the presence of productivity shocks. The model is calibrated to fit key aggregates of the US economy between 1962 and 1980. Chang's results indicate that a higher corporate income tax reduces investment and the capital stock in the steady state and has a smoothing effect on output, on hours worked, and on inventories, but it increases the variability of investment in equipment and structures. A random ITC has a destabilizing effect on investment, increasing the variability of both equipment and structures investment by 23 and 40 percent, respectively.[55]

This is similar to the predictions of our model. As a point of comparison, considering the variability of investment expenditures in our model for the nearly perfectly positively correlated case (Γ_3^{pc}) with

[54] However, the CV's for the simulated path of investment expenditures are typically lower than the CV's for the one-step ahead response. Moreover, Table 3 shows that we also cannot rank the variability of long-run investment expenditures across the individual NC distributions. The reason is that the CV's reported in column (7) are averaged across all values of γ and α (whereas those in columns (5) show the variability of investment expenditures across different values of γ *averaged* over the possible α values). Thus, over longer horizons, the endogenous response of investment to changes in γ and α tends to mitigate the impact of differences in persistence in the ITC distributions on the overall variability of investment expenditures.

[55] The ITC also has a tendency to delay investment in structures.

the IID case, we find that the variability of investment (measured by the CV of the short-run response) increases by nearly 30%. (Over longer horizons, taking into the endogenous response of investment, we find that the variability of investment expenditures in nearly 65% greater in the random relative the near certainty case.)[56] In a related study, Auerbach and Hassett (1992) estimate the impact of tax policy on investment for the period 1956-1988 using a structural approach and assuming endogenous policy. In their framework, greater instability in the sum of future costs of capital will tend to make investment more unstable. They show that the net effect of tax policy on investment has been destabilizing but that the de-stabilizing effects on structures investment is much smaller.

We conclude this section by noting that the persistence of tax policy matters for investment behavior. We find that less persistent tax policy leads to higher average investment in both the short-and long-run but that less persistent tax policy is also more destabilizing. Our model delivers both qualitative and quantitative evidence in this regard. Thus, when setting tax policy instruments, policy-makers face a trade-off in terms of higher versus more stable investment.

4.2 Sensitivity Analysis

In this section, we vary the persistence of the demand shock, its variance, the discount factor, and the elasticity of demand. Table 3 reports the response of investment under alternative experiments.[57] A discussion of the sensitivity results requires a brief discussion of the long run changes of various experiments. In the long run, the firm will invest to achieve its long-run capital stock. If demand and

[56]Chang also shows that a combined corporate income tax-ITC policy package increases the variability of aggregate investment, equipment investment, and structures investment by respectively 44, 56, and 53 percent.

[57]Whenever we vary the parameters of the $\ln(\alpha_t)$ process, we calculate the new set of grid points for α_t and the associated transition probabilities.

tax policy uncertainty are irreducible, then the firm's long run capital stock will be stochastic.[58] The stationary distribution of the capital stock which describes the long run behavior of the model can be shown using standard methods.[59] The impact of increases in risk and changes in persistence on the long run desired capital stock follow from the rest of our analysis up to this point. In particular, any changes in the distribution of the ITC or the stochastic tax wedge that reduce investment expenditures or alter its variability will have similar effects on the distribution of the long run capital stock.

Table 2 shows that an increase in demand uncertainty lowers investment across all experiments.[60] For all three distributions, investment in the short-run falls in response to a MPS shift in the distribution for the demand shock. However, the largest decline occurs for the PC case. More precisely, including the states for which investment does not occur, investment falls by 5.1% (from $294 billion to $279 billion) under Γ_2^{pc}). By contrast, investment only falls by 1.7% and 3.1% for the IID and NC cases. In the short run, firms respond to the increased riskiness in their environment by reducing investment expenditures at both extensive and intensive margins, conditional on investing. In the long-run, investment falls by 4.6%, 4.8%, and 4% for the PC, IID, and NC cases in response to a MPS in α_t. Although investment falls as firms' capital stocks converge to a lower mean in the long run, the optimal adjustment of the capital stock implies that the long-run average investment response is similar across PC, IID, and NC. Another effect of the increased variability in demand is to increase the variability of investment. For all the cases considered in the sensitivity analysis, an increase in demand variability increases the CV

[58] We can define the ergodic set for the firm's long run desired capital stock as $[K^L, K^H]$, where K^L and K^H solve

$$\bar{p}^I = \beta E V_k \left((1-\delta) K + I(K, \underline{\alpha}, \bar{p}^I) \right), \tag{4.1}$$

$$\underline{p}^I = \beta E V_k \left((1-\delta) K + I(K, \bar{\alpha}, \underline{p}^I) \right), \tag{4.2}$$

where where $\underline{p}_I$, $\underline{\alpha}$, and $\bar{\alpha}$, $\bar{p}_I$ denote the lowest and highest values of the investment price and demand shock, respectively.

[59] For an application in the context of the irreversible investment model, see Demers (1991) or Altug, Demers, and Demers (1999).

[60] We conduct a MPS exercise by doubling σ_α, holding constant the mean of α_t. Using the properties of the log-normal model, $E'(\alpha_t) = E(\alpha_t)$ provided $\mu'_\alpha = \mu - \frac{1}{2}[(\sigma'_\alpha)^2 - \sigma_\alpha^2]$.

of both the short- and long-run investment response. Thus, firms experiencing greater variability in their demand process reduce their investment levels immediately and over longer horizons. In addition, greater variability in demand induces greater variability in investment and hence, in the distribution of the long-run capital stock.

