AMERICAN PHILOSOPHICAL QUARTERLY
Volume 38, Number 2, April 2001

FREE ASSUMPTIONS AND THE LIAR PARADOX

Patrick Greenough

I. OVERVIEW

According to what Parsons (1984) has dubbed the "Standard Solution" of the liar paradox, a sentence that says of itself that it is false is a sentence that lacks a truth-value. More sophisticated versions of the Standard Solution take such sentences to be neither *definitely* true nor *definitely* false (McGee 1989, 1991; Soames 1999). The advertised goal of all such proposals is to identify a principled reason to refuse to assert that the liar sentence is (definitely) true/false. In this paper, it is argued that while the *form* of the Standard Solution is correct, the reasons why a speaker should refuse to assert that the liar sentence is (definitely) true/false have been systematically misidentified hitherto. An alternative solution (one that retains the shape but the not the substance of the Standard Solution) is developed based on the insight that it is improper to even *suppose* the liar sentence to have a truth-status (true or not) on the grounds that supposing a liar sentence to be true/not-true essentially defeats the *telos* of supposition in a readily identifiable way. On that basis, one can block the paradox by restricting the *Rule of Assumptions* in Gentzen-style presentations of the sentential sequent-calculus. The lesson of the liar paradox turns out to be that not all assumptions are for free.

II. THE FORM OF THE STANDARD SOLUTION

A sentence that says of itself that it is false is a sentence that lacks a truth-value. Such is the key thesis of what Parsons (1984) has dubbed the "Standard Solution" of the liar paradox.[1] For all its endurance the Standard Solution has proved hard to stabilize. The familiar stumbling block has been the strengthened liar sentence—the sentence that says of itself that it is not true.[2] One natural response to the strengthened liar paradox is to strengthen the Standard Solution in some appropriate fashion. The most sophisticated attempt in this general direction has been given by McGee (1989, 1991). The key idea is to draw a distinction between truth and *definite* truth. A sentence that says of itself that it is not true is a sentence that is neither *definitely* true nor *definitely* false. For McGee, sentences of this sort are "unsettled" in truth-value—the rules that determine their correct usage give "bizarre and conflicting answers" (1991, p. 8). But any strengthened solution of this general type generates its own form of the strengthened liar sentence—the sentence that says of itself that it is not definitely true.[3] The great merit of McGee's proposal is that steps are taken to address this form of the strengthened liar without recourse to an essentially richer metalanguage. Whether this proposal succeeds (and there are serious, but perhaps not insuperable, doubts on that score) is not the immediate concern in this paper.[4] The real interest of the Standard Solution (in either its simple or strengthened guise) is whether the shape of the strategy invoked in order to combat the paradoxes provides the basis for a successful solution.

The strategic form of the Standard Solution (simple or strengthened) is more or less based on the following rationale: the liar sentence has some characteristically problematic feature (call it the *L*-property). In virtue of this feature, this sentence ought to receive a particular evaluative property (call it the *E*-property) that in turn requires us both to refuse to perform the speech act of *S*-ing that this sentence is true, and to refuse to perform the speech act of *S*-ing that this sentence is false. Parsons's version of the Standard Solution, for instance, runs as follows: liar sentences are in some way "defective" (*L*-property), such that they lack a truth-value (*E*-property), such that

> having discovered that a sentence or proposition does not have a truth-value, we want to reject it, *not* to assert a related sentence (its negation) which we also wish to reject. (Parsons 1984, p. 144)

The same strategic template is also employed in sophisticated versions of the Standard Solution. For McGee, liar sentences are governed by conflicting rules of application (*L*-property), such that they are neither definitely true nor definitely false (*E*-property), such that of their truth-status one should say '"I do not know," without intending to intimate that there is any fact of the matter there to be known (p. 218). Soames (1999) has likewise recently employed the same general strategy. For Soames, the rules governing the use of liar sentences are only "partially defined" (*L*-property), such that liar sentences are neither *determinately* true nor *determinately* false (*E*-property), such that

> there will be no possible grounds for accepting either the claim that the truth predicate applies to them or the claim that it does not. Because of this, both the claim that such sentences are true and the claim that they are not true must be rejected, thereby blocking the usual paradoxical results. (Soames 1999, p. 164)

Just as with Parsons, Soames takes the speech act of *rejecting* a sentence to be distinct from the speech act of asserting the negation of this sentence. This latter speech act is usually known as the speech act of *denial*—an act we will encounter again below (see Parsons 1984 for a good discussion of the distinction between rejection and denial).

The problem of the strengthened liar, in all its many guises, then becomes: no matter what *E*-property we identify as justifying both the principled refusal to perform the speech act of asserting that the liar sentence is (definitely) true and the principled refusal to perform the speech act of asserting that it is (definitely) not true, this very property (when fully expressible in the language) permits the reinstatement of some form of the paradox. Indeed Soames (pp. 176–181) concedes that his own approach does not in the end have the resources to combat a strengthened liar sentence of the form "This sentence is not determinately true." (Note that what

Soames means by *determinate* truth informally coincides with what McGee means by *definite* truth.)

III. The Proposal

The *shape* of the Standard Solution *feels* right, even though it has proved difficult to correctly identify the *L*-property and *E*-property that will turn the trick without either reintroducing the paradox in some refined form or without recourse to an essentially richer metalanguage. The nub of such a solution is that possession of the *L*-property is an obvious defect of language. The best response to this defect is a principled silence. In this paper, it will be argued that both the *L*-property and *E*-property together with the particular speech act of *S*-ing that the liar sentence is true/false (a speech act we must refuse to perform) have all been misidentified hitherto. Rather than nominate the liar sentence as neither (definitely) true nor (definitely) false, in the usual manner, it is put forward that it is illegitimate to *suppose* the liar sentence to be true and illegitimate to *suppose* the liar sentence to be false (not-true). Significantly, the *E*-property here identified is not truth-theoretic. Truth does not, and arguably should not, play any substantive role in our dissolution of the liar. In this respect the proposal advanced in this paper is deflationist. It is often thought that a deflationist theory of truth is more compromised than most with respect to the liar paradox simply because no (substantive) truth-theoretic resources are available on a deflationary view (Simmons 1999 explicitly expresses this view, though it is implicit in many reactions to deflationism). It is my hope to show that just the opposite is the case. It is rather the bringing to bear of truth-theoretic resources (such as truth-value gaps) that proves to be problematic and ultimately self-defeating. This is to say that the proposal argued for in this paper is not merely compatible with deflationism, it provides both a positive and novel reason to accept a deflationary conception of truth.[5]

If it is (in a sense to be defined below) illegitimate to suppose that liar sentences have a truth-status, then which speech act of *S*-ing that the liar sentence is true/not-true should we refuse to perform upon discovering this feature? A simple rule governing suppositions runs thus: only suppose what it is legitimate to suppose. A corresponding rule runs: refrain from supposing what it is illegitimate to suppose. (These rules will actually turn out to require qualification—but more of that below.) On the basis of this latter rule one ought to refuse to suppose that the liar sentence is true and refuse to suppose that the liar sentence is false (not-true). This is in contrast to the usual formulations of the Standard Solution where the focus is on the speech act of *rejection*, the speech act of refusing to assert. Soames (1999) asks:

> In what sense do we reject these claims? At a minimum, we must not assert them. However there is more to it than that. We must also hold that it would be a mistake to assert them. (p. 171)

There is indeed more to it than that: we must also hold that it would be a mistake to even *suppose* such sentences to be true/not-true. Merely to refuse to assert that the liar sentence is true/not-true is itself an insufficient response to the paradox. A speaker may refuse to assert that the liar sentence is true while nonetheless supposing for the sake of argument that it is true. If this speaker does suppose this for the sake of argument, a paradoxical derivation can be given. The focus on the speech act of *rejection* is a red herring. Refusing to assert the liar sentence is a necessary but not a sufficient response to the paradox. In refusing to suppose *P* (on the grounds that it is improper to suppose that *P*) a speaker is committed to refusing to assert *P*, but not conversely. The speech act of refusing to suppose that the liar sentence has a truth-status (true or not), is however both necessary and sufficient to block the paradox, as we shall see.

