

5 Questions: Philosophy of Logic

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1. *Why were you initially drawn to the philosophy of logic?*

I was an undergraduate in both mathematics and philosophy at the University of Keele in the 1960s. We studied a little logic in both subjects, but there was barely any attempt to connect logic with philosophy—but one course in philosophy of mathematics with Alan Treherne sparked my interest in logic enough for me to go to the Mathematics department at the University of Bristol for an intensive Masters course in mathematical logic. My doctorate at Oxford was in philosophy of language, with connections to linguistics inspired by the Chomsky boom then raging even in Oxford. Identifying a subject of ‘philosophy of logic’ came slowly, needing to be carved out between philosophy of language and philosophy of mathematics. In particular, logic was very much directed towards its use in mathematics or the analysis of language. It was largely a dogmatic discipline, not welcoming any philosophical interference. The first books with the title *Philosophy of Logic* are those by Quine [12] in 1970 and Putnam [11] in 1972, but what really first led me into the subject were two publishing events of 1975, of Kripke’s ‘Outline of a theory of truth’ and of the first volume of Anderson and Belnap’s *Entailment: the logic of relevance and necessity* [2]. Both were a revelation, in overturning what seemed to be a dogma.

The first dogma was that the logical paradoxes were really a dead subject. The story went that the set-theoretic paradoxes had caused a revolution in the foundations of mathematics around 1900, but were solved by the creation of axiomatic set theory by Zermelo and others; and the semantic paradoxes, distinguished from the other paradoxes by Ramsey, had been solved by Tarski by distinguishing object language from metalanguage. Kripke challenged this orthodoxy, revealing the hidden costs of implausibility of the Tarskian proposal. Kripke’s ideas opened up a whole research programme in developing his own account—which was indeed programmatic, as his use of ‘outline’ in his title emphasized—though I was never particularly attracted by his positive proposal, which famously retained the “ghost” of Tarski’s hierarchy. But it was revelatory in showing that what was the right response to the paradoxes was still a live issue.

Discovering Anderson and Belnap’s work was even more instructive, not least in its immediate effect. I read their book while completing my doctoral thesis, and I was immediately weaned from philosophy of language to philosophy of logic and have never been back. What perhaps was most exciting was the thought that we could turn the tables on logic, so that instead of logic’s dictating what philosophy must think (e.g., that conditionals must be truth-functional), philosophy could examine logic’s presuppositions and find them wanting. At the same time, it seemed to me that Anderson and Belnap’s programme had problems of its own at its base, and that things I’d learned in logic at Bristol, particularly in proof theory, could be applied to the questions Anderson and Belnap had raised about

the correct account of logical consequence. Indeed, the focus on logical consequence, rather than logical truth, recorded a shift in view which, even though one can trace it back to Gentzen and Tarski, was only then becoming the norm.

2. *What are your main contributions to the philosophy of logic?*

At the same time as encountering these iconoclastic ideas in the philosophy of logic, I was also developing an interest in what is known as medieval logic. This is a bit of a misnomer, since it covers philosophy of logic and philosophy of language in the middle ages as much as, if not more than, logic itself. Since then, I have worked and thought and published in contemporary philosophy of logic and medieval logic about equally.

My main contributions in contemporary ideas have been in relevant logic, in the theory of paradox, and in what is now termed ‘inferentialism’. The idea of relevant logic (sometimes known as ‘relevance logic’) is that in a valid consequence, the premises must be relevant to the conclusion. But it does not try to formalize that connection by a “relevance” filter. That would be a mistake, though one that is often made, especially by critics but even by some practitioners. Suppose that we said that an inference is valid if it is truth-preserving and the premises satisfy the “relevance” requirement of being relevant to the conclusion. Now take a truth-preserving inference which does not satisfy the relevance test, and suppose the premises are true. Then since the argument is truth-preserving, it follows that the conclusion must be true. But since the premises fail the relevance test, we are apparently not supposed to infer the conclusion—it’s not a “relevantly valid” inference. That way lies madness. The main spur to my work in relevant logic was to find a better and more appropriate account of logical consequence which would reveal that really truth-preserving inferences were already relevant. This research culminated in my book *Relevant Logic* [14] (out of print but available online at http://www.st-andrews.ac.uk/~slr/Relevant_Logic.pdf), and my work since then has been guided by adherence to the belief that relevant logic gives the correct account of logical consequence.