Second, we find that across different values of ρ, the impact of persistence in the ITC on the level and variability of investment becomes increasingly (statistically) significant as the persistence of the demand shock diminishes. (That is, while the qualitative results are the same across the different values of ρ, the standard errors of the estimates diminish very significantly as the value of ρ decreases.) In line with this result, we find that the CV's of the long run investment response for the PC, IID, and NC cases become much closer as ρ increases from 0.75 to 0.85. These results imply that while demand uncertainty is an over-riding concern with respect to the irreversible investment decisions of firms, the more persistent are demand shocks, the more difficult it will be to identify the role of tax policy on firms' irreversible investment decisions. The persistence of the demand process for firms can have important implications for their profitability and long-run performance. In their analysis of export dynamics for developing countries, Mody and Yilmaz (1997) show that the countries which have developed long-term buyer-seller relationships will be less likely to suffer from a decline in their exports during an economic downturn, and to have high or positive persistence in export growth. We might also expect large, profitable firms to face more persistent demand shocks. Our results suggest that an aggregate approach which fails to control for the persistence of demand shocks across heterogeneous firms will find less significant impacts of tax policy on investment.

A *lowering* of the real interest rate increases the stochastic steady for the economy but it does not alter its variability. We note that in the transition from the old to the new stochastic steady state, the average level of investment expenditures increases in both the short run and over longer horizons. Investment nearly doubles for all values of the ITC in the short-run for the IID, PC, and NC cases.

There is also a significant increase in the long-run average level of investment as firms move to a new stochastic steady state with a higher average long-run capital stock. Along the transition path, there is also a significant decline in the variability of investment expenditures for all three ITC distributions. The CV's for the one-step investment response and the long-run average investment response are all lower compared to the baseline case. By contrast, an increase in interest rates facing firms (perhaps due to a general increase in risk) is also likely to lead to more adverse effects arising from tax policy uncertainty. As firms transit to a new steady state with lower capital stocks, we would expect significant declines in the average level of investment, and an increase in its variability along the transition path. These results are derived without taking into account general equilibrium considerations. If these are present, then investment and real interest rates are endogenously determined, and the impact of an increase in risk that lowers investment might have different effects on the real interest rate. In Coleman (1997), for example, an increase in uncertainty concerning sectoral shocks, and hence the future return to capital, has an overall ambiguous effect on current aggregate investment, but it reduces the real interest rate due to the desire to smooth consumption.[61]

Changes in the elasticity of demand have two separate effects on investment. First, an increase in the elasticity of demand increases the steady-state capital stock. As ε changes from -3.607 to -5.00 (so that the elasticity of demand increases), we find that investment rises significantly in the short-run, and also over the longer horizon as firms invest more in the transition to the new stochastic steady state. By contrast, average investment declines significantly in response to a decrease in the elasticity of demand (which occurs when ε changes from -3.607 to -2.65). Increases in the elasticity of demand also reduce the variability of investment expenditures along the transition path to the new steady as

[61]Faiq (2001) also analyzes a growth model with multiple capital goods. He compares the long-run levels of investment in a model with irreversibility and in a fully flexible model economy. He also draws implications about the price of risk and the risk-free rate. See also Sargent (1980) for an early analysis of a general equilibrium model with investment irreversibility.

firms invest more for all levels of the capital stock and all realizations of demand and tax policy. By contrast, declines in the elasticity of demand increase the variability of investment expenditures along the transition path as firms firms refrain from investing as they converge to a new stochastic steady state with a lower desired capital stock.

Changes in the elasticity of demand also alter the impact of investment irreversibility. Caballero (1991) has argued that for a monopolist with a CRS production function as the elasticity of demand becomes infinite ($\|\varepsilon\| \to \infty$) the marginal revenue of capital Π_K is independent of K_t. In this case, the irreversibility constraint becomes irrelevant so that an increase in uncertainty raises investment.[62] However, uncertainty may have an impact on irreversible investment decisions if the elasticity of demand faced by an industry is less than infinite.[63] Thus, for more competitive firms or industries, we would expect tax policy to affect investment decisions directly through the cost effect; the information effect and the option value of waiting that it induces becomes irrelevant as the marginal revenue of capital becomes independent of the capital stock. From Table 3, we note that the incidence of the binding irreversibility constraint falls by 30%, 44%, and 27% for the IID, PC, and NC cases in the short-run as the demand curve facing firms becomes more elastic. Thus, as the irreversibility constraint matters less and less for firms, we find that firms adjust their capital stock faster to its desired level.

In summary, changes in the persistence of demand shocks have little affect on the average level of investment but they increase the standard errors of the investment response. The impact of an increase in price risk is to reduce investment expenditures in the short-run and over longer horizons and to increase the variability of investment expenditures and of the long-run capital stock. As the elasticity of demand falls, firms reduce their investment as the economy converges to a lower stochastic steady state. By contrast, a decline in the interest rate or an increase in the elasticity of demand raises the

[62] This is similar to Hartman's (1972) conclusion.

[63] Dixit (1989), Hopenhayn (1992), Leahy (1993) and Caballero and Pindyck (1996) analyze investment in a competitive market equilibrium with entry and exit.

stochastic steady state, and stimulates investment. Finally, a change in the elasticity of demand also changes the impact of irreversibility on investment decisions. In the limiting case of a perfectly elastic demand, the irreversibility constraint becomes irrelevant, and firms' investment response approaches their investment response in a frictionless capital stock model.

4.3 Increases in Risk

An examination of U.S. tax policy shows that there have been a number of significant changes during the postwar period. These changes have affected the persistence of tax policy, changed its variability, and altered its mean values. We use data on the stochastic tax wedge that is aggregated across 23 classes of equipment assets to gauge the impact of these changes. This approach tends to obscure the differential tax treatment of alternative assets. However, it allows us to examine some of the broad shifts in tax policy that occurred in the postwar period.

