CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS
WORKING PAPER SERIES

CDMA04/01

Interest Rate Bounds and Fiscal Policy

Jagjit S. Chadha[*]
University of St Andrews

Charles Nolan[†‡]
University of St Andrews

OCTOBER 2004
Forthcoming in *Economics Letters*

ABSTRACT

When the monetary authority controls the short-term interest rate we find that under a regime of permanent (and even persistent but temporary) deficits that a *strict* upper bound on the feasible interest rate sequence is present. More generally, the satisfaction of the fiscal authority's present value budget constraint in the presence of a deficit sequence, means that monetary and fiscal decisions cannot be independent. This is an important caveat to the results in McCallum (1984)

JEL Classification: E63; E52; E31.
Keywords: Intertemporal macro; monetary policy and fiscal policy interactions.

[*] CDMA, Tel: +44 (0)1334 462440. Email: Jagjit.Chadha@st-andrews.ac.uk
[†] CDMA, Tel: +44 (0) 1334 462425. Email: Charles.Nolan@st-andrews.ac.uk
[‡] Corresponding author. We are grateful for discussions with Sean Holly, and for comments from an anonymous referee. All errors are due to the authors.

CASTLECLIFFE, SCHOOL OF ECONOMICS & FINANCE, UNIVERSITY OF ST ANDREWS, KY16 9AL
TEL: +44 (0)1334 462445 FAX: +44 (0)1334 462444 EMAIL: AWT2@ST-AND.AC.UK
WWW.ST-ANDREWS.AC.UK/ECONOMICS/CDMA/CDMA.SHTML

1. Introduction

The seminal contribution of Sargent and Wallace (1981) convinced most macroeconomists that the effective conduct of monetary policy will be hampered by imprudent fiscal policy. In characterizing monetary and fiscal policy as *infinite sequences* of decisions, constrained ultimately by a transversality condition, the potential tensions were brought into sharp relief. With sufficiently high interest rates a permanent sequence of deficits might result in the government being unable to place its debt in the market. In order to meet the present-value budget constraint (PVBC), seigniorage revenue may need to rise. The monetary authority loses *de facto* control of the price-level, either now or in the future and there results a repudiation of the escalating debt via inflation.

However, McCallum (1984) demonstrated that permanent deficits need not have inflationary consequences when the deficit is defined to *include* interest payments. In other words, fiscal policy can be separated from monetary consequences providing the fiscal authority responds to the correct state variable in formulating its sequence of instrument choices. However, both these analyses and the literature that followed,[1] assumed that the monetary authority's instrument is the monetary base and that prices are perfectly flexible. Whilst both assumptions have been useful, their removal offers new insights.

Woodford (1997) shows that modelling the monetary authority as controlling the short-term nominal interest rate - which in the presence of sticky prices implies control of the short-term real rate - is consistent with a determinate (locally) unique rational expectations equilibrium. In this note, therefore, we adopt the perspective that monetary authorities can influence the short-term real interest rate.[2]

[1] See, for example, Persson and Tabellini (1994) for an introduction.
[2] Taylor (1999) also argues that monetary policy can be usefully characterised as a feedback

Section 2 outlines the PVBC analysis for two alternate fiscal policy regimes and their implications for feasible interest rate sequences. Section 3 considers some extensions of our argument. Section 4 concludes and offers some remarks on our main result, that a fiscal policy that runs permanent deficits *always* ends up constraining the feasible sequence of interest rates.

2. The Analysis

Consider a deterministic economy, in which wealth takes one of two forms: money, which earns no interest, and one-period nominal, riskless bonds, which do earn interest.[3] The one-period public sector flow budget constraint is given by:

$$\frac{B_t}{(1+i_t)} = B_{t-1} + P_t(G_t - T_t) - (M_t - M_{t-1}), \tag{2.1}$$

where B_t is the nominal quantity of debt redeemed at the start of $t+1$, i_t is the nominal interest rate between period t and $t+1$, P_t is the aggregate price level, $(G_t - T_t)$ is the real primary deficit in period t, and $(M_t - M_{t-1})$ is seigniorage raised in period t. A central assumption is that the monetary-fiscal sequences avoid Ponzi schemes,[4] such that,

$$\lim_{T\to\infty} B_{t+T}\left(\prod_{j=0}^{T}(1+i_{t+j})\right)^{-1} = 0. \tag{2.2}$$

rule for the short term nominal interest rate. As well as outlining the 'Taylor principle' he examines the general scope of the finding through time and across several countries.