IV. Bivalence and Illegitimate Suppositions

If liar sentences are not legitimately supposable then how does this feature impact upon bivalence? It is familiar that the principle of bivalence receives a strict and a generalized formulation. The former formulation states that every unambiguous sentence that says that something is the case is either true or false; the latter that such sentences are either true or not true. Strict bivalence is nonetheless compatible with the possibility of what we might call *anodyne* truth-value gaps. Sentences that express questions, commands, or exclamations are neither true nor false, but obviously these sentences do not impugn strict (nor generalized) bivalence—they are anodynely gappy. The same goes for well-formed but meaningless declarative sentences. (A sentence is "gappy" in the non-anodyne sense when it says that something is the case but lacks a truth-value.) Bivalence (strict or generalized) is relevant only to sentences (or utterances) that represent the world as thus and so (Williamson 1994, pp. 187–88). It is tempting to conjecture that every sentence that is not legitimately supposable must thereby be anodynely gappy. But this thought is too hasty. It depends on just *why* a sentence is not legitimately supposable. Meaningless sentences are indeed not legitimately supposable, and of course these sentences are compatible with, but not subject to, both forms of bivalence. Arguably, however, not all sentences that fail to be legitimately supposable are thereby meaningless. The key thesis of this paper is that liar sentences are both meaningful and not legitimately supposable. (In the next section, considerations are advanced in favor of the left conjunct of this claim, while in Sections VI through VIII arguments are given in favor of the right.) Once we make room for such a possibility, then a solution to the liar becomes a genuine prospect.

But if liar sentences are meaningful and yet not legitimately supposable, should we then conclude that they thereby satisfy generalized bivalence but not strict bivalence? Such a thought might be driven by reflection on the following conditionals:

(C1) L is true $\rightarrow$ L is legitimately supposable

(C2) L is false $\rightarrow$ L is legitimately supposable

(where L = "L is not true," and where '$\rightarrow$' is the material conditional). Given C1 and C2, together with the key thesis of this paper, namely, that L (and "L is not true") are not legitimately supposable (and the validity of *modus tollens*), it follows that liar sentences are gappy in the non-anodyne sense. If this were so, then the proposal in hand would collapse into the simple Standard Solution and would thus fall foul of the strengthened liar paradox. To secure the proposal, we must find grounds for rejecting C1 and C2.

If we (provisionally at least) take seriously the possibility that, in addition to the semantic values *true* and *false*, meaningful declarative sentences can also take the "intermediate" semantic value *not legitimately supposable*, then under the most natural interpretation C1 and C2 are to be evaluated as not legitimately supposable. In more detail, if we grant (pending further argument below) that the antecedents of C1 and C2 are indeed not legitimately supposable, then the consequents of these conditionals are, accordingly, false. Under all of the most familiar three-valued matrices for the material conditional, i.e., those given by Lukasiewicz (1930), Bochvar (1939), and Kleene (1952), a conditional with a false consequent but an "intermediate" antecedent takes the intermediate value. Since C1 and C2 are not legitimately supposable they are not warrantedly assertible—they should not be accepted, and the proposal in hand does not collapse into the Standard Solution.

One key feature of note here is that the contrapositives of C1 and C2 are likewise not legitimately supposable (they have true antecedents but intermediate consequents). This has the result that it is not legitimate to suppose (and so not legitimate to assert) that the intermediate semantic status excludes truth or excludes falsity. But on that basis it then looks tempting to say that a sentence can fail to be legitimately supposable but nonetheless remain either true or false (where falsity for our purposes is equivalent to non-truth). However this is not so. Generalized bivalence is best formulated as a conjunction of two principles:

> *Principle of valence*: Every meaningful declarative statement has a truth-status.
>
> *Principle of two truth-status*: There are two sorts of truth-status: *true*, *not-true*[6]

The thesis that it is not legitimate to suppose that that the liar sentence has a truth-status entails that it is not legitimate to suppose (and hence to assert) the *principle of valence*. Since there is no (overt) worry with the *principle of two truth-status*, then on the plausible assumption that a conjunction with one true conjunct and one intermediate conjunct must take the intermediate value, then generalized bivalence is not legitimately supposable and so not legitimately assertible, where crucially, this does not entail that bivalence is deniable—that its negation is assertible.[7] And so, in addition to the triad of positions *anodynely gappy*, *non-anodynely gappy but not strictly bivalent*, and *non-anodynely gappy but strictly bivalent*, there is a further status that meaningful but non-legitimately supposable sentences may take, namely a status for which all forms of bivalence are themselves not legitimately supposable. We have glimpsed how such a proposal impacts upon classical semantics. Now we must endeavor to secure the thesis that liar sentences are indeed meaningful.

V. Is the Liar Sentence Meaningful?

To answer this question in detail would require more space than is available here, so what follows is just an outline of how the arguments might run. The immediate evidence strongly suggests that there is no particular reason to doubt that liar sentences are devoid of content. The sentence "This sentence is not true" is certainly grammatical. Nor would it seem to represent a category mistake, for the right kind of predicate is predicated of the right category of thing. Furthermore, each word would also seem to bear its usual meaning, and there ought to be no particular worry concerning self-referenceæjust as the (false) sentence "This sentence contains ten words" says that something is the case, so does the liar sentence. With these observations in mind, it is surprising to find how many authors have thought that liar sentences (or utterances of liar sentences) fail to represent the world as thus and so.[8] At first sight, such a "no-proposition" view of liar sentences seems susceptible to a version of the strengthened liar paradox. If a liar sentence fails to say that something is the case (i.e., fails, for all intents and purposes, to express a proposition) then it lacks a truth-value by default since it cannot be a *bona fide* truth-bearer. Accordingly, it seems one can run the strengthened liar paradox given in note 2 against such a proposal. But this is too quick, for the rule of *truth-introduction* employed there was in fact stated too simply. This rule (and the corresponding rule of *truth-elimination*) should rather be stated, respectively, as follows:

> If ϕ says that something is the case, then from $\ulcorner\Gamma \vdash \phi\urcorner$ one can infer $\ulcorner\Gamma \vdash$ 'ϕ' is true$\urcorner$

> If ϕ says that something is the case, then from $\ulcorner\Gamma \vdash$ 'ϕ' is true$\urcorner$ one can infer $\ulcorner\Gamma \vdash \phi\urcorner$

These rules ensure that semantic ascent and descent are permitted if it is first given that 'ϕ' says that something is the case (cf. Williamson 1994, pp. 187–8). Since the sentence that says of itself that it is not true does not say that something is the case, then the consequent of these rules is not validated and no strengthened liar paradox is derivable. Is this no-proposition response at all cogent?

There are two conspicuous problems with the no-proposition response to the liar paradox. Firstly, it would appear that in any case one can reconstruct the paradox in terms of propositions rather than sentences. Let 'Π' stand for the proposition that Π is not true. Assume that Π is true. Then given what 'Π' stands for, this is just to say that the proposition that Π is not true is itself true. Given the 'equivalence thesis' (i.e., the proposition that Π is true if and only if Π) then we can infer that Π is not true. Contradiction. Conclude (by negation-introduction) that: Π is not true. But given the equivalence thesis we can now infer that the proposition that Π is not true is itself true, and given that 'Π' stands for the proposition that Π is not true this is just to say that Π is true. Paradox.