We estimated processes for the stochastic tax wedge for three different periods: 1946-1961, 1962-1986, and 1987-1996. For 1946-1961 and 1987-1996, the stochastic tax wedge appears to fluctuate randomly around a constant mean. However, for the period 1962-1986, the stochastic tax wedge is better characterized by a stationary $AR(1)$ process. For example, estimating an $AR(2)$ process, we find that the coefficient on the second lag is not significantly different from zero:

$$\ln(\zeta_t) = \underset{(0.125)}{-0.4303} + \underset{(0.214)}{0.6637}\ln(\zeta_{t-1}) - \underset{(0.208)}{0.3083}\ln(\zeta_{t-2}) + \epsilon_t,$$

with standard errors in parentheses. Second, we note that there has been a significant increase in the variability of the stochastic tax wedge for the period 1962-1986 compared to the pre-1962 or post-1986 period. For 1962-1986, the standard deviation for the stochastic tax wedge is 0.0271 while for 1951-1961 and 1987-1996, it is 0.006 and 0.007, respectively. This variability reflects changes in the ITC, changes in depreciation allowance provisions, and changes in the corporate tax rate. Third, there is a decline in the after-tax costs of investing after 1962, the average value falling from 0.56 to 0.51 and a significant increase after 1986, with the mean value rising to 0.66.

The combination of changes that occurred for the distribution of ζ_t would be expected to have opposing effects on investment. To isolate these effects, we consider three experiments. First, we ask how much would investment have changed if the stochastic tax wedge was IID, and there occurred an increase in tax risk. Second, we ask how much investment would have changed had the tax wedge been a positively correlated process, as suggested by the data, and tax risk increased due to greater variability in government spending and tax revenues in the period 1962-1996. Finally, we repeat the same exercise under the assumption that stochastic tax wedge is negatively correlated. We conduct these experiments holding the average after-tax costs of investing constant, and then discuss how investment would have changed had this quantity been lower.

For the first experiment, we assume that $\ln(\zeta_t)$ is an IID process with normally distributed errors. We consider two standard deviations for this process, the first being appropriate for the period 1951-1961 and the second being appropriate for the period 1962-1986.

$$(1)\quad \ln(\zeta_t) \sim N(\mu_\zeta, \sigma_\zeta^2);\quad \mu_\zeta = -0.6;\quad \sigma_\zeta = 0.011 \text{ and } 0.0347.$$

For the second experiment, we allow the logarithm of the stochastic tax wedge to be positively correlated (PC). Then

$$(2)\quad \ln(\zeta_{t+1}) = -0.15 + 0.75\ln(\zeta_t) + \epsilon_{t+1};\quad \epsilon_{t+1} \sim N(0, \sigma_\epsilon^2);\quad \sigma_\epsilon = 0.0347.$$

This process would appear to characterize the period 1962-1986, since the logarithm of actual tax wedge for that period has an unconditional mean of -0.66 and an unconditional standard deviation of 0.0524. Finally, we conduct a counterfactual experiment by assuming the stochastic tax wedge is negatively correlated (NC). When doing this, we keep the unconditional mean and variance of the stochastic tax wedge fixed at the values for PC.

$$(3)\quad \ln(\zeta_{t+1}) = -1.05 - 0.75\ln(\zeta_t) + \epsilon_{t+1}; \quad \epsilon_{t+1} \sim N(0, \sigma_\epsilon^2) \quad \sigma_\epsilon = 0.0347.$$

The continuous distributions for $\ln(\zeta_t)$ are approximated with a discrete distribution using 10 values for the grid and discrete probabilities computed as an application of the Tauchen method. The corresponding set of points for ζ_t when $\sigma_\zeta = 0.0525$ is given by $P = \{0.445, \ldots, 0.549, \ldots, 0.677\}$.[64]

The first part of Table 4 shows the impact of increases in risk in tax policy. First, suppose that the process for tax policy had been similar to the one that prevailed prior to 1962 so that $\ln(\zeta_t)$ evolved according to (1) with $\sigma_\zeta = 0.011$, then the average level of investment would have been \$258 billion (taking into account states in which the firm did not invest). However, an examination of US data shows that the variability of tax policy increased in the post-1962 period. Under the assumption that $\sigma_\zeta = 0.0524$, we note from Table 4(a) that the average level of investment falls to \$211 billion. This corresponds to a 18% decline investment in response to an approximately five-fold increase in the standard deviation of the tax policy shock. We observe similar declines in the one-step ahead investment response as we alter the remaining parameters of the model. Specifically, investment falls by 18% or 19% as we change the persistence of the demand shock or increase its variability. It decreases by 20% when the real interest falls from 5.6% to 2.4% or when the elasticity of demand increases from 3.607 to 5.[65]

[64] Even with 10 discrete points, the autoregressive parameter and the standard deviation are estimated quite accurately. For the IID process, when the actual standard deviation is 0.011, the estimated standard deviation is 0.0105 and when the actual standard deviation is 0.0524, its estimate is 0.0502. For the PC case, the conditional standard deviation and autoregressive parameter are estimated as 0.07357 and 0.0353 while for the NC case, they are estimated as -0.7307 and 0.0246.

[65] In Table 4, we report the average response of investment across all possible values of the stochastic tax wedge. Since we approximate a continuous log-normal process with a discrete process, there is also a "current effect" of a riskier distribution for tax policy. Thus, investment falls when firms find themselves in a tax state lower than average, and it tends to increase if firms finds themselves in a tax state higher than average. On net, average investment falls in response

Over longer horizons, the main impact of a MPS in the stochastic tax wedge is an increase in the variability of investment expenditures. While an MPS in tax policy would also be expected to reduce the long-run average response of investment, the results reported in column (4) of Table 4(a) show that the current effect tends to dominate the risk effect if firms are able to adjust their capital stocks optimally over the long run.[66] Nevertheless, for the different cases reported in Table 2, we do find a significant increase in the CV's of the long-run investment response ranging from 48% to 61%, depending on the individual case considered.

Proposition 4 in Section 3 implies that for a continuous IID process, a MPS unambiguously reduces investment for each value of ζ_t. If we could hold all the other parameters of the tax policy process fixed, the increase in variability for the tax policy process observed after 1962 would be equivalent to an MPS experiment for the distribution for ζ_{t+1}, in which we increase the variance of ζ_{t+1} holding its mean constant.[67] Thus, our results indicate that an increase in the variability of tax policy of the magnitude observed in the data would have led to a significant decline in the average level of investment.