My work on paradox is more recent, and has been inspired by my research in medieval logic. My early work in medieval logic should probably be classed as philosophy of language, trying to make sense of the specifically medieval notion of supposition. A term “supposits” for an object or range of objects if it stands for those objects in a sentence, so the role of supposition as a technical term is similar to that of reference in modern semantics, though broader, for it also covers the logical behaviour of quantified and general terms. I’d been aware, at least since the publication of George Hughes’ translation of and commentary [8] on John Buridan’s chapter on “insolubles” in his *Sophismata* of the medievals’ treatment of the semantic paradoxes, but felt that Buridan’s analysis, though interesting as an alternative to Tarski’s and Kripke’s, was no more convincing than theirs. It slowly became apparent, however, that, although most scholarly work on the medieval account of paradoxes had focussed on Buridan, the most

seminal idea in medieval times was due to Thomas Bradwardine, writing some ten or twenty years earlier. It was with the publication of Bradwardine's *Insolubilia* in the 1320s that the whole approach to the insolubles (which include the semantic paradoxes, but also epistemic paradoxes and other puzzles) underwent a sea change. Before Bradwardine's attack on it, the dominant position was restrictionism, that the paradoxes resulted from an illicit form of self-reference. Bradwardine's devastating criticism overturned this doctrine, and his positive idea, that the paradoxes are implicitly self-contradictory in somehow asserting their own truth as well as their falsehood, is found in some form or other in almost all succeeding treatments in the middle ages. I believe that Bradwardine's idea is still viable, and have tried to develop it and apply it to a range of paradoxes in several recent publications, as well as editing and translating Bradwardine's *Insolubilia* into English [4]. One might call it the "multiple-meanings" solution, since Bradwardine claims, indeed, proves, that the paradoxes signify or mean many things besides what they overtly say.

A third area of philosophy of logic to which I've contributed is logical inferentialism, the idea that logical constants should be given a proof-theoretic rather than a model-theoretic semantics. I was introduced to the background to this idea in my studies with John Mayberry for my Masters thesis but it lay dormant until I was faced with the challenge of giving a relevant semantics for the connectives in my analysis of the foundations of relevant logic. A recurrent complaint in the 1960s about relevance logic was that it had no semantics. The same complaint had been directed at modal logic in the 1950s, famously answered by Kripke in his possible-worlds semantics. Meyer and Routley, among others, rose to this challenge by formulating the ternary semantics for relevant logic. Just as Kripke modelled the unary modal operators with a binary relation of relative possibility, so the binary entailment operator was modelled by a ternary relation, but, ternary relations being so much less intuitive than binary ones, this analysis was not the public relations success that Kripke's work had managed for modal logic. Indeed, though model theory can yield significant technical results, it can only translate one meaning into another, or even distort it. My response was to propose giving the meaning of the relevant connectives proof-conditionally (in the final chapter of *Relevant Logic* [14]). The idea goes back ultimately to Gentzen, subsequently developed by Prawitz and Dummett. At its heart lies the proposal that meaning be given in the introduction-rules for a logical constant, which encapsulate its assertion-conditions, requiring that the elimination-rules should be justified by that meaning and so should lie in an appropriate relation of harmony with them. I've tried to spell this out in the notion of "general-elimination harmony" (a term coined by Nissim Francez and my colleague Roy Dyckhoff)—see [13].

3. What is the proper role of philosophy of logic in relation to other disciplines, and to other branches of philosophy?

Where there are two distinct activities and two different names, it's useful to use one name for the one, the other for the other, but unfortunately, there's been no

consistent use of the terms 'philosophy of logic' and 'philosophical logic'. I prefer to use the former for the philosophical discipline of examining logical notions, the latter for the development of logical concepts and methods for philosophical purposes (contrasted with mathematical logic): thus philosophical logic will embrace modal logic, substructural logic, many-valued logic, mereology, and so on, whereas mathematical logic has included set theory, recursion theory and suchlike. Of course, these are not hard and fast divisions and can overlap. Philosophy of logic, in contrast, is philosophy, like philosophy of science and philosophy of art, with its distinct subject matter. Its role is to examine the concepts of logic, including consequence, quantifiers, identity, proof, model and so on. It can clearly overlap with philosophy of language, when looking at concepts like name, predicate and proposition, or with metaphysics, in examining identity, part and whole. Sharp distinctions are not important, so much as polarities of focus.