[3] The following analysis does not model the behavior of the private sector, as the main points can be made without doing so.

[4] The no-Ponzi game restriction is consistent with optimal private sector behavior. O'Connell and Zeldes (1988) find that no rational individual will hold the liabilities of a government that attempts to run a Ponzi game. That is because the welfare of any individual holding such government debt for any period will be strictly lower than under an alternate feasible consumption program. Had we modelled the representative agent side of the model we could have generated a slightly different form of this restriction where the consumption Euler equation would have been used to substitute the marginal utility of wealth for the interest rate term. See, for example, McCallum (1984).

Whilst we do not model explicitly the behaviour of private agents, it is known that their optimal consumption-saving programme will also be characterized by conditions analogous to (2.2).[5] We take (2.2) to be sufficient to ensure that the PVBC is satisfied, and that given the level of outstanding liabilities at the start of any time period the ensuing intertemporal sequence of net surpluses plus seigniorage is sufficient to meet those liabilities.

Let T_t denote the period t tax yield. We will analyze fiscal rules (regimes) of the form

$$T_t = \lambda_t G_t - \frac{(M_t - M_{t-1})}{P_t} + \gamma \frac{B_{t-1}}{P_t}. \tag{2.3}$$

Fiscal policy is characterised by the sequence $\{(\lambda_{t+s}, \gamma_{t+s})\}_{s=0}^{T}$, that is by choices on size of the deficit, $(1-\lambda)G$, and the rate at which debt is retired, γ. We assume that γ is constant and $0 < \gamma < 1$, so that the government retires a portion of outstanding debt. For simplicity, we further assume that seigniorage is rebated lump sum to agents. The particular rules that we analyze will then be indexed simply by restrictions on the sequence $\{\lambda_{t+s}\}_{s=0}^{T}$. Before we proceed we need to clarify the implications of (2.2) for our class of fiscal policy rules. First, given the restrictions on γ, the fiscal authority, looking forward from any time t, will always do enough to repay the outstanding debt in existence at the start of time t. Consequently, fiscal solvency hinges on the present value of future surpluses and deficits (in $t+1, t+2, \ldots$). So, we need to clarify the implications of (2.2) for this sequence. It turns out that as time $T \to \infty$ the fundamental requirement for

[5] This brief comment hardly does justice to the issues arising from the incorporation and intepretation of (2.2) in an economic model. For instance, as the recent debate over the fiscal theory of the price-level demonstrates, whether one views (2.2) merely as an equilibrium condition of an economic model, as opposed to a requirement regardless of the price or interest sequence, has profound implications for issues such as price determinacy under interest rate pegs, exchange rate determinacy and the necessity of base money to the central bank's control of inflation. See Janssen, Nolan and Thomas (2002) for a discussion of some of these issues. For the purposes of this paper we do not adopt a fiscalist stance. In other words, we view (2.2) as holding for all feasible price and interest rate sequences, and not just equilibrium sequences.

fiscal solvency on any monetary-fiscal programme is that:[6]

$$\sum_{s=0}^{T}\left[\left\{\prod_{j=0}^{s-1}(1+i_{t+j})\right\}^{-1}(1-\gamma)^{T-s}(1-\lambda_{t+s})P_{t+s}G_{t+s}\right]\to 0. \tag{2.4}$$

In other words, the discounted sum of net government liabilities must tend to zero.