Next, we consider a MPS shift in the distribution of ζ_{t+1}, conditional on ζ_t, for the PC and NC cases. For this purpose, we double the variance of $\ln(\zeta_{t+1})$ while keeping conditional mean of ζ_t constant. Table 3(a) shows that investment drops in the short-run, and the variability of investment expenditures increases for all parameter configurations when firms face a riskier environment. For the one-step ahead investment responses, investment falls by 5% to 6% for the PC case, and by 4% and 5% for the NC case. The decline in investment in the short-run is of a similar magnitude as we change the persistence and variability of demand shocks, increase the real interest rate, or alter the elasticity of demand.[68]

to a MPS in tax policy, which is what we report in Table 4.

[66] We would expect this current effect to disappear as we approximate the continuous process with an increasingly larger set of discrete grid points so that the probability of the end-points becomes increasingly smaller.

[67] To ensure that the distributions satisfy the conditions for a MPS in level form, that is, $E'(\zeta_t) = E(\zeta_t)$ we assume that $(\sigma'_\zeta)^2 = 2\sigma_\zeta^2$ and that $\mu'_\zeta = \mu_\zeta - (1/2)[(\sigma'_\zeta)^2 - \sigma_\zeta^2]$.

[68] Specifically, investment falls on average by 4% for NC and by around 5-6% for PC for each of these experiments.

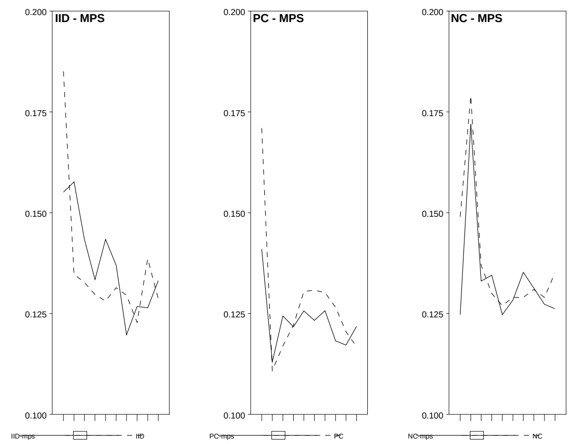

Figure 6: Long-run Response to a MPS for Lognormal Models

Table 4 shows that an increase in tax risk invariably reduces the average level of investment in the long run, and increases its variability, especially for the NC cases. For the baseline case, average investment falls from \$128 to \$123 billion for the PC cases, and from \$139 to \$134 billion for the NC case. Likewise, the CV for the NC case increases from 1.43 to 1.51. Changes of similar magnitudes are observed in the sensitivity analysis, especially if firms are facing unexpectedly low interest rates or more elastic demand curves. Finally, we note that an increase in tax risk for the PC and NC cases also leads to an inter-temporal substitution effect of investment over longer horizons. To examine this, we graph the simulated time paths for investment expenditures for the baseline and MPS experiments in Figure 6 as a function of the median capital stock. For the IID, PC and NC cases, we observe that investment initially falls in response to a MPS, and that it becomes more variable. For the PC case, the decline in investment at dates t and $t+1$ is followed by a small increase at date $t+2$. Thus, we find that while the overall effect of an increase in tax risk is to reduce investment, changes in the conditional mean of the after-tax costs investing induced by the initial MPS may lead to some investment expenditures being shifted over time.

Hassett and Metcalf (1999) have argued that an increase in tax risk when the ITC follows a geometric Brownian motion (which corresponds to a random walk in our framework) increases the median time to investing. However, when the ITC follows a two-state Poisson process, they show that a MPS leads to a decline in both the median time to investing and *ex post* average hurdle price ratio, that is, a MPS in the ITC leads to an increase in investment. Their analysis is based on a model of project choice given an initial capital stock. They attribute their findings to the non-stationarity of the process driving the ITC. By contrast, we have shown for stationary processes that are IID, PC, and NC, an increase in risk leads to a immediate *decline* in investment. All of our results are qualitatively in line with this prediction. In the long run, an increase in tax risk leads to a lower mean and to greater variability in the distribution of the long-run capital stock. In the next section, we demonstrate that Hassett and Metcalf's conclusions can be attributed to the impact of a "current effect" of a more spread out distribution for tax policy.

4.4 Increases in Risk: Further Results

4.4.1 A Two-State Poisson Model

We now assume that the ITC evolves according to a Possion process. We first assume that the tax credit can take on two possible values (or "states"), $\gamma^H > \gamma^L$. Conditional on being in each tax state, the number of changes in the ITC expected in a given time interval is equal to λ^i, and the probability of observing at least one change in the ITC in the next period is $1 - e^{-\lambda^i}$ for $i = L, H$. With this notation, the probability of staying in a high ITC state is given by $e^{-\lambda^H}$, and the probability of moving to a low ITC state conditional on being in a high ITC state at time t, is $1 - e^{-\lambda^H}$. Thus the transition probability matrix is given by

$$\begin{bmatrix} e^{-\lambda^L} & 1 - e^{-\lambda^L} \\ 1 - e^{-\lambda^H} & e^{-\lambda^H} \end{bmatrix}.$$

Alternatively, we can say that the expected duration of a given state i is $1/\lambda^i$. Note that when $\lambda^H = \lambda^L$, the (unconditional) probability of being in any one state is 0.5. When $\lambda^H \neq \lambda^L$, say, $\lambda^L = 0.35, \lambda^H = 0.2$, the long-run unconditional probabilities of each state are no longer equal, and the state with a longer expected duration also has a higher unconditional probability of occurrence, namely, 0.3804 and 0.6196, respectively. [69]

In the simulations, we first assume that $\lambda^L = \lambda^H = 0.35$, which implies that the expected duration of either state is approximately three years. In this case, the probability of remaining in the currently observed state is 0.7047 and the probability of transiting to a different state is 0.2953. We consider seven distributions for the tax rate, which are listed in Table 5. Distribution (i) constitutes a "certainty" case. A larger spread between γ^L and γ^H implies an increase in risk regarding the tax state. Thus, conditional on the low state being observed at time t distributions (iv) and (v) constitute a MPS over distribution (i), that is, conditional on the low state, the expected mean is 0.5 but the variability is higher. Likewise, conditional on the high state being observed at time t, distributions (vi) and (vii) constitute a MPS over distribution (i). Distributions (ii) and (iii) are unconditional MPS's over distribution (i).