The second problem turns on the possibility of contingent liar sentences.[9] If I inscribe on my whiteboard the sentence "Some sentence on this whiteboard is not true," then whether this sentence counts as liar-like depends on the contingent fact as to whether or not there is more than one sentence inscribed on the whiteboard.[10] Suppose I rub out all other sentences bar this one sentence. While we should expect such a change to affect the *E*-property we take this sentence to have, we should not expect any such change to affect whether or not this contingent liar sentence says that something is the case. According to certain versions of the simple Standard Solution, for instance, rubbing out all other sentences on the board will affect whether or not the sentence in hand has a truth-value, but will not affect whether this sentence has truth-conditions. In more neutral terms, changes in the world can affect whether or not a statement is warrantedly assertible, but these changes ought not to have any direct impact on whether the statement has warranted assertibility conditions. Of course much more could be said about this matter, but there is at least a strong *prima facie* case to think that liar sentences are meaningful.[11]

Thus far nothing has been said as to what sort of *L*-property liar sentences possess to justify the evaluation that it is illegitimate to suppose that such sentences have a truth-status (true or not). The claim developed below is that non-contingent liar sentences possess a distinctive logical form—a form that inevitably undermines the *telos* or goal of the speech act of supposition. It proves possible to identify a syntactic (rather than a truth-theoretic) *L*-property of non-contingent liar sentences that dictates that a speaker must not suppose such sentences to have a truth-status. To secure this claim we must first survey some salient features of the speech act of supposition. To this end it is useful to begin by comparing the speech act of supposition with that of assertion.

VI. Supposition and Assertion: Teleology

In what follows, it is merely necessary to uncover those aspects of supposition that are directly relevant to a dissolution of the paradox.[12] First some preliminaries. The term "supposition" is ambiguous. On the one hand, we can speak of supposition as a species of speech act, and on the other, we can speak of the sentence or proposition that is the object of that speech act. In what follows, it is used to refer to the former. It is also germane to speak of suppositions in a broader senseæas acts of linguistic inscription and as a mental acts that an individual can perform without necessarily uttering sounds. One can think of utterances that say that something is the case (i.e., assertions, suppositions, conjectures, etc.) as being the primary bearers of truth-values, or one can think of these acts as bearing truth-values only insofar as they express propositions or have as their objects meaningful declarative sentences. For the sake of convenience, we can take declarative sentences to be the primary truth-bearers, simply because the debate concerning the liar paradox has conventionally dealt with the problems attending liar *sentences*.

Supposition is a goal-directed activity. In supposing, quite simply, we are interested in establishing what follows from what. Supposition in this sense, as we should expect, is governed by teleological norms. Teleological accounts of assertion are familiar from the writings of Dummett (1959; 1973, p. 320; see also Priest 1987, pp. 77–79; 2000, pp. 309–10). The point or goal of assertion, on these accounts, is to utter true sentences—to hit the truth. The teleological norm governing assertion thus runs: in asserting, aim to say what is true. Making assertions for Dummett and Priest is usefully compared with a game: to utter truths is to win, while to utter falsehoods is to lose. Call this the *truth-account* of assertion. (Assertion is here more or less conceived in the Fregean sense as the "outer" manifestation of the mental act of judgment, an act whose attitudinal correlate is belief. A more refined view might maintain that while the *telos* of judgment/belief is truth, the *telos* of assertion is truth *plus* the communication of truth.) A stronger teleological account says that it is constitutive of assertion that the *telos* of assertion is knowledge (Williamson 2000, p. 1, expresses a version of this stronger view by saying that "the point of belief is knowledge.") The teleological norm on this account runs: in asserting, aim to say what you know to be true. Call this the *knowledge-account* of assertion. This is not the place to defend this account in detail, but the knowledge account is surely more compelling. Though it's harder to win at the game of assertion on the knowledge account, we do not want to win at this game by accident: our assertions are required to be reliability right—a condition that the truth account cannot enforce.

What then of the *telos* of supposition? Suppositional reasoning is intimately connected with the categorical assertion of conditional claims. This fact is reflected in the validity of the deduction theorem: $A \vdash B$ if and only if $\vdash A \rightarrow B$ (where '$\rightarrow$' is the material conditional, and "$A \vdash B$" abbreviates "B is provable (in some unspecified proof-theory) given A," and where "$\vdash A \rightarrow B$" abbreviates " '$A \rightarrow B$' is a theorem, i.e., provable on no assumptions"). It makes no sense to speak of *mere* supposition. In supposing some sentence A, one is interested in giving valid proofs of what follows from A. Generally, we suppose some sentence A in order show *whether or not* some sentence B is provable from A.[13] In particular, we aim to be in a position to assert "$\vdash A \to B$" truly or be in a position to assert "$\nvdash A \to B$" truly. So, the teleological norm governing the supposition of A (in order to see whether B follows) runs: aim to be in a position to truly assert that B is provable from A or to be in a position to truly assert that B is *not* provable from A. Call this the *truth-account* of supposition. In contrast, the stronger knowledge account of suppositional reasoning says that in supposing A, one must be in a position to know that B is provable from A or be in a position to know that B is *not* provable from A. To win at supposition, it is not enough for one to truly assert whether or not B follows from A. One loses at supposition if one's assertion that $A \vdash B$ is indeed true, but where one's belief that $A \vdash B$ could easily have been wrong—one does not want to win at the game of supposition by accident. Again, this is not the place to defend such a knowledge account in detail, but for this reason alone, the knowledge account is surely more cogent. It is crucial to note that the goal of supposition is stated as an *exhaustive* disjunctive condition: aim to either know that $A \vdash B$ or know that $A \nvdash B$, where one fails to satisfy this goal if one is in neither epistemic position.

VII. Suppositional Inaptitude and the Supposition Test

One may fail to satisfy the point of suppositional reasoning for a variety of reasons. One may fail to be in a position to know that B follows from A or to know that B does not follow from A, simply through limitations on one's powers of logical deduction. Sometimes the very integrity of the supposed sentence is the root reason for failing to win at the game of supposition. This occurs, for instance, in the case of supposing sentences that do not bear a proper content. In supposing the sentence "Jim is slithy mimsy brillig and generous" (call this A) in order prove whether or not the sentence "Jim is generous" (call this B) logically follows, one cannot know that $A \vdash B$ or know that $A \nvdash B$ since, even though the inference is *formally* valid, and "B" is a truth-bearer, the sentence "A" is plainly gibberish and so not a proper truth-bearer. Here we should rather say that it is not our reasoning that is at fault *per se*, but the very supposition of the sentence that features as antecedent. In this case, there is *in principle* no warrant to accept/deny *all* conditionals in which a meaningless statement features as antecedent—we are not in a position to know that $\vdash A \to B$, nor in a position to know that $\nvdash A \to B$. We are thus entitled to say that the speech act of supposing A is *essentially* improper. It thus pays at this point to introduce some terminology to refer to those sentences that may, for whatever reason, essentially defeat the goal of supposition. Say that

> a sentence "A" fails to be *supposition-apt* if there is *in principle* no warrant for a speaker to accept or deny *all* conditionals in which "A" is the antecedent.

This effectively characterizes what we may call *generic* supposition-inaptness (a more specific characterization will be given in a moment). We may say that a sentence is supposition-apt just in case it is not supposition-inapt (just in case, that is, there is *in principle* some knowledge conferring warrant to accept that $\vdash A \to B$ or some knowledge conferring warrant that $\nvdash A \to B$).

Sentences may be supposition-inapt for a variety of reasons. Ungrammatical sentences, sentences that embody category mistakes, nonsensical sentences, and so forth, are all supposition-inapt. These are all sentences that fail to say that something is the case. However, lack of proper content is a sufficient but not a necessary condition of suppositional inaptitude. We should also allow that meaningful sentences may essentially defeat the *telos* of supposition. One way in which this might occur is when one has *both* a warrant (or reason) to accept that $\vdash A \to B$ *and* a warrant (or reason) to accept that $\vdash \sim(A \to B)$ and so a warrant (or reason) to accept that $\nvdash A \to B$ (given that warrants transmit over the entailment from "$\vdash \sim(A \to B)$" to "$\nvdash A \to B$"). Hence, one cannot be in a position to know that $\vdash A \to B$ since the evidence one has for $\nvdash A \to B$ (such as a putatively valid proof) defeats the possibility of this knowledge, but neither can one be in a position to know that $\nvdash A \to B$ since the evidence that one has for $\vdash A \to B$ (such as a putatively valid proof) defeats the possibility of this knowledge also. One fails to win at the game of supposition in such a case.