2.1. Fiscal Regime (1): A Balanced Budget

Suppose that the government is prohibited from running fiscal deficits at any time, and in each period retires a portion of outstanding debt (γB_{t-1}), and meets all current period expenditure, denoted P_tG_t. Fiscal policy is thus the sequence $\{(\lambda,\gamma)\}_{s=0}^{T}$ with $\lambda = 1$ and $0 < \gamma < 1, \quad \forall s$. Monetary policy is the sequence of one period decisions denoted by $\{i_{t+s}\}_{s=0}^{T}$. In period t the tax yield is:

$$T_t = G_t - \frac{(M_t - M_{t-1})}{P_t} + \gamma\frac{B_{t-1}}{P_t}. \tag{2.5}$$

Using (2.5) in (2.1) reveals that

$$\frac{B_t}{(1+i_t)} = (1-\gamma)B_{t-1}. \tag{2.6}$$

Iterating on this expression demonstrates that such a fiscal rule satisfies the no Ponzi game condition *independently* of monetary policy (the sequence of interest rates), so

$$\lim_{T\to\infty} B_{t+T}\left(\prod_{j=0}^{T}(1+i_{t+j})\right)^{-1} = \lim_{T\to\infty}(1-\gamma)^{T+1}B_{t-1} = 0. \tag{2.7}$$

[6] To show this, substitute (2.3) into (2.1) and iterate forward, successively substituting for period debt.

To confirm this, set $\lambda = 1 \; \forall s$ in equation (2.4).[7] The balanced budget fiscal rule is clearly a special case as it assumes that the fiscal authority will never run a primary deficit from period t onwards. We go now to another extreme which has been the focus of much attention.

2.2. Fiscal Regime (2): Permanent Deficits

The existence of a permanent deficit implies $0 < \lambda < 1$. We continue to assume that there is a lower bound on taxes determined by the the debt repayment parameter γ. The fiscal rule is now:

$$T_t = \lambda G_t - \frac{(M_t - M_{t-1})}{P_t} + \gamma \frac{B_{t-1}}{P_t}. \tag{2.8}$$

Substituting (2.8) into (2.1) yields

$$\frac{B_t}{(1+i_t)} = (1-\gamma)B_{t-1} + (1-\lambda)P_t G_t. \tag{2.9}$$

The public sector is now running a deficit in every period. This policy is sustainable *if and only if* the following expression goes to zero in the limit:

$$\begin{aligned} B_{t+T}\left(\prod_{j=0}^{T}(1+i_{t+j})\right)^{-1} &= (1-\gamma)^{T+1}B_{t-1} + \\ &(1-\lambda)\sum_{s=0}^{T}\left[\left\{\prod_{j=0}^{s-1}(1+i_{t+j})\right\}^{-1}(1-\gamma)^{T-s}P_{t+s}G_{t+s}\right]. \end{aligned} \tag{2.10}$$

Our conclusions under in Section 2.1 demonstrate that we require the second term on the right-hand side of this expression to converge to zero. As (2.10) is a

[7] Note that γ need not be a 'big' number. In fact, as is clear from the analysis in Canzoneri, Cumby and Diba (2001), it could in fact be identically zero for a large, but finite, number of time periods whilst still ensuring solvency.

special case of (2.4) it will be convenient to make some simplifying assumptions. A useful special case is when the sequence of nominal government expenditures is fixed:

$$(1-\lambda)P_{t+s}G_{t+s} = (1-\lambda)\overline{PG} \qquad \forall s. \tag{2.11}$$

We can now see why a sequence of permanent fiscal deficits leaves monetary policy hamstrung. Substituting (2.11) into (2.10) we note that the second expression on the right hand side of (2.10) becomes

$$(1-\lambda)\overline{PG}\sum_{s=0}^{T}\left[\left\{\prod_{j=0}^{s-1}(1+i_{t+j})\right\}^{-1}(1-\gamma)^{T-s}\right]. \tag{2.12}$$

For a given rate of debt retirement (γ), the implication for monetary policy is clear: it must drive the expression in square braces to zero. Alternatively, for any given interest rate sequence, γ must be sufficiently accommodative. An illuminating example of the implication for monetary policy is where interest rates are set at the level given in equation (2.13)

$$i_{t+s} = \left\{(1-\gamma)^{-2} - 1\right\} \qquad \forall s \geq 0. \tag{2.13}$$