When comparing the investment response under the alternative distributions, it is important to note that in addition to the risk effect there also exists a "current level" of the tax state. Suppose that $z = 0.40$ on average. Then, shifting from a distribution with $\gamma^L = \gamma^H = 0.1$ to one in which (conditional on observing γ^L at time t) $\gamma^L = 0.08$ and $\gamma^H = 0.15$ next period implies that an increase in the tax wedge $\zeta = (1 - z - \gamma)$ from 0.50 to 0.52, or a 4% increase in the after-tax costs of investing. Hence, a firm that observes the low state at time t is induced to wait given that there is also a positive probability of transiting to a state next period with $\gamma = 0.15$ (and a 13.5% fall in the after-tax costs of investing as the tax wedge falls from 0.52 to 0.45.

Now consider the case in which a firm faces shift in the distribution with $\gamma^L = \gamma^H = 0.1$ to one in

[69] Note that since the variance is also given by λ, a lower value of λ implies both a longer expected duration and a smaller variance. Hence the lower the value of λ, the less uncertain the firm is as to the expected duration of that state.

which (conditional on observing γ^H at time t) $\gamma^H = 0.05$ and $\gamma^H = 0.121$. In this case, the tax wedge ζ falls from 0.50 to 0.479, implying a 4.2% fall in current costs. Furthermore, the spread in the new distribution implies that the firm could be facing a 14.8% *increase* in the tax wedge (from 0.479 to 0.55) in the future if there occurs a transition to a low state. There is both a greater incentive (lower current costs) to invest and a stronger disincentive to wait (higher prospective future costs).

Looking at Table 5, we note that the "current effect" dominates the risk effect across all distributions. It is apparent that conditional on the high state being observed, investment increases rapidly as the value of γ^H increases, since the firm wants to take advantage of the very favorable cost conditions. The larger the percentage increases in γ^H, the larger proportionate increase in investment. For example, as γ^H increases from 0.10 to 0.121 or 0.15 to 0.29 (corresponding to increases of 21.1% and 33.3%), investment increases by close to 50% (from \$395 to \$584 billion in the first case and \$737 and \$1.1 billion in the second, including the states for which investment is zero). By contrast, smaller percentage increases in γ^H are accompanied by smaller percentage increases in investment. As γ^H increases from 0.121 to 0.142 (a 17% increase), investment increases by 20% (from \$584 billion to \$699 billion). At the extensive margin, we notice the same effects at work: the incidence of the irreversibility constraint steadily diminishes as γ^H increases. When calculating the response of investment conditional on being in the low state, we notice that the level of investment conditional on γ^L is also very sensitive to the value that γ^H takes, but is inversely related to it. Thus, as γ^L falls from 0.08 to 0.05, we notice that investment falls by 21%, 24%, and 54% depending on whether γ^H equals 0.121, 0.15, or 0.219. The lower the value of γ^H, the less the value to the firm of waiting to invest. Hence it invests more in the low state than it otherwise would.

Note that in this comparison across distributions, if the current value of γ_t is kept constant at 0.05, we can look at the impact of a change in the distribution (the risk effect) without the "current" effect. Notice also that conditional on γ^L, the distribution $(0.05, 0.219)$ FSD dominates $(0.05, 0.15)$ which in turn FSD dominates $(0.05, 0.121)$. Hence the result on investment accords with our theoretical findings, according to which investment falls for an FSD improvement in the distribution. Specifically, we notice that investment conditional on $\gamma^L = 0.05$ falls from \$128 to \$95 to \$44 billion as γ^H increases from 0.121 to 0.15 to 0.219. The results concerning the extensive margin confirm these effects: conditional on being in the low state, the incidence of the binding constraint increases from 56 to 60 to 71 as we move from distribution *(vi)* to *(iii)* to *(v)*. We therefore observe that investment becomes more variable as policy becomes more variable. (i.e., as the spread in the ITC increases.) This conclusion is in line with Bizer and Judd (1989).

In column (5), we report the average level of investment, which takes into account that on average each ITC state is observed half of the time. According to this measure, we notice that investment tends to first decrease and then increase as the spread in the ITC increases. This is due to the "current effect" in the high state: investment in the high state increases substantially as the value of γ^H increases. At the same time, investment in the low state is similarly sensitive to the value of γ^H and decreases as the latter increases (we may term this the "risk effect'). Average investment displays both effects at work. We nevertheless observe that average investment does fall steadily with a MPS in the case of distributions *(vi)* and *(vii)*.[70]

We also vary the expected duration by assuming that $\lambda^i = 0.2$. In this case, the expected duration of the tax state is 5 years. We consider an alternative set of distributions for γ such that conditional on the low state, the distributions in (*iv*) and (*v*) constitute a MPS over distribution (i), and conditional on the high state, distributions (*vi*) and (*vii*) constitute a MPS over (i). As can be seen from Table 5, analyzing the case for $\lambda^L = \lambda^H = 0.2$ yields the same qualitative results on investment (and on average investment) as we increase the spread in the ITC. The qualitative results on investment (and on average investment) are also the same for the case for $\lambda^L = \lambda^H = 0.693$ where the expected duration of both

[70] The key condition for average investment to fall with a spread seems to be that the tax incentive in the high state not be too large.

states is only 1.5 years. We also vary the expected duration of the low and high ITC states separately, and consider the cases $(\lambda^L = 0.2, \lambda^H = 0.35)$ and $(\lambda^L = 0.35, \lambda^H = 0.2)$ so that one of the states is more persistent than the other. As we compare the investment response across these cases conditional on the low state, we note that the higher the duration, the higher the level of investment in the low state. Conditional on $\lambda^L = 0.05$, we note as the expected duration of the low state increases from 1.5 to 3 to 5 years, the average level of investment, conditional on investing, increases from $152 to $238 to $244 billion.[71] With regards to investment conditional on the high state, we note that investment tends to vary with the duration of the low state relative to the duration of the high state. Thus, the level of investment in the high state is higher when the duration of the low state exceeds that of the high state, and vice versa. For example, for $\lambda_i = 0.35$ and 0.02, the average levels of investment in the high state, conditional on investing, are equal. (See the distributions (*ii*) for parts (I) and (II) of Table 5.) By contrast, when $\lambda^L \neq \lambda^H$, the investment is higher in the high state, the higher the duration of the low state (as in the case with $\lambda^l = 0, 2$ and $\lambda^H = 0.35$ compared to $\lambda^L = 0.35$ and $\lambda^H = 0.2$.) Firms invest more in the high state when they know that should there be a switch to a low state, they will face high cost conditions for a long time.