Such observations suggest a more specific formulation of suppositional inaptitude that will enable us to isolate the particular *L*-property possessed by liar-sentences in virtue of which they are supposition-inapt. We can do this by submitting the suppositional credentials of declarative sentences to the following test:

> *The Supposition Test.* A sentence "*A*" fails to be supposition-apt if, for *all* sentences "*B*," one can establish that
>
> (i) $\vdash_{NK+} A \to B$, and
>
> (ii) $\vdash_{NK+} \sim(A \to B)$.

First some minor comments on this test: (a) Note that "$A \vdash_{NK+} B$" abbreviates "*B* is provable given *A*, in classical logic (plus the rules of truth-introduction and truth-elimination)." (b) If a sentence is supposition-inapt in this more specific sense then it will be supposition-inapt in the generic sense defined above, but not vice versa. (c) Strictly speaking, one ought to add a third condition to the effect that the rules of proof in NK+ are beyond reproach (as indeed they are—apart that is, from the rule of assumptions as we shall see). This ensures that when (i) and (ii) are satisfied, we are indeed blaming the suppositional credentials of "*A*" rather than the system of proof itself. (d) This test is relevant only to non-contingent liar-sentences, for in deriving a paradox from a contingent liar sentence the premise set is never empty—it must contain some relevant contingent assumption. (For the sentence "some sentence on this page is not true" to be paradoxical, it must depend on the contingent assumption that there is only one sentence on this page.)[14]

But what exactly does this test amount to? Clause (i) says that *B* is derivable from *A*, while clause (ii) entails that *B* is *not* derivable from *A*. If both clauses are satisfied then clearly something has gone wrong—but what? The obvious response is to blame our proof-theory—that some rule of proof in NK+ does not preserve the designated value and requires restriction in some appropriate fashion. (In fact the culprit is indeed the rule of assumptions, as we shall see below.) More informally, satisfaction of (i) and (ii) shows that we are essentially prevented from finding out what any of the logical consequences of the sentence "*A*" are. When the *telos* of supposition is defeated in this absolute we can say that "*A*" is supposition-inapt: there's no point in supposing a sentence if one can never be in a position to demonstrate what follows from this sentence.[15] Given the above discussion, we are now in a position to show that contradictions pass the supposition test while liar sentences characteristically do not.

VIII. Testing the Suppositional Credentials of Liar Sentences

The most that can be inferred from the supposition of "P & ~P" is that $\vdash_{NK+}(P\&{\sim}P) \rightarrow P$ and $\vdash_{NK+}(P\&{\sim}P) \rightarrow {\sim}P$. Clause (ii) is not satisfied, and so contradictions (which themselves contain no liar sentences) pass the supposition test. If we could additionally establish that $\vdash_{NK+}(P\&{\sim}P)$, then the matter would be different, for then (given *modus ponens*) we could infer $\vdash_{NK+}{\sim}P$, and given $\vdash_{NK+}(P\&{\sim}P)$, this is to show that $\vdash_{NK+}{\sim}((P\&{\sim}P) \rightarrow P)$, and so clause (ii) would be satisfied also. This is exactly what happens with the liar paradox. However, to show that liar sentences are supposition-inapt in this way we need to first address a preliminary puzzle.

Clauses (i) and (ii) are very demanding in the sense that it must be shown that *B* both is and is not derivable from *A*, for *all* substitution of the sentential variable *B*. But surely we should expect a sequent L is true $\vdash$ ~(Jam is red and Jam is not red) to be a valid sequent, and, given the deduction theorem, we should likewise expect it to be the case that $\vdash_{NK+}$ L is true $\rightarrow$ ~(Jam is red and Jam is not red).[16] Hence, "L is true" ought to be able to feature as the antecedent of certain unproblematic conditionals. If it can do so, then "L is true" passes the supposition test (contrary to the advertised aims of the proposal). However in NK+, *thinning* (i.e., the structural rule of dilution/weakening: *from* $\ulcorner \Gamma \vdash_{NK+} B \urcorner$ *infer* $\ulcorner \Gamma, A \vdash_{NK+} B \urcorner$) is valid, and so liar-susceptibility can be shown to be "infectious." For instance, one can show that the supposition that "L is true v ~(Jam is red and Jam is not red)" gives rise to paradox if one admits thinning.[17] This infectiousness indicates why it is pertinent to ensure that clauses (i) and (ii) hold for every substitution for *B*.

To show that a liar sentence fails the supposition test we need to move in two stages. Firstly, we need to show that clause (i) and (ii) are satisfied for all the "relevant" putative consequences of liar sentences; secondly, we need to show that these are satisfied for all the "irrelevant" putative consequences of such sentences. The distinction between relevant and irrelevant is roughly that intended by relevance logicians as we shall see. A sentence *B* is a "putative" logical consequence of *B* when there is a proof-theoretically valid demonstration that *B* follows from *A* in NK+. As indicated above this does not mean that *B* is a *bona fide* logical consequence of *A* as there may be a proof-theoretically valid demonstration that *B* does not follow from *A*.[18] Let's turn to look at a relevant putative logical consequence of the liar sentence.

Let the logical form of the liar sentence be given by the usual equality L = "L is not true." Let "*A*" stand for the sentence "L is not true," and let "*B*," the putative candidate consequence of *A*, abbreviate this very same sentence. Then suppose

(1) L is not true.

Then by conditional-introduction we can straightforwardly show that $\vdash_{NK+} A \rightarrow B$:

(2) L is not true $\rightarrow$ L is not true.

Clause (i) is satisfied. But we also need to establish $\vdash_{NK+}{\sim}(A \rightarrow B)$. For this it suffices to establish both $\vdash_{NK+}A$ *and* $\vdash_{NK+}{\sim}B$. Given the rule of truth-introduction, from line (1) we can infer:

(3) "L is not true" is true.

Given that L = "L is not true," then by substitution in (3) we infer:

(4) L is true.

Contradiction. So, by negation-introduction we can infer:

(5) ~(L is not true)

which establishes $\vdash_{NK+}$ ~*B*. By the rule that allows us to infer ⌜'ϕ' is not true⌝ from ⌜~ϕ⌝ we now derive

(6) "L is not true" is not true.

Given that L = "L is not true," then by substitution in (6) we infer:

(7) L is not true.

which establishes that $\vdash_{NK+}$*A*. Given that '→' is the material conditional, to establish both $\vdash_{NK+}$*A and* $\vdash_{NK+}$~*B* is to establish that $\vdash_{NK+}$~(*A* → *B*).

The result generalizes. Let "*A*" represent the sentence "L is not true," and let "*B*" represent the sentence "'L is not true' is true." By conditional introduction on lines (1) and (3) we can establish that $\vdash_{NK+}$*A* → *B*. Given that at line (7) we have $\vdash_{NK+}$*A*, and at line (6) we have $\vdash_{NK+}$~*B*, then we have again shown that $\vdash_{NK+}$~(*A* → *B*). Take another example, namely the sequent $\vdash_{NK+}$ L is true → (L is true v *P*). We have already proved that $\vdash_{NK+}$ L is true, and the sequent itself is easily provable. We now need to prove that ~(L is true v *P*). But since from lines 5 and 7 it in any case follows that from $\vdash\perp$, then by *ex falso quodlibet* we can derive $\vdash_{NK+}$~(L is true v *P*).

Clearly, for any liar-like sentence *A*, and for any putative relevant consequence of this sentence *B*, we can always demonstrate that clauses (i) and (ii) are both satisfied—at least if we allow ourselves the full resources of NK+, including the classical spread law. Consequently, liar sentences fail to pass the supposition test, at least for all "relevant" substitutions for *B*. For liar sentences to fail to pass the supposition test in full generality, then for *all* substitution for *B* (be they relevant putative consequences or an irrelevant putative consequences) one must likewise be able to establish that (i) $\vdash_{NK+}$ *A* → *B*, and (ii) $\vdash_{NK+}$ ~(*A* → *B*). For instance, in order to ensure that "'L is true' is supposition-inapt," we at the very least need to establish that $\vdash_{NK+}$ L is true → *B*, $\vdash_{NK+}$ L is true → ~(*B*), and $\vdash_{NK+}$ L is true, where "*B*" ranges over the classical theorems. More than that, we need to let "*B*" range over such irrelevant consequences as "the moon is made of cheddar cheese," and the like. One can secure this result in a variety of ways, but I shall use the "paradoxes of strict implication."