If monetary policy follows this path then expression (2.12) can be written as

$$(1-\gamma)^{T}\sum_{s=t}^{T}\left[(1-\gamma)^{s-t}(1-\lambda)\overline{PG}\right], \tag{2.14}$$

where the expression in square braces converges to

$$\frac{1-\lambda}{\gamma}\overline{PG}. \tag{2.15}$$

Consequently, as $T \to \infty$ expression (2.14) tends to zero. Although it is clear that (2.13) is not unique,[8] in the spirit of McCallum (1984) we find that (2.13) is a

[8] There are a number of ways to see this non-uniqueness. Perhaps the most obvious is to note that if $i_{t+s} = \left\{(1-\gamma)^{-2} - 1\right\}$ $\forall s \geq 0$ is a feasible equilibrium sequence then so too must be $i_{t+s} = \left\{[2(1-\gamma)]^{-2} - 1\right\}$ $\forall s \geq 0$.

sufficient condition for permanent primary deficits to be a feasible fiscal policy. More importantly, we find that permanent fiscal deficits, such as Regime (2), place an upper bound on the sequence of interest rates and so do not imply separability in the feasible set of monetary and fiscal choices. The intuition is simply that the bound increasingly constrains the interest rate sequence as the rate of debt retirement falls.

3. Some Extensions

The discussion above holding nominal government expenditure fixed does not indicate that we need to assume an extreme form of price rigidity. What is critical, as we now make explicit, is that, for a given value of γ, the monetary authority needs sufficient control over the real interest rate. We continue to assume that government expenditure is constant. Rewriting solvency condition (2.4) in real terms yields

$$(1-\lambda)\overline{G}\sum_{s=0}^{T}\left[\left\{\prod_{j=0}^{s-1}\frac{(1+\pi_{t+1+j})}{(1+i_{t+j})}\right\}(1-\gamma)^{T-s}\right]. \tag{3.1}$$

As in the previous example, the expression in square braces must tend to zero to ensure fiscal solvency. Expression (3.1) can usefully be re-written as

$$(1-\lambda)(1-\gamma)^{T}\overline{G}\sum_{s=0}^{T}\left[\left\{\prod_{j=0}^{s-1}\frac{(1+\pi_{t+1+j})}{(1+i_{t+j})}\right\}\left(\frac{1}{1-\gamma}\right)^{s}\right]. \tag{3.2}$$

A sufficient condition for this expression to reach zero in the limit is simply that the term in square braces is convergent, as opposed to having a zero limiting value.[9] This will be the case as long as the following requirement is met infinitely

[9] See Rudin (1976), Theorem 3.3(c), page 49.

often:[10]

$$i_s - \pi_{s+1} < \gamma \qquad \forall \;\; s \geq T. \tag{3.3}$$

This expression has a very obvious interpretation; it requires that the fiscal authority eventually repay a sufficient portion of the debt each period.[11] An alternative interpretation, is that the debt retirement schedule places an upper bound on the feasible real interest rate sequence.

Finally, we note that even when deficits are merely persistent, the above arguments go through. That is consider a deficit $D_t = \rho D_{t-1}$, where $\rho > 1$ and $D_t \equiv (1-\lambda)P_t G_t$. A restriction analogous to (3.3) occurs:

$$(1-\rho) + i_s - \pi_{s+1} < \gamma \qquad \forall \;\; s \geq T. \tag{3.4}$$

Expression (3.4) shows the constraint on monetary policy is clearly eased, but is still not entirely absent either.

4. Conclusions

Our results complement those of Sargent and Wallace (1981) and especially McCallum (1984). The latter showed that incorporating the interest burden into the arithmetic of fiscal solvency is important for the independence of monetary policy. However, if instead we view monetary policy as control of the short term real interest rate, the constraint imposed on monetary policy by a permanent deficit takes the form of an upper bound on the interest rate sequence. And even under less extreme fiscal policies, such as a temporary but persistent deficit, monetary conduct may be hampered. This latter result may also shed some light

[10] We are essentially drawing on d'Alembert's ratio test. This says that for a convergent series: $\underset{n\to\infty}{lim\ sup}\ |a_{n+1}/a_n| < 1$. In the text, however, we are effectively unwinding the unstable roots forward to ensure convergence.