Finally, comparing the case with $\lambda^L = \lambda^H = 0.2$ with the case where $\lambda^L = \lambda^H = 0.35$ and $\lambda^L = \lambda^H = 0.693$ for each of the ITC spreads, we note from Table 5 that the more persistent the distribution, the less variable investment is. As the expected duration of each state increases from 1.5 to 3 to 5 years, the CV's for the ITC spreads (0.05,0.15) and (0.00,0.20) decline from (1.31,1.28) to (1.19,1.24) to (1.11,1.18), respectively. (See Table 5, column (4).) Thus, greater stability in policy reduces the variability of investment. These conclusions are in accord with our findings from Section 2

[71] In Table 5, columns (1) and (2) show that conditional on the low state being equal to 0.05, the probability of investing for the distributions in parts (I), (II), and (III) are 0.4, 0.39, and 0.29 while the average levels of investment (including those states for which investment is zero) are 0.095, 0.095, and 0.044. Based on these numbers, we find $E(I_t|I_t > 0$ from the relation $E(I_t) = Pr(I_t > 0)E(I_t|I_t > 0)$, as stated in the text.

on persistence.

These conclusions are based on the one-step ahead or short-run responses for investment. We can also examine the long-run response for investment reported in columns (6) and (7) of Table 5. First, we note that investment is higher under distributions for which the favorable tax incentives are perceived to be more temporary. As the expected duration of each state decreases from 5 to 3 to 1.5 years, the average level of investment increases from \$207 to \$220 to \$225 billion for the (0.05,0.15) spread, and from \$273 to \$284 to \$297 billion for the (0.00,0.20) spread. Second, we note that the "current effect" of the high ITC state under more spread distributions tends to dominate the "risk effect" of the low ITC state in the long run. However, this effect is more pronounced, the less temporary is the perceived favorable tax incentive. When the expected duration of each ITC state is 3 years, conditional on $\gamma^L = 0.05$, the long-run average investment response increases from \$185 to \$220 to \$325 billion as γ^H increases from 0.121 to 0.15 to 0.219, and conditional on $\gamma^L = 0.00$, the average investment response increases from \$207 to \$284 billion as γ^H increases from 0.142 to 0.20. When the expected duration of each ITC state increases to 5 years, we note that average investment tends to decrease for the spreads in (*vi*) and (*vii*) of Part (II) relative to the certainty case, with the long-run average level of investment falling to \$171 billion and \$176 billion from \$179 billion. Furthermore, for the case with $\lambda_i = 0,2$, the average level of investment for the different spreads is, in general, lower compared to the case with $\lambda_i = 0.35$.

These findings are confirmed in Figure 7, which displays the long-run responses of investment for each of the spreads in parts (I), (II), and (III) of Table 5 for the median capital stock. First, we find that the initial response of investment is higher for more persistent distributions. Second, we note that when the expected duration of each ITC state is 1.5 or 3 years, the investment response is higher for almost all models relative to the certainty case at all horizons. (The one exception to this is model (*vi*) for $\lambda_i = 0.35$.) By contrast, when the expected duration of each ITC increases to 5 years, we find that the investment response is, in general, lower and less variable compared to the less persistent distributions. (For some models, i.e. models (*vi*) and (*vii*), investment is lower relative to the certainty case for some horizons.) Since firms know that they will be in the good state with high probability, there is less incentive for them to increase their investment to take advantage of a currently high tax incentive. Likewise, if they are in the bad state, they expect the current tax state to continue with high probability. Consequently, they do not face an incentive to delay investment in the expectation that conditions will be better tomorrow. Thus, we see that the impact of the "current effect" of a more spread out distribution for the ITC depends not only on its magnitude but also on its persistence.

4.4.2 A Three-State Poisson Model

We end this section by considering a three-state case with γ_L, γ_M and γ_H, where γ_M is a middle level of ITC. The transition matrix may be written as follows:

$$\Gamma_2^P = \begin{bmatrix} e^{-\lambda_L} & p^{LM}\left(1-e^{-\lambda_L}\right) & \left(1-p^{LM}\right)\left(1-e^{-\lambda_L}\right) \\ p^{ML}\left(1-e^{-\lambda_M}\right) & e^{-\lambda_M} & \left(1-p^{ML}\right)\left(1-e^{-\lambda_M}\right) \\ \left(1-p^{HM}\right)\left(1-e^{-\lambda_H}\right) & p^{HM}\left(1-e^{-\lambda_H}\right) & e^{-\lambda_H} \end{bmatrix}.$$

In this expression, conditional on being in the low state (and conditional on a change occurring), there is now a positive probability p^{LM} of moving to a middle state, and $(1-p^{LM})$ probability of moving to a high state. In our simulations, we take $p = 0.5$ for all cases.

In the two-state Poisson model, firms face limited uncertainty. While they do not know the duration of each state with certainty (they only know the average duration) conditional on the current state, firms know the direction of the change as well as the magnitude of this change with certainty. In the three-state Poisson model, conditional on being the low or the high states, the firm knows the direction of the change but not the magnitude. By contrast, in the middle state, it knows how much the ITC will change, conditional on moving to a low or high state, but it does not the direction of the change.