Let *A* be the sentence "L is not true" as before. Line (5) effectively establishes that $\vdash_{NK+}$~*A*. Given the *Rule of Necessitation*, we can then infer that $\vdash_{NK+}\Box$~*A*. Given the paradoxes of strict implication, we can then infer both that $\vdash_{NK+}$*A* ≺ *B* and $\vdash_{NK+}$*A* ≺ ~*B* (for all *B*). Since strict implication ≺ entails material implication → then this is just to establish that $\vdash_{NK+}$*A* → *B* and $\vdash_{NK+}$ *A* → ~*B*. Since we have already shown that $\vdash_{NK+}$*A*, then by *modus ponens* we can now infer that $\vdash_{NK+}$~*B*, which gives us $\vdash_{NK+}$~(*A* → *B*), and so both clauses (i) and (ii) are satisfied for any sentence *B*, be it relevant or not.[19] So, it is illegitimate to suppose a sentence for which we are never in a position to accept or deny what putatively follows (relevantly or irrelevantly) from that sentence. Liar sentences are essentially unfit to enable the *telos* of supposition to be satisfied. But on that basis of having that *L*-property, how can one appropriately restrict the proof theory of NK+? To answer that question we must return to our comparative analysis of supposition and assertion.

IX. Supposition and Assertion: Constitutive Rules

Teleological norms for speech acts do not necessarily coincide with the norms codify by what Williamson (1996) has called the *constitutive* rules that govern such acts. These rules specify the norms that *essentially* and *uniquely* govern each speech act.[20] On the Williamsonian model, each and every speech act can be identified by reference to the rules that are constitutive for it, and in turn we can evaluate the performance of a particular speech act on the basis of its constitutive rule. As Williamson (p. 491) puts it:

> Constitutive rules do not lay down necessary conditions for performing the constituted act. When one breaks a rule of a game, one does not thereby cease to be playing that game. . . . Likewise, presumably, for a speech act: when one breaks a rule of assertion, one does not thereby fail to make an assertion. One is subject to criticism precisely because one has

> rather than the truth or falsity of any assumptions made; hence [this rule] allows us to make any assumptions we please—the job of the logician is to make sure that any conclusion based on them is validly based, *not* to investigate their credentials.

These remarks are instructive. Lemmon is of course right to stress that a speaker can legitimately suppose a sentence that is false, for the logician's business is first and foremost to establish what follows from what. In proving the law of non-contradiction, for example, we must indeed first suppose a sentence that is *necessarily* false. It is a further issue whether in supposing *P* the logician is free to ignore *P*'s *non*-truth-theoretic credentials. The matter is of course clearer in the case of the *sentential* as opposed to the *propositional* calculus. Propositions just are *bona fide* supposable truth-bearers by default, so goes the thought, while declarative sentences may fail to say that something is the case. Lemmon, I'm sure, would have agreed that the Rule of Assumptions in the sentential calculus would require restriction in order to accommodate those declarative sentences that, for whatever reason, fail to express propositions. In *sentential* logic we thus need to state the Rule of Assumptions as follows: we are permitted to introduce at any stage in a proof any declarative sentence we choose as a premise of the argument only if that sentence is *supposition-apt*. In sequent calculus form this rule is to be given as follows:

Rule of Assumptions. $$\frac{}{\Sigma \vdash \Sigma}$$

(Provided the sentence Σ is supposition-apt)

That is, from the "null sequent" (or the empty sequent ...⊢...) we can infer the sequent that $\Sigma \vdash \Sigma$ only if the credentials of Σ are in order—that, is only if Σ is supposition-apt. Should we wish to work exclusively in a propositional sequent calculus then worries over sentences that fail to express propositions can be put aside. However, the possibility of the propositional version of the liar paradox given above, together with the key thesis of this paper—that liar sentences/propositions are meaningful but fail to be supposition-apt—means that we need to keep this restriction in place even when stating the rule of assumptions with respect to reasoning with propositions. In order to have a general theory of suppositional inaptitude, however, it is necessary to persevere with the sentential calculus.

The Rule of Assumptions as stated above effectively incorporates into our proof-theory the following constitutive rule governing suppositions:

> *The S-rule:* One must: suppose that *P* only if *P* is supposition-apt.[21]

The question now arises: have we stated this rule strongly enough? In making any speech act one represents oneself to have the authority to do so. In supposing *P*, one represents oneself as *knowing* that *P* is supposition-apt, as knowing that *P* is fit to enable the *telos* of supposition to be satisfied. But this suggests that the following stronger rule is in fact correct:

> *The S-rule*:* One must: suppose that *P* only if one *knows* that *P* is supposition-apt.[22]

In supposing *P*, a speaker implies, but does not assert, that she knows that *P* is supposition-apt. Arguably, it would be an (albeit artificial) Moorean paradox to suppose the sentence "*P*" while simultaneously canceling the implicature that one has the authority to make this supposition by asserting (perhaps using the convention of holding up a certain flag) "I do not know that *P* is supposition-apt." One does something wrong in simultaneously supposing a sentence but disavowing any knowledge that this sentence is fit to satisfy the *telos* of supposition. If something less than knowledge that *P* is supposition-apt made supposing *P* permissible then one could enact the Moorean paradox with impunity, but one cannot. The *S*-rule* thus encodes the injunction: Don't suppose *P* if you don't know that *P* is supposition-apt. In other words, we have to replace the slogan *all assumptions are for free* with the slogan that only knowledge that *P* is supposition-apt permits the supposition that *P*. Thus it would appear that the stronger *S*-rule* is the correct constitutive rule governing supposition. Accordingly, it seems we must modify the Rule of Assumptions to the effect that: we are permitted to introduce at any stage in a proof any declarative sentence we choose as a premise of the argument only if that sentence is *known* to be *supposition-apt*. Thus:

*Rule of Assumptions.** $\dfrac{\quad}{\Sigma \vdash \Sigma}$

(Provided the sentence Σ is *known* to be supposition-apt)

It now ought to be clear where we go wrong in the paradoxical derivation. One breaks the constitutive *S*-rule* (and indeed the *S*-rule) in supposing liar sentences to be true or in supposing them to be not-true. This is, first and foremost, a mistake at the level of speech acts—a pragmatic mistake. The revised Rule of Assumptions* represents a way of accommodating the possibility of this pragmatic mistake into our proof theory. Once this accommodation is made, then we are in a position to say that in attempting to truth-evaluate the liar sentence we go wrong at the very first step in applying the Rule of Assumptions* to this sentence. Since this rule is indeed a rule of proof, it is the reasoning that is at fault in the liar paradox. The suppositional credentials of liar sentences are such that it is illegitimate to suppose them: the lesson of the liar is that not all assumptions are for free.[23] On that basis, we have found a principled reason to refuse to suppose that L is true and a principled reason to refuse to suppose that L is not true. But are matters really so straightforward?

X. The Strengthened Liar Sentence and the Revenge Problem

The Standard Solution has, throughout its various guises, been a conspicuous failure owing to the problem of the strengthened liar paradox. Is there a form of strengthened liar sentence that might regenerate the paradox for this version of the Standard Solution? Any sentence that says of itself that it is not supposition-apt seems to be a good candidate for a strengthened liar sentence for this proposal. Let the logical form of this candidate strengthened liar sentence be represented by the equality SL = "SL is not supposition-apt" and suppose for the sake of argument that

(1) SL is not supposition-apt.

By the rule of truth-introduction we infer

(2) "SL is not supposition-apt" is true.

and by substitution we derive

(3) SL is true.