[11] Actually this expression is an approximation, since we ignore the cross term: $[(p_{t+1}/p_t) - 1] \times \gamma$.

on why some monetary policy makers, such as at the European Central Bank, may support strict controls on the fiscal policies of member states.

References

[1] Canzoneri, Matthew B, Robert E. Cumby and Behzad T. Diba (2001), "Is The Price Level Determined by the Needs of Fiscal Solvency?", *American Economic Review*, Vol. 91, No. 5, pp.1221 - 1238.

[2] Janssen, Norbert, Charles Nolan and Ryland Thomas (2002), "Money, Debt and Prices in the UK: 1705-1996", *Economica*, August, vol. 69, no. 275, pp. 461-479(19).

[3] McCallum, Bennett T. (1984),."Are Bond-financed Deficits Inflationary? A Ricardian Analysis", *The Journal of Political Economy*, Vol. 92, No. 1. (Feb., 1984), pp. 123-135.

[4] O'Connell, Stephen A. and Stephen P. Zeldes (1988), "Rational Ponzi Games", *International Economic Review*, Vol. 29. pp. 431-450.

[5] Persson, Torsten and Guido Tabellini (1994), eds. *Monetary and Fiscal Policy, vols 1* and *2,*Cambridge MA: MIT Press.

[6] Rudin, Walter (1976), *Principles of Mathematical Analysis*, Third Edition, McGraw-Hill International Edition.

[7] Sargent, Thomas J. and Neil Wallace (1981), "Some Unpleasant Monetarist Arithmetic", *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall, 1-17.

[8] Taylor, John B., (1999), *Monetary Policy Rules*, Chicago: University of Chicago Press, NBER.

[9] Woodford, M (1997), "Control of the Public Debt: A requirement for Price Stability?", in *The Debt Burden and Monetary Policy*, eds: G. A. Calvo, and M. A. King, ., London: MacMillan.

About the CDMA

The **Centre for Dynamic Macroeconomic Analysis** was established by a direct grant from the University of St Andrews in 2003. The Centre funds PhD students and a programme of research centred on macroeconomic theory and policy. Specifically, the Centre is interested in the broad area of dynamic macroeconomics but has a particular interest in a number of specific areas such as: characterising the key stylised facts of the business cycle; constructing theoretical models that can match these actual business cycles; using these models to understand the normative and positive aspects of the macroeconomic policymakers' stabilisation problem; the problem of financial constraints and their impact on short and long run economic outcomes. The Centre is also interested in developing numerical tools for analysing quantitative general equilibrium macroeconomic models (such as developing efficient algorithms for handling large sparse matrices). Its affiliated members are Faculty members at St Andrews and elsewhere with interests in the broad area of dynamic macroeconomics. Its international Advisory Board comprises a group of leading macroeconomists and, ex officio, the University's Principal.

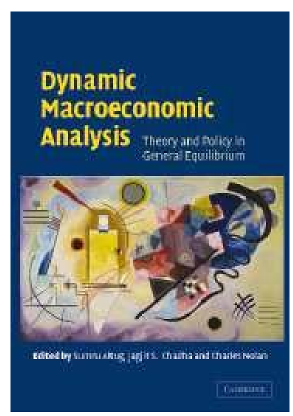
RECENT WORKING PAPERS FROM THE CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS

Number	**Title**	**Author(s)**
CDMA04/01	Interest Rate Bounds and Fiscal Policy	Jagjit S. Chadha (St Andrews) and Charles Nolan (St Andrews)

The new book, available now!

Dynamic Macroeconomic Analysis: Theory and Policy in General Equilibrium
Edited by Sumru Altug, Jagjit S. Chadha, Charles Nolan

Cambridge University Press
ISBN:0521534038



For information or copies of working papers in this series, or to subscribe to email notification, contact:

Alex Trew
Castlecliffe, School of Economics and Finance
University of St Andrews, Fife KY16 9AL
Scotland, UK

Email: awt2@st-and.ac.uk; Phone: +44 (0)1334 462445; Fax: +44 (0)1334 462444.