4. What have been the most significant advances in the philosophy of logic?

The most significant advance in philosophy of logic in recent decades has been the recognition that logical consequence is the predominant notion, and one worthy of close examination both as to its foundations and to its identification. Although the study of consequence goes back at least to Gentzen, and arguably much earlier (Bolzano, the medievals, indeed, to Aristotle), and alternative accounts of consequence include the proposed revisions of the intuitionists (with the identification of a specifically intuitionist notion of consequence by Heyting in 1930, though rejected by the founder of intuitionism, Brouwer, himself) and of the relevantists (stemming from Ackermann's famous paper 'Begründung einer strengen Implikation' [1] of 1956), the thought that logic is identical with logical truth lingered well into the second half of the twentieth century. The steps from logical truth to the focus on the consequence relation between two formulae, then to that between a set of formulae and a formula, and finally to its (possibly) most general form in the relation between sets of formulae (usually dubbed "multiple-conclusion" consequence) was slow and hard won. Even now the legitimacy of multiple-conclusion consequence is highly tendentious.

The attitude to logical revision has also changed dramatically since Quine posed his "deviant's dilemma" [12] in 1970: "when he tries to deny the doctrine he merely changes the subject." Possibilities have opened up in at least two dimensions. The rearguard action tried to identify logic with so-called first-order classical logic, resisting extensions to higher-order logic and to modal and other intensional logics. These are now much more readily accepted as part of the proper study of logic, with important applications both in philosophy and in other disciplines, such as computer science. They are what Susan Haack [6] called "extended logics"; in addition there are what she called deviant logics, such as intuitionist and relevant, and also now dialethic, non-contractive and other logics which seek not to extend first-order logic, but to overturn it. Clearly, the extended logics can live happily side by side; recent proposals for a logical pluralism try to suggest that deviant logics can also co-exist, identifying different consequence

relations as appropriate for different purposes, but all equally good and equally right. Whether this proposal is coherent seems to me to be a vital matter for future research.

One of the main spurs to logical revision was the implicational paradoxes, but most recent grounds for logical revision (and logical pluralism) have been the logical paradoxes. As already mentioned, research into the semantic paradoxes was re-awakened by Kripke's classic paper [10] of 1975. The paradoxes seem to force a choice between revising our theory of truth or our logic. Tarski chose to constrain the account of truth by imposing a hierarchy of truth-predicates. Kripke at first glance appears to do neither: he retains classical logic (albeit, allowing truth to be a partial predicate) and he rejects the hierarchy, seeking a universal notion of truth. In reality, both logic and truth are compromised. The "ghost" of the hierarchy means that several semantic concepts cannot be expressed (in the object language), e.g., 'paradoxical', and even the truth predicate seems artificially constrained. Moreover, as subsequently interpreted, the classical logic of Tarski has been replaced by a three-valued logic, or a super-valuational logic whose consequence relation is non-standard.

These limitations have led to new avenues of research in recent years, and opened the possibility of further logical revision in light of the paradoxes. Even if there is less consensus about the right approach to the paradoxes, there is now a lively debate. Much of this has focussed on recognition of the importance of Curry's paradox, though its full force, and its distinctness from the Liar paradox, is not always appreciated. Curry's aim in [5] was to investigate how the problems with Russell's paradox (of the set of all sets which are not members of themselves) might be replicated in a logic without negation. Take an arbitrary sentence A , and consider the set of all sets such that if they are self-membered then A . A semantic version of the paradox was identified subsequently: by diagonalization, there is a sentence C equivalent (or even identical) to 'If C is true then A '. An argument turning on the truth-equivalence, *modus ponens*, contraction (or absorption) and conditional proof establishes A , whatever A was. If A is \perp , the absurd sentence, then C is effectively the Liar (since not- B is equivalent to 'if B then \perp '); more interestingly, the argument derives triviality (arbitrary A) directly without a detour through a contradiction and the spread law (if A and not- A then B), so rejecting the spread law and possibly accepting true contradictions does not suffice to avoid triviality. Either some other logical principles must go (e.g., contraction) or (as I myself favour) the truth-equivalence must be qualified.

