

PY1003: Introduction to Logic
Example Class 8

Multiple Generality and Identity

1. Translate the following arguments into the language of predicate logic. Use truth trees to demonstrate whether or not each argument is valid. If an argument is invalid, exhibit a counterexample.

- (i) Everyone is related to everyone. Therefore, someone is related to herself.
- (ii) There is someone who loves everyone. So, everyone is loved by someone or other.
- (iii) Everyone is a mother. Therefore, Susan is her own mother.
- (iv) Some people are arrogant. Nobody likes anyone who is arrogant. So some people are not liked by anyone.
- (v) Jekyll wears a hat, but Hyde does not. So Jekyll and Hyde are not the same person.
- (vi) Everyone is identical to everyone. Victoria is rich. So everyone is rich.

2. Use truth trees to establish whether or not the following sets are consistent. If a set is consistent, exhibit a model.

- (i) $\{\forall x(Fx \rightarrow \forall y Rxy), \neg Rab, Fa\}$
- (ii) $\{(Fa \wedge Fb) \rightarrow a=b, b=c, Fa \wedge \neg Fc\}$
- (iii) $\{\forall x \exists y Lxy, \exists x \neg \exists y Lyx\}$
- (iv) $\{\forall x(Px \rightarrow \neg x=x), Pa\}$

3. Show that the identity relation is

- (i) transitive
- (ii) symmetric