PY1003: Introduction to Logic
Example Class 6

Trees with Predicate Logic (I)

(A) Which of the following sets of sentences are consistent? Use truth-trees to justify your answers.

(i) {Everyone at the party is happy. Sarah is not happy. Sarah is at the party.}

(ii) {Everyone at the party is drinking wine or beer. Some people who drink beer are violent. Nobody at the party is violent}

(iii) {Somebody is drinking wine. Everyone is drunk. There is at least one sober person who is drinking wine.)

(B) Which of the following arguments are valid? Use truth trees to justify your answers. (Note that these arguments should be familiar from the last example class.)

(i) All whales are mammals. No fish are mammals. So no whales are fish.

(ii) All cats are animals. Some animals have tails. So some cats have tails.

(iii) Only the brave deserve freedom. Some who deserve freedom are Christians. All Christians are blessed in the sight of God. Therefore, some of the brave are blessed in the sight of God.

(C) Which of the following sequents are valid? Use truth-trees to justify your answers.

(i) $\forall x(Fx \rightarrow Gx) \vdash \forall x(\neg Gx \rightarrow \neg Fx)$

(ii) $\forall x(Fx \rightarrow Gx), \forall x(Gx \rightarrow Hx) \vdash \forall x(Fx \rightarrow Hx)$

(iii) $\exists x(Fx \wedge Gx), \forall x(\neg Hx \rightarrow \neg Gx) \vdash \exists x(Fx \wedge Hx)$

(iv) $\forall x(Fx \vee Gx), \vdash \forall xFx \vee \forall xGx$

(v) $\exists x(Fx \leftrightarrow Gx) \vdash \forall xFx \vee \exists xFx$