The real difference between the two-state and three-state cases occurs when the firm is in the middle state, and we consider a distribution of the ITC. If γ^M remains constant, a change in the distribution is not accompanied by the current level effect. Hence, having isolated the risk effect, the impact of a MPS in the distribution of the ITC should, according to our theoretical discussion, lead to a decline in investment conditional on γ^M being observed at date t.

The simulation results in Table 7 unambiguously support this conclusion. For example, for the case where $\lambda^L = \lambda^M = \lambda^H = 0.35$, (case (i)) the level of investment conditional on observing γ^M at time t falls by 13% with the first spread and by an additional 20% with the second spread. For the case where $\lambda^L = \lambda^M = \lambda^H = 0.2$ and each state lasts for five years, investment in the middle state is less sensitive to increases in the ITC spread: investment declines by 6% with the first spread and by 11% with the second spread. In this case, the longer duration of the middle state increases investment in that state, and this effect mitigates the impact of a spread in the ITC. More interestingly, we note that conditional on γ^M being observed at time t, the case where $\lambda^L = \lambda^M = \lambda^H = 0.35$ is itself a MPS over the case where $\lambda^L = \lambda^M = \lambda^H = 0.2$ for *a given* spread in γ. We thus expect (and observe) investment in the latter case to be lower than in the former for all spreads. All of the results in Table 7 support this conclusion. Conditional on the middle state being observed, we note that the average level of investment falls from \$372 to \$345 billion for the first spread, and from \$331 to \$275 billion for the second spread.

We also examine the impact of a MPS on the time path of investment. For this purpose, we simulate the long-run response of investment conditional on the middle ITC being realized at all dates. From column (6) of Table 7, we note that investment declines with a MPS in the ITC for both spreads, regardless of the expected duration of the ITC state. However, the decline in investment is greater for ITC processes that are perceived to be less temporary. When the expected duration of each ITC spread is 3 years, investment declines by 9.5% with the first spread, and by 14.8% with the second spread.

Increasing the expected duration of each state to 5 years, we find that investment declines by 3.4% with the first spread, and by 8.1% with second spread. Similar to the 2-state Poisson model, we also find that the more persistent the ITC process, the less difference there is between the long-run investment paths across different ITC spreads. Figure 8 plots the time paths of the investment response by varying the expected duration of the ITC state from 5 to 3 years to approximately 9 months (corresponding to $\lambda_i = 0.2, 0.35$, and 1.0986. We observe that investment is more variable and falls by a greater amount in response to a MPS for ITC distributions that are less persistent. This finding is similar to our results for the 2-state Poisson model,

Comparing these results with those obtained by Hassett and Metcalf (1999), we note first that, in line with their observations for the 2-state Poisson model, investment tends to bundle up in the high state. However, in contrast to their conclusion that a MPS *increases* investment, we observe a decline in average investment for some spreads in the distribution of the ITC. Comparing our results of the two-state Poisson model with the three-state Poisson model, in contrast to Hassett and Metcalf, we conclude that, in the presence of irreversible investment, a mean-preserving increase in the riskiness of the ITC lowers investment in both the short run and over longer horizons when the ITC follows a stationary Poisson process. However, the impact of a more spread out distribution for the ITC depends on the persistence of the underlying process.

5 Conclusions

The notion that investment behavior is highly sensitive to firms' perceptions of changes in their economic environment is a notion that dates back to Keynes (1936).[72] In this paper, we have considered changes

[72]Keynes notes the following: "The outstanding fact is the extreme precariousness of the basis on which our estimates of prospective yield have to be made. Our knowledge of the factors which govern the yield on an investment some years hence is usually very slight and often negligible. If we speak frankly, we have to admit that our basis of knowledge for

in persistence, changes in volatility, and increases in risk arising from changes in tax policy on firms' irreversible investment decisions. We have shown that tax incentives that are perceived as being more temporary lead to higher investment on average but also that higher policy variability (or lower policy persistence) increases the variability of investment expenditures. Furthermore, we have shown that an increase in tax risk around an unchanged mean leads to a decline in investment and to an increase in its variability in the short run and over longer horizons. Thus, our results assign an important role for firms' perceptions about changes in tax policy.

The more recent investment literature has emphasized the deleterious effects of increases in risk and uncertainty on firms' investment decisions. Our results indicate that the findings of Hassett and Metcalf (1999) - which suggest that increases in risk may stimulate investment - can be attributed to the impact of a "current" effect of a change in tax policy as firms seek to take advantage of higher than average tax incentives. The investment stimulus is greater, the more temporary such incentives are perceived to be. However, consistent with other studies, less persistent policy is also likely to be more destabilizing. Thus, we find that policy-makers face a fundamental trade-off when setting tax policy in terms of higher versus more stable investment.

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estimating the yield ten years hence of a railway, a copper mine, a textile factory, the goodwill of a patent medicine, an Atlantic liner, a building in the City of London, amounts to little and sometimes nothing; or even five years hence. In fact, those who seriously attempt to make an estimate are so often so much the minority that their behavior does not govern the market." (pp. 149-150.)

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Appendix

Lemma 1 *Assume $\tilde{\gamma}_t$ is positively serially correlated. V_K is increasing in γ.*

Proof. The proof is by backward induction and uses the fact that V can be viewed as the limit of a sequence of value functions corresponding to finite horizon problems. Consider a sequence of value functions $\{V^N(K,\gamma)\}$ defined by $V^0 \equiv 0$, and $V^N = T\left[V^{N-1}(K,\gamma)\right]$ where a superscript on V, K, I or on γ denotes the number of time periods left until the end of the horizon. Observe that $V^1 \equiv (1-\tau)\Pi(K^1)$ where $K^1 = (1-\delta)K^2 + I^2$. Therefore, $I^1 \equiv 0$. Also,

$$V^2 = \max_{I_2}\{(1-\tau)\Pi(K^2) - p^{I2}I^2 + \beta E[V^1(K^1,\gamma^1) \mid \gamma^2]\}$$

and

$$\begin{aligned} V_K^2 &= (1-\tau)\Pi_K(K^2) + (1-\delta)\beta E V_K^1\left[\left((1-\delta)K^2 + I^2, \gamma^1\right) \mid \gamma^2\right] \\ &= (1-\tau)\Pi_K(K^2) + (1-\delta)\beta(1-\tau)\Pi_K\left((1-\delta)K^2 + I^2(\gamma^2)\right). \end{aligned}$$