If we allow that it is sufficient for SL to be supposition-apt that it be true then we can further infer

(4) SL is supposition-apt,

which contradicts (1) and so by negation-introduction we infer

(5) ~(SL is not supposition-apt),

which is just to demonstrate that the sentence that says of itself that it is not supposition-apt is false—this sentence *is* in fact supposition-apt (since it is false), contrary to what it says of itself. (Cf. the sentence that says of itself that it contains ten words.) Unlike other forms of the Standard Solution the proposal in hand does not appear to regenerate the paradox in some refined form. This provides a strong *prima facie* reason for thinking that a proposal of this sort is along the right lines. Is there a "revenge problem" for this proposal?

In the literature on the liar, the revenge problem (for whatever proposal in hand) has in general been confused with the problem of the strengthened liar paradox. Roughly, a solution suffers from the revenge problem when it has pathological but not necessarily inconsistent consequences (see note 9). It would appear that there is a form of revenge problem for the solution proposed here, and it can be framed as follows: in order to test whether a sentence passes the supposition test one must first suppose this sentence in order to demonstrate what its (putative) logical consequences might be. If a sentence fails to pass the test then it is illegitimate to suppose it—but that was just what we needed to do in order to test its credentials via the supposition test. Briefly put, we seem to be supposing the liar sentence in order to show that it is not legitimately supposable, but if it is not legitimately supposable then we are not entitled to suppose it *tout court*. Even though this revenge problem would appear to be a pragmatic rather than a logical paradox, it nonetheless demands a response.

One thought might be that the *S*-rule* (and indeed the *S*-rule) are stated too strongly. Independently of any worries concerning the liar paradox, there are grounds to think that both these rules are too prohibitive in any case. Consider the possibility that a certain very complex (but grammatical) sentence Ø encodes a category mistake. Such a sentence fails to say that something is the case. Suppose that a speaker nonetheless believes Ø to be supposition-apt and performs the speech act of supposing Ø in order to see what logically follows. Given the *S*-rule and the *S*-rule*, this speaker has done something wrong in supposing Ø to be true. Yet there may be no other way of revealing that Ø encodes a category mistake other than by supposing it to be true/not-true and applying rules of inference to its sentential and sub-sentential structure. In response to this possibility, one might try to considerably weaken the *S*-rule as follows:

> *The S-rule**:* One must: suppose that *P* only if one does not know that *P* fails to be supposition-apt.

This weaker rule encodes the injunction: don't suppose *P* if you know *P* fails to be supposition-apt. Thus, for instance, in the absence of any warrant for believing *P* to be essentially unfit for suppositional reasoning we are free to suppose *P* with impunity. But this weaker rule just seems too liberalæsurely there ought to be something impermissible about supposing meaningless sentences to be true! A better response to this problem is to distinguish between the primary and secondary goals of supposition.

The primary teleological norm governing suppositions is the norm distinguished hitherto: in supposing *P*, in order to see if some sentence *Q* logically follows, aim to have a warrant either to deny or accept that $\vdash P \rightarrow Q$. In supposing the sentence Ø to be true one will indeed essentially fail to satisfy this primary goal of supposition. Nonetheless, in such cases a secondary norm may legitimately come into force, namely: in supposing *P*, aim to have a warrant to either accept or deny the thesis that *P* is supposition-apt. In such cases, the rules of the game of supposition have changed and a speaker is accordingly permitted to suppose *P* at least insofar as they are now aiming to satisfy a different goal—the goal of testing the suppositional credentials of *P*. It seems we can give a constitutive rule to accommodate this secondary teleological norm as follows:

> *The P-rule:* One must: suppose that *P* only if one has presupposed that *P* is supposition-apt.

This rule looks rather cumbersome, but its effect is to permit a speaker to suppose sentences that are supposition-inapt, and indeed that might be *known* to be supposition-inapt. Consider again the complex category mistake Ø. Suppose I know that Ø is a category mistake but I want to communicate this fact to others by supposing it to be true and then subjecting it to certain rules of inference in order to reveal its pathological nature. In this case, not only does one break both the *S-rule* and *S-rule** in supposing Ø, but also the weaker *S-rule***. The *P-rule*, on the other hand, is not broken. This rule encodes the weak injunction: do not suppose *P* if you have not firstly presupposed that *P* is supposition-apt. One represents oneself to have the authority to suppose sentences that are potentially, actually, or actually known to be supposition-inapt simply on the basis that one undertakes a commitment to discharge the presupposition that these sentences are supposition-apt if these sentences reveal (or re-reveal) themselves to be essentially unfit for suppositional reasoning.

On the plausible assumption that a speech act can be identified by the teleological and constitutive rules that uniquely and essentially govern that act, then we can thus distinguish two species of the speech act of supposition: supposing *P* in order to see what the logical consequences of this sentence are, and supposing *P* in order to establish whether *P* is supposition-apt. The thought now goes that just as we can legitimately suppose the sentence Ø, at least insofar as we are bound by a teleological norm to uncover the suppositional credentials of Ø, we can likewise legitimately suppose that L is true in order to test the suppositional credentials of L. If this is right, then there is no genuine revenge problem for a view of this sort.

XI. Semantic Closure

Lastly, we may ask if the proposed solution is able to meet the requirements of semantic closure? The aim of the solution developed here is to indeed show that a semantically closed language can after all be consistent once we appropriately restrict the Rule of Assumptions. But we must take care what is meant by semantic closure. For Tarski, a language is semantically closed when

> the language in which the antinomy is constructed contains, in addition to its expressions, also the names of these expressions, as well as semantic terms such as the term "*true*" referring to sentences of this language; we have also assumed that all sentences which determine the adequate usage of this term can be asserted in the language.[24]

Others, such as Herzberger (1970, p. 26), go much further, and argue that a semantically closed language should at the very least contain "the *means* for recording the truth-value of each of its own sentences" (my italics). The theory just offered is semantically closed in Tarski's sense, but not strictly in Herzberger's. Merely for the language to contain its own truth-predicate does not in itself entail that the language contains the expressive means for assigning truth-values to its sentences. On the proposed theory, a speaker is essentially prevented from assigning truth-values to liar sentences, for it is improper to suppose that such sentences have truth-values—a claim that falls short of asserting that they lack truth-values. This might seem an unwelcome consequence. As Parsons (1984, pp. 148–9) notes, it is “commonly said, regarding the paradoxes, that one must ‘buy consistency at the price of expressive completeness’.” But this is a trade-off the deflationist is perfectly willing to pay. Given that on a deflationary view, the semantics of the language is not given by a truth-conditional theory of content but by a theory in which truth plays no explanatory role, then it is grist for the deflationist’s mill that the theory of truth is expressively incomplete (see note 11). The real expressive adequacy of a semantics for the language must reside at the level of the sentences that, to paraphrase Tarski, determine the adequate usage of the terms in the language. The sentence “L is not supposition-apt” is a sentence of just this sort—and we have seen that the language can contain such sentences without fear of generating further paradox. If this is correct, then truth does not and should not play any explanatory role in the dissolution of the liar paradox.

University of St. Andrews

NOTES

Earlier versions of this paper were presented at Logica ‘99, Liblice Chateau, Czech Republic, 25 June 1999, and at the 11th International Congress of Logic, Methodology, and Philosophy of Science, Krakow, Poland, 24 August 1999. Thanks to Stuart Shapiro and Alan Weir on the former occasion, and to Adam Rieger on the latter, for valuable feedback. I am also very grateful to Peter Clark, Jesper Kallestrup, Katherine Hawley, Fraser MacBride, Patrice Philie, Duncan Pritchard, Sven Rosenkranz, and, especially, Crispin Wright, for their very helpful comments on the first draft, and to Bradley Armour-Garb, John Kearns, Stephen Read, Oliver Schulte, Hartley Slater, and two anonymous referees for this journal, for detailed and invaluable comments on the final version. Additional thanks go to Paul Horwich and Hartry Field for email feedback on their respective positions on the semantic paradoxes.

1. The Standard Solution seems goes back at least as far back as Bochvar (1939), and has been defended by numerous authors such as Martin (1967), van Fraassen (1968), Skyrms (1970), and Parsons (1984).

