

**PY1003: Introduction to Logic**  
**Example Class I**

**Sentential Logic Revisited**

(1) Translate the following sentences into the language of sentential logic. Remember to provide a key for your translation of the atomic sentences.

- (i) If the president is shot, then there will be a coup and the government will be overthrown.
- (ii) The president will be shot unless he bribes the assassin and he won't do that unless he has a million pesos.
- (iii) If the president is shot there will be riots, but the government won't be overthrown.
- (iv) Provided that you attend the example classes and the lectures, you'll pass the exam.
- (v) If you neither study hard nor attend the lectures, you won't pass the exam unless you bribe the examiner.
- (vi) If enough people show up, we will take a vote. Otherwise, we'll reschedule the meeting.
- (vii) I won't pass the exam if I don't study, and even if I study I'll probably fail.
- (viii) You can vote if and only if you are a British citizen over the age of eighteen.

(2) Translate the following arguments into the language of sentential logic. Use truth-tables to determine their validity. Where an argument is invalid, exhibit a counterexample.

- (i) If the points are damp, no spark will be generated. If no spark is generated, the engine won't start. Therefore the engine will start only if the points are dry.
- (ii) If I am snowed in, I will miss classes. If I miss classes, the course will be behind schedule. Therefore, the course will stay on schedule only if I am not snowed in.
- (iii) If Min is at home, then Henry certainly isn't. If Min is at home, then so is Henry. So Min is not at home.
- (iv) Jackson will be convicted if he is guilty or if the jury is biased. The jury won't be biased. So if Jackson is guilty, then he won't be convicted.
- (v) If the government rigs the election, there will be riots. However, the government will play fair only if it is guaranteed victory, and it isn't guaranteed victory unless it rigs the election. So there will be riots.

(vi) I'll fail the exam whether or not I study. So I'm going to fail.

(3) Which of the following formulae are tautologies? Which are contradictions? (Use truth-tables to answer these questions.)

(i)  $A \vee \neg(A \wedge C)$

(ii)  $A \vee (\neg A \wedge C)$

(iii)  $(A \vee B) \leftrightarrow \neg(\neg A \rightarrow B)$

(iv)  $A \wedge (\neg A \vee B)$

(v)  $A \wedge \neg(\neg A \vee B)$

(vi)  $A \leftrightarrow \neg A$

(vii)  $((A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C)))$

(4) Many of the logical constants are interdefinable. For instance,  $A \leftrightarrow B$  is logically equivalent to  $(A \rightarrow B) \wedge (B \rightarrow A)$ . In fact, any one of the following pairs of logical constants can be used to define all of the others:

$\{\neg, \rightarrow\}$

$\{\vee, \neg\}$

$\{\wedge, \neg\}$

Using the first pair, attempt to define all the remaining logical constants. The formula that you come up with (the *definiens*) should be logically equivalent to the formula you are trying to define (the *definiendum*.) Show that this is so in each case, using a truth-table.