In medieval (philosophy of) logic, the most significant advance in recent years has been a better understanding of the medieval logical genre of obligations. There are many extant treatises on obligations, from around 1200 right through to the late middle ages, but scholars were very puzzled by them for many years. They are theoretical treatises giving different versions of rules for a curious form of disputation, though no records of any actual such disputations exist. In an obligational disputation, the Opponent presents a *casus* (a hypothetical situation)

and a proposition, usually false in that situation; if the Respondent agrees to take part, the Opponent fires a series of further propositions at the Respondent, which the Respondent may only grant, deny or doubt, according to a strict set of rules, and depending on the initial proposition and the *casus*. What, however, was the point of such disputations, and why were the rules constructed as they were? There is still disagreement on the first question, but the second has yielded to detailed research, revealing essentially just two (or perhaps three) main types of theory, the standard theory (*responsio antiqua*—the old response) and a rival theory (*responsio nova*—the new response) formulated by reason of dissatisfaction with certain aspects of the standard theory. The old response had a dynamic nature, which led in some cases to changes of response in the course of a disputation. This seems to have unsettled some theorists, and the new response speaks to this issue, but at the cost of robbing the theory of some of its more exciting aspects. Sense has emerged out of darkness, but there is much more research needed.

5. What are the most important open problems in philosophy of logic, and what are the prospects for progress?

The times seem right for a thousand flowers to bloom, for the eclecticism of logical pluralism, if sense can be made of the idea of multiple equally good consequence relations. The main challenge is to avoid pluralism about consequence collapsing into pluralism, or even relativism, about truth. (Of course, if truth is relative, then so too is truth-preservation, and then consequence is also relative.) Beall and Restall [3] proposed to take the model-theoretic formulae for truth-preservation, that a valid consequence preserves truth in a model, and replace ‘model’ by ‘case’ or ‘situation’ as a way of capturing the necessity and formality of consequence. But does this relativize truth to ‘truth in a case’; and if we now restrict cases to, e.g., consistent cases, or complete cases, or generalize it to inconsistent and incomplete cases, is there any justification for denying that consequence is truth-preservation in all cases, rather than just those which support classical logic, or intuitionistic logic or any but the weakest logic validated by the largest class of models?

Inferentialism has its own pluralists, often advocating a wide class of substructural logics as equally good. But Quine’s challenge is ever present: if meaning is given by the logical rules, do not different rules (different logics) necessarily entail different meanings? One response is to separate the operational rules (as meaning-determining for the connectives) from the structural rules (as defining the underlying logic). There is much work to be done here, first, to produce a viable inferentialist account of logical consequence (monistic or pluralist); secondly, to show its superiority to the model-theoretic account (as well as showing the inadequacy of the latter).

Kripke’s arguments in ‘Naming and necessity’ [9] produced a revolution across philosophy in showing that the concepts of analyticity, necessity and apriority were not obviously, indeed, perhaps not in fact, co-extensive. (His observation

was not unprecedented—see, e.g., Sloman’s paper [15] in *Analysis* 1967—but it was his that were effective). One upshot has been to recognise sentences such as ‘I am here now’ as logically true, and context-sensitive expressions as requiring logical treatment. But a viable context-sensitive account of logical consequence is still wanting. The creation of two-dimensional semantics, separating the context of assessment from the context of utterance, was crucial, but the proper articulation has yet to be accepted into the mainstream.

There has been much technical work on truth theories inspired by Kripke’s suggestions for dealing with paradox. The verdict is still open, however, on whether logic should be revised in light of the paradoxes, indeed, whether logic should be revised for other than strictly logical reasons, or whether the revisions should be in the account of truth, leaving logic intact. Are there really true contradictions, can or should they be saved from triviality, or do the paradoxes turn on insensitivity to contextual factors or blindness to the multiple meanings of expressions? There are many many questions to pursue. The only doubt is whether it will be possible to recognise the correct conclusions among the plethora of proposals.

References

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