2. Let the logical form of this sentence be given by the equality SL = “SL is not true.” If SL lacks a truth-value then SL is not true (and not false). Given the rule of *truth-introduction* (i.e., from $\ulcorner\Gamma\vdash\phi\urcorner$ infer $\ulcorner\Gamma\vdash$ ‘ϕ’ is true$\urcorner$) then we can derive “SL is not true” is true. By substituting “SL” for “‘SL is not true’” we then derive that SL is true—contrary to the claim that SL lacks a truth-value.

3. Let the logical form of this sentence be given by the equality DL = “DL is not definitely true.” If DL is unsettled in truth-value then DL is not definitely true (and not definitely false). Given a rule that, following Heck (1993, p. 203), we may call DEF* (i.e., from $\ulcorner\Gamma\vdash\phi\urcorner$ infer $\ulcorner\Gamma\vdash$ ‘ϕ’ is definitely true$\urcorner$), then we can derive that “DL is not definitely true” is definitely true. But by substituting “DL” for “‘DL is not definitely true’” we then derive that that DL is definitely true—contrary to the claim that DL is unsettled in truth-value.

4. Heck (1993) has suggested (in connection with using a "definitely" operator to model higher-order vagueness) that DEF* (see note 3) is not a valid rule of inference under indirect, i.e., subordinate, proofs such as conditional proof and *reductio ad absurdum*. Might the same ploy be used against insulating the sentence DL from paradox? This suggestion is not unattractive. However it seems that one can nonetheless reconstruct the paradox in the following way: Suppose (for the sake of argument) that DL is definitely true, then by substitution we can infer "DL is not definitely true" is definitely true. By the rule of what we may term *def-elimination* (i.e., from $\ulcorner\Gamma\vdash$ 'ϕ' is definitely true$\urcorner$ infer $\ulcorner\Gamma\vdash\phi\urcorner$) we can derive that DL is not definitely true, which contradicts our original supposition, and hence we can rigorously prove (by negation-elimination) that DL is not definitely true. Crucially, as McGee (1991, p. 221) notes, "whatever we can prove rigorously is definitely true," which is to say that DEF* ought to be valid under the scope of indirect proofs when the premise set Γ is empty. If so, then we can infer that "DL is not definitely true" is definitely true, and by substitution we can likewise rigorously prove that DL is definitely true. McGee's response to this formulation of the paradox is to reject the validity of the inference rule *def-elimination* under the scope of indirect proofs (McGee, p.222). This has the result that the one cannot prove the schema: "ϕ" is definitely true $\supset \phi$ (a schema that is nonetheless provable for Heck). For doubts about the tenability of this restriction see Priest (1994, p. 388). There are also further problems with McGee's proposal. Mills (1995) has argued that McGee is unable to give a convincing interpretation of what it is to be "unsettled" in truth-value. Relatedly, there are also very general doubts as to whether "definitely" can bear a non-epistemic sense (Williamson 1994, pp. 194–95).

5. Horwich (1998a, p. 77) rightly recognizes that a deflationist cannot employ truth-value gaps for theoretical work in semantics. Field (1992, p. 322, fn. 1), strangely, is less sure of this. Since both Field (1994b) and Horwich (1998a, p. 79) admit a notion of definite/determinate truth in order to account for such pathologies as vagueness, it might then be thought that they ought to readily endorse some version of the sophisticated Standard Solution of the liar paradox (indeed Soames is a deflationist of sorts). However, in his (1994a, p. 250, fn. 1, fn. 2), Field is content to largely ignore the semantic paradoxes. This is remedied in a forthcoming postscript to this paper, where Field proposes a paraconsistent revision of classical logic along the lines given by Priest (1998). Bradley Armour Garb (forthcoming) has independently argued that dialetheism provides the best deflationary response to the liar paradox. In a similar vein, Priest (1999, p. 307) has recently argued that "*Honest* deflationism is not only compatible with dialetheism, it leads in its direction." Horwich (1998a, pp. 40–42) in contrast gestures toward solving the liar paradox by demanding that Tarski's T-schema (or the equivalence schema: the proposition that p is true iff p) be restricted in some appropriate fashion (though Horwich does not detail how such a restriction is to be effected). Truth, for Horwich, is then to be defined by the maximal consistent set of such instances.

6. Cf. Sanford (1976, p. 196).

7. It should also be noted that the principle of generalized bi-exclusion (which says that no meaningful declarative sentence is both true and not true) is likewise not legitimately supposable.

8. See, e.g., Bar-Hillel (1957) for an early statement of this view.

9. A proposal of this sort also suffers from what has come to be known as the "revenge problem." Roughly speaking, any solution to the liar paradox suffers from the revenge problem when it has pathological (but not necessarily inconsistent) consequences. (Generally speaking most authors run the revenge problem together with the problem of strengthened liar paradox, when in fact the former need not entail the latter.) If SL is meaningless such that SL is not true then, given substitution, we can also assert that "SL is not true" is not true. Since we cannot employ the rule of

truth-introduction, this falls short of being a proper contradiction. It nonetheless remains pathological since in asserting that SL is not true, a speaker does not thereby assert that "SL is not true" is true. Hence the Fregean platitude that to assert *that P* is to assert *that it is true that P* would appear to be a casualty of a proposal of this general sort.

10. The example is adapted from Mackie (1973, p. 294).

11. The deflationist will in fact argue that the driving thesis behind the *no-proposition* view is the truth-conditional conception of meaning and understanding. The impossibility of truth-evaluating the liar sentence is taken to be evidence that this sentence fails to have truth-conditions and so fails to say that something is the case. But it is well known that the truth-conditional conception of meaning has unpalatable consequences. Furthermore, alternative theories of meaning and understanding are available in which grasp of meaning does not entail grasp of truth-conditions, and in which truth plays no substantial explanatory role. One can give a theory of content via reference to warranted assertibility conditions, or one might seek to give a use-theoretic, or conceptual role, model of meaning and understanding. This is not the place to defend such deflationary theories of content, but see Field (1994b) and Horwich (1998b). Arguably, such accounts provide added confirmation that liar sentences do indeed bear content.

12. The best work on supposition has been done by Cargile (typescript); Dummett (1976, pp. 309–10); Green (2000); (Kearns (1997), and Kearns (typescript).

13. There need be no requirement that a speaker must have some particular sentence *B* in mind. Occasionally we wish to suppose a sentence when not having a very fixed idea of what its putative logical consequences might be. (I am indebted to Patrice Philie for stressing this point.)

14. A complete treatment of how a proposal of this sort extends to contingent liar sentences is the subject of a sequel to this paper.

15. One might be tempted to think that liar sentences are not legitimately supposable on the grounds that in supposing a liar sentence to be true one can show in NK+ that every sentence is derivable (since one can show that $\vdash\bot$ and the e*x falso quodlibet* is valid). But it's not the classical spread principle *per se* that enables us to say that the supposition of liar-sentences is illegitimate. The provability of everything is only relevant once we recognize the teleological norms governing supposition. The provability of everything is a necessary but not a sufficient condition for "L is true" to count as suppositionally inapt.

16. One reason for this expectation is that these sequents contain what we might call "mixed" formulas. The sentence "Jam is red" belongs to the non-semantic fragment of the object language, while "L is not true" belongs to the semantic fragment (we are here assuming semantic closure—see the last section). On that basis, we are given a syntactical guarantee that "Jam is red" is not liar-susceptible, and so we should expect it to be subject to the theorems and inferences of classical logic.

17. Proof: (let "*P*" abbreviate "Jam is red"). Assume L is true v ~(*P*& ~*P*), and assume L is true. By &I and &E, one then infers that L is true (which now depends on both assumptions). From there one can derive $\bot$ and so, by ~I on the second assumption infer that L is not true, from which one can then derive $\bot$. One then uses ~I to reject the first assumption to yield ~(L is true v ~(*P*& ~*P*)), and by de Morgan, we can derive L is not true & ~~(*P*& ~*P*). From the left conjunct of this we derive $\bot$ (which rests on no assumptions). Paradox. If ~(*P*& ~*P*) is immune from liar-susceptibility then one would expect L is true v ~(*P*& ~*P*) to be likewise immune, but it's not.