We need to consider two cases: (a) $I^2(\gamma^2) = 0$, and (b) $I^2(\gamma^2) > 0$. If $I^2(\gamma^2) = 0$, V_K^2 is trivially decreasing in γ^2. If $I^2(\gamma^2) > 0$, then

$$V_{K\gamma^2}^2 = (1-\delta)\beta(1-\tau)\Pi_{KK}\frac{dI^2}{d\gamma^2} \quad \text{where } \frac{dI^2}{d\gamma^2} = \frac{-p^{k2}}{\beta(1-\tau)\Pi_{KK}}.$$

Thus, $V_{K\gamma^2}^2 = -(1-\delta)p^k < 0$.

Now, given that $K^2 = (1-\delta)K^3 + I^3$,

$$V_K^3 = (1-\tau)\Pi_K(K^3) + (1-\delta)\beta E V_K^2\left[\left((1-\delta)K^3 + I^3, \gamma^2\right) \mid \gamma^3\right]$$

and

$$V_{K\gamma^3}^3 = (1-\delta)\beta E[V_{KK}^2 \mid \gamma^3]\frac{dI^3}{d\gamma^3} + (1-\delta)\beta\int V_K^2 dG_{\gamma^3}(\gamma^2 \mid \gamma^3)$$

where

$$\frac{dI^3}{d\gamma^3} = \begin{cases} \frac{-p^{k3} - \beta\int V_K^2 dG_{\gamma^3}(\gamma^2|\gamma^3)}{\beta E[V_{KK}^2 \mid \gamma^3]} & \text{if } I^3 > 0 \\ 0 & \text{if } I^3 = 0. \end{cases}$$

We need to consider two cases: $I^3 = 0$ and $I^3 > 0$.

(a) Now, suppose that $I^3 > 0$. $V_{K\gamma^3}^3$ reduces to $V_{K\gamma^3}^3 = -(1-\delta)p^k < 0$.

(b) Suppose next that $I^3 = 0$. $V^3_{K\gamma^3}$ reduces to:

$$V^3_{K\gamma^3} = -(1-\delta)\beta \int V^2_{K\gamma}\left((1-\delta)K^3, \gamma^2\right) G_{\gamma^3}(\gamma^2 \mid \gamma^3) d\gamma_3 \leq 0$$

where we have used integration by parts and where the sign follows since $V^2_{K\gamma} \leq 0$ and $G_{\gamma^3} \leq 0$. That is, since V^2_K is decreasing in γ^2, $V^3_{K\gamma^3} \leqslant 0$ if $G(\gamma^2 \mid \gamma^{3\prime})$ dominates $G(\gamma^2 \mid \gamma^3)$ by FSD for all $\gamma^{3'} \geq \gamma^3$ .Thus, V^3_K is decreasing in γ^3. Now, assume that $V^4_K, \ldots, V^{N-1}_K$ are decreasing in γ. Proceeding analogously, we can show that V^N_K is decreasing in γ. Therefore, the limit function V_K is decreasing in γ. ■

Proof of Proposition 1

Proof. Let I^*_t denote the optimal investment given G and $\hat{I}_t$ the optimal investment given $\widehat{G}$.

$$\begin{aligned} &\int V_K\left((1-\delta)K_t + I^*_t, \widehat{\gamma}_{t+1})\right) d\widehat{G}(\gamma_{t+1}|\gamma_t) \\ &\leq \int V_K\left((1-\delta)K_t + I^*_t, \gamma_{t+1}\right) dG(\gamma_{t+1}|\gamma_t) \leq p^I_t \end{aligned}$$

where the first inequality follows by an application of Lemma 1 and the second by the Kuhn-Tucker condition for the optimal choice of I^*_t. By the concavity of V in $K_{t+1}, \hat{I}_t \leq I^*_t$. ■

Lemma 2 *Assume $\widetilde{\zeta}_t$ is independently distributed. V_K is concave in ζ.*

Proof. The proof is by backward induction. Proceeding analogously to the proof of Lemma FSD we obtain: $V^2_K = (1-\tau)\Pi(K^2) + (1-\delta)\min\left\{\zeta^2 p^k, \beta(1-\tau)\Pi_K\left((1-\delta)K^2\right)\right\}$ is piecewise linear and, therefore, concave in ζ^2. Assume V^i_K, $i = 3, \ldots, N-1$ concave in ζ^i. $V^N_K = (1-\tau)\Pi_K(K^N) + (1-\delta)\min\left\{\zeta^N p^k, \beta E V^{N-1}_K\left((1-\delta)K^N, \zeta^{N-1}\right)\right\}$ is piecewise linear and therefore concave in γ^N. Thus, the limit function is concave in ζ. ■

Lemma 3 *Let $\widetilde{\zeta}_t$ be positively serially correlated. If $\widetilde{\zeta}_{t+2}$ is a stochastically increasing and concave function of $\tilde{\zeta}_{t+1}$, V_K is concave in ζ.*

Lemma 4 *Let $\widetilde{\zeta}_t$ be negatively serially correlated. Assume $V_{K\zeta} > 0.V_K$ is concave (convex) in ζ if $F_{\zeta_t\zeta_t}(\zeta_{t+1}|\zeta_t) > (<) 0$.*

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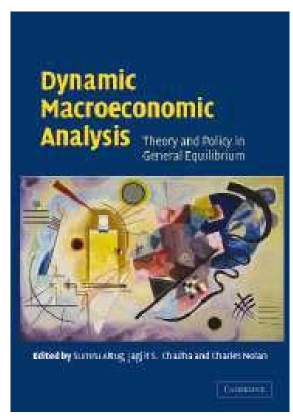

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