PY1003 Introduction to Logic
Lecture 2: Sentential Logic Revisited I

Atomic sentences and compound sentences

Consider the following sentence:

If Smith is convicted then either he is guilty or the jury is biased.

This sentence is built up from three smaller sentences:

Smith is convicted
Smith is guilty
The jury is biased

Some terminology

An atomic sentence: a basic sentence, not built up out of other sentences
A compound sentence: a sentence built up out of one or more smaller sentences
A (sentential) connective [note the small 's']: A phrase which is used to 'connect' sentences to create a longer sentence, e.g.:
Examples of English connectives:

If…. then
… and …
It is not the case that….
…. or …

Sentence letters

Ross is in his office or Ross is in the library
How do we translate this sentence into the language of Sentential logic? We begin by deciding on a capital letter to stand in for each of the atomic sentences:

A: Ross is in his office
B: Ross is in the library

These letters are called sentence letters.
So at this first stage the target sentence becomes:

A or B

NB: Sometimes caution is required when picking out sentence letters to begin a translation.

1. Ross is in his office or Ross is in the library.
2. Ross isn't in his office.
3. So he's in the library.

How many sentence letters are needed to translate this argument?

A: Ross is in his office
B: Ross is in the library
??? C: He's in the library

Sentential connectives

The next stage is to replace the English-language connectives with the symbols that we use to represent them in Sentential Logic:

It is not the case that A	$\neg A$	(negation)
A or B	$A \vee B$	(disjunction)
A and B	$A \wedge B$	(conjunction)
If A then B	$A \rightarrow B$	(conditional)
A if and only if B	$A \leftrightarrow B$	(biconditional)

The Sentential Logic connectives are also often called *logical constants*. This is because, unlike sentence letters which can be used to stand for various different sentences, the connectives always mean the same thing.

More terminology

All the connectives we'll be dealing with – all the ones that can be translated using Sentential Logic connectives – are *truth-functional*. This means you can work out the truth-value (true or false) of a compound sentence from the truth-values of the atomic sentences (or *atoms*). The truth-value of the compound is a function of the truth-value of the atoms.

E.g. You only need to know the truth-value of A and the truth-value of B in order to work out the truth-value of $A \wedge B$.

Some connectives are not truth-functional, e.g.:

Michael believes that …

You need to know more than the truth-value of *It's raining* to work out the truth-value of *Michael believes that it's raining*.

Sentences in a logical language are often referred to as *formulae*. *Atomic formulae* are just atomic sentences; similarly for *compound formulae*.

Sentential connectives have a fixed number of slots for smaller sentences. Most of the Sentential connectives are *binary* or two-slot connectives:

$A \wedge B$

$A \vee B$

$A \rightarrow B$

$A \leftrightarrow B$

Negation is the only *unary* (one-slot) connective in Sentential:

$\neg A$

Brackets and scope

Consider how we translate this sentence:

(1) I will either stay in and study or else go for a curry.

A: I will stay in
B: I will study
C: I will go for a curry

(2) $A \wedge B \vee C$

Two readings of (2):
(a) I will either stay in and study, or else I will go for a curry.
(b) I will stay in, and I will either study or go for a curry.

In the language of Sentential we distinguish between these two readings by using brackets:
(2a) $(A \wedge B) \vee C$
(2b) $A \wedge (B \vee C)$

Compare the two formulae:
1. $(A \wedge B) \vee C$
2. $A \wedge (B \vee C)$
The binary connective $\wedge$ is used differently in the two cases. We call this a difference in scope. The scope of a connective is a matter of how much of the sentence it operates on, or *governs*. In 1, the $\wedge$ only governs a small chunk of the sentence – the $A \wedge B$. In 2, it governs the whole sentence.
In 1 the $\wedge$ has narrow scope and in 2 it has wide scope. In 1, it is the v which has wide scope. In 2, the v has narrow scope.

A rule to help with determining scope:

> The scope of a binary connective in a sentence is that part of the sentence enclosed in the closest pair of matching brackets within which the connective lies, if there is such a pair, or else the entire formula, if there is not.

(Remember that this only applies to binary connectives, not to negation.)

The connective with the widest scope in a sentence is called the sentence's *main connective*.

We will use the convention whereby no brackets are needed to indicate the scope of the main connective, when that connective is binary:

$(A \wedge B) \vee C$

Some people prefer to use one pair of brackets for each binary connective:

$((A \wedge B) \vee C)$

In effect, we do the same but leave out the brackets that would come with the main connective.
(NB: See below for what happens when the main connective is our unary connective, negation)

Some examples:
$(B \leftrightarrow C) \rightarrow A$
$A \lor (B \lor (C \land D))$

If you do not indicate scope properly, you will not end up with well-formed (properly grammatical) formulae.
Some examples of formulae which are ill-formed due to scope ambiguity:
$B \leftrightarrow C \rightarrow A$
$A \lor (B \land C \land D)$

Some other examples of ill-formed formulae (with reasons):
$(B \land C) \rightarrow d$ (There are no lower-case letters in the language of Sentential Logic)

$A \neg B$	($\neg$ is a unary connective)
$\rightarrow B$	($\rightarrow$ is a binary connective)
$A \land \lor B$	(too many connectives)

<u>Scope and negation</u>
Consider the sentence
(3) Apples aren't very nice, and tangerines are.
A: Apples are very nice
B: Tangerines are very nice
(4) $\neg A \land B$
Is this sentence ambiguous?
(4a) $(\neg A) \land B$
(4b) $\neg (A \land B)$
But in fact there is no ambiguity. There is a convention that '$\neg$' always takes the narrowest possible scope.
Some examples:
$\neg A \rightarrow B$
$\neg (A \rightarrow B)$
$A \rightarrow (A \land \neg (B \lor C))$
$A \rightarrow (A \land (\neg B \lor C))$

Translating a whole argument

We use the turnstile symbol, $\vdash$, to symbolise the move from premises to conclusion.

Example:
Ross is in his office or Ross is in the library
Ross isn't in the library
Therefore Ross is in his office

A v B, ¬A ├ B

This way of presenting an argument is sometimes called *sequent form.*

The complete language of Sentential Logic consists of:
1. The capital letters A, B, C ... etc.
2. The logical constants: ¬, v, ∧, →, ↔
3. The turnstile: ├
4. Matching pairs of brackets: (…)

If you want to read more, please try:
Colin Howson, *Logic with Trees*, ch. 1, section 2
Graeme Forbes, *Modern Logic*, ch. 2, sections 1-4

An important fact:
By the definition of validity, any argument which is such that its premises cannot all be true is valid. Because if it can't be that its premises are all true, it can't be that it's premises are all true and its conclusion is false. Which means it is valid.