18. A sentence *B* is a *bona fide* semantic consequence of *A* only if (a) $\vdash_{NK+}A\rightarrow B$ (b) $\nvdash_{NK+}\sim(A\rightarrow B)$. It might be thought that clause (b) is superfluous since it looks like it follows from (a). But that entailment is predicated on the assumption that from the metalinguistic statement "$\vdash_{NK+}A\rightarrow B$"

one can assert that $A \rightarrow B$, and from the metalinguistic statement " $\nvdash_{NK+} \sim(A \rightarrow B)$" one can infer that $\sim(A \rightarrow B)$ and derive a paradox from which one infers that $\nvdash_{NK+} \sim(A \rightarrow B)$. But these inferences are valid only if "$\vdash$" is already factive, so to speak, and to have that property it must already be answerable to semantic consequence, which is to beg the very question at issue.

19. Alternatively one could use *ex falso quodlibet*, i.e., from $\ulcorner \vdash_{NK+} \bot \urcorner$ infer $\ulcorner A \urcorner$. Given that we can use the liar paradox to establish that $\vdash_{NK+} \bot$, then given EFQ we can establish both that $\vdash_{NK+} B$ and $\vdash_{NK+} \sim B$. And by two applications of conditional-introduction on any line in which a liar sentence *P* is supposed, we can establish that $\vdash_{NK+} A \rightarrow B$ and $\vdash_{NK+} A \rightarrow \sim B$, and since we already have $\vdash_{NK+} B$ then the same result follows.

20. As Williamson says: "The normativity of a constitutive rule is not moral or teleological. [T]he criticism that one has broken a constitutive rule of an institution [is not] the criticism that one has used it in a way incompatible with its aim" (1996a, pp. 491–2). For example, I may fail to satisfy the teleological norms that may govern the game of chess (e.g., the norms: aim to win, aim to exercise your mind, aim to pass the time, etc.) without thereby breaking any of the rules constitutive of chess.

21. As stated, the condition C(*P*) given here is essential but not *unique* to supposition. To ensure that this rule essentially and uniquely governs the act of supposition we also need to build into this rule the proviso that it is not necessary that *P* be true or even warranted, or indeed that *P* be believed by the speaker. Since this proviso plays no part in the discussion that follows I will omit its further mention.

22. This knowledge requirement might be thought to be too strong. But there is actually no reason to demand that knowledge of supposition-aptness should take the form of a rigorous proof, or even be reflectively accessible to the subject. On the Williamsonian model of assertion, for example, one can satisfy the *A-rule* without knowing that one has done so. The point carries over to issues connected with supposing contingent liar sentences. While the postcard paradox, and analogues, are easily recognizable as such, most contingent liar paradoxes lurk unseen. In general however, even in the absence of any overt evidence of the supposition-aptness of *P*, one suspects that it is not an *easy* possibility that one be mistaken about the supposition-aptness of *P*. Think of the assertion at time t1 of "what you, the reader, judged to be the case at time t2 is false," where you asserted at time t2 the sentence "what Patrick judged to be true at t1 is true." It's not an easy possibility that I should form the false belief that my assertion is supposition-apt. Where I lack evidence of the supposition-aptness of *P*, the world remains, in general, supposition-friendly.

23. Of course not all deductive systems contain the Rule of Assumptions—axiomatic presentations being the most notable example. Since for every proper axiomatic presentation of the sentential calculus there is a corresponding natural deduction presentation then this ought to be no obstacle to the full generality of this proposal. In this respect, consider the simplest template of the liar paradox whereby a substitution of Tarski's T-schema is "L is not true" is true iff L is not true, and where by substitution this yields the paradoxical biconditional L is true iff and L is not true. Since Tarski's T-schema is a derived principle requiring the rules of truth-introduction and truth-elimination, together with conditional proof, and the rule of assumptions, then this simple template presents no special problem. To restrict the rule of assumptions to non-liar sentences is just to restrict Tarski's T-schema likewise, providing just the sort of principled constructive restriction of the T-schema (and the equivalence schema) that Horwich (1998a, p. 42) hopes for (see note 5 above).

24. Tarski (1944) in Linsky (1952, p. 20).

REFERENCES

Armour-Garb, B. Forthcoming. “Deflationism and Dialetheism.”
Bar-Hillel, Y. 1957. “New Light on the Liar. *Analysis* 18: 1–6.
Cargile, J. Typescript. “Supposing for the Sake of Argument.” July 1999.
Dummett, M. A. E. 1976. *Frege: Philosophy of Language.* London: Duckworth.
Field, H. 1992. “Critical Notice of *Truth* by P. Horwich.” *Philosophy of Science* 59: 321–330.
_______ . 1994a. “Deflationist Views of Meaning and Content.” *Mind* 103: 247–285.
_______ . 1994b. “Disquotational Truth and Factually Defective Discourse.” *Philosophical Review* 103: 405–452.
Field, H. Forthcoming. “Postscript to ‘Disquotational Truth and Factually Defective Discourse’.”
Green, M. S. 2000. “The Status of Supposition.” *Nous*.
Haack, S. 1978. *Philosophy of Logics.* Cambridge: Cambridge University Press.
Heck, R. 1993. “A Note on the Logic of (Higher-Order) Vagueness.” *Analysis* 53: 201–8.
Herzberger, H. 1970. “Truth and Modality in Semantically Closed Languages.” In *The Paradox of the Liar,* ed. R. L. Martin. New Haven: Yale University Press.
Horwich, P. 1990. *Truth.* Oxford: Blackwell Publishers.
_______ . 1998a. *Truth,* 2nd ed., Oxford: Oxford University Press.
_______ . 1998b. *Meaning.* Oxford: Oxford University Press.
Kearns, J. T. 1997. “Propositional Logic of Supposition and Assertion.” *Notre Dame Journal of Formal Logic* 38: 325–349.
_______ . Forthcoming. “A Full System of Illocutionary Propositional Logic.”
Lemmon, E. J. 1965. *Beginning Logic.* London: Thomas Nelson and Sons.
Linsky, L., ed. 1952. *Semantics and the Philosophy of Language.* Urbana: University of Illinois Press.
Mackie, J. L. 1973. *Truth, Probability and Paradox.* Oxford: Clarendon Press.
McGee, V. 1989. “Applying Kripke’s Theory of Truth.” *Journal of Philosophy,* pp. 530–539.
_______ . 1991. *Truth, Vagueness, and Paradox.* Indianapolis: Hackett.
Mills, A. 1995. “Unsettled Problems with Vague Truth.” *Canadian Journal of Philosophy* 25: 103–117.
Moore, G. E. 1962. *Commonplace Book: 1919–1953.* London: Allen and Unwin.
Parsons, T. 1984. “Assertion, Denial, and the Liar Paradox.” *Journal of Philosophical Logic* 13: 137–152.
Priest, G. 1994. Review of *Truth, Vagueness, and Paradox* by Vann. McGee. *Mind* 103: 387–391.
_______ . 2000. “Truth and Contradiction.” *The Philosophical Quarterly* 50: 305–19.
Rescher, N. 1969. *Many-Valued Logic.* New York: McGraw-Hill.
Sanford, D. 1976. “Competing Semantics of Vagueness: Many Values versus Super-Truth.” *Synthese* 33: 195–210.
Simmons, K. 1999. “Deflationary Truth and Liar.” *Journal of Philosophical Logic* 28: 455–488.
Skyrms, B. 1970. “Return of the Liar: Three-valued Logic and the Concept of Truth.” *American Philosophical Quarterly* 7: 153–161.
Soames, S. 1999. *Understanding Truth.* Oxford/New York: Oxford University Press.
Tarski, A. 1944. “The Semantic Conception of Truth.” *Philosophical and Phenomenological Research* 4: 341–76. Also in Linsky 1952.
Unger, P. 1975. *Ignorance a Case for Scepticism.* Oxford: Oxford University Press.
Williamson, T. 1994. *Vagueness.* London/New York: Routledge.
_______ . 1996. “Knowing and Asserting.” *Philosophical Review* 105: 489–523.
_______ . 2000. *Knowledge and its Limits.* Oxford: Oxford University Press.