

PY1003: Introduction to Logic
Example Class 3

Trees for sentential logic I

(A) Decide the consistency of the following sets of Sentential formulae using a truth tree. If a set is consistent, exhibit a model.

- (i) $\{A \rightarrow B, B \leftrightarrow \neg C, A \rightarrow C, A\}$
- (ii) $\{A \rightarrow B, B \rightarrow C, A \rightarrow \neg C\}$

(B) Decide the consistency of the following sets of English sentences by translating into Sentential and using a truth tree. If a set is consistent, exhibit a model.

- (i) {If Moriarty has escaped then either Holmes has bungled or Watson's on the job. Holmes has not bungled unless Moriarty has escaped. Watson is not on the job. Moriarty has not escaped if and only if Holmes has bungled.}
- (ii) {Moriarty will not escape unless Holmes acts. We shall rely on Watson only if Holmes does not act. Holmes does not act. It is not the case that Moriarty will escape unless we rely on Watson.}

(C) Determine which of the following formulae are tautologies. Use a tree to justify your answer.

- (i) $A \rightarrow (B \rightarrow A)$
- (ii) $(A \rightarrow \neg A) \rightarrow A$

(D) Determine which of the following formulae are contradictions. Use a tree to justify your answer.

- (i) $A \leftrightarrow \neg A$
- (ii) $A \leftrightarrow (\neg A \vee B)$
- (iii) $\neg ((A \rightarrow B) \vee (B \rightarrow A))$