

PY1003 Introduction to Logic
Lecture 1

1. Arguments

An argument consists of one or more **premises** and a **conclusion**. You should already be practiced at spotting the premises and conclusions of arguments.

The premises of an argument are usually intended to 'support' the conclusion in some sense. However, what kind of support they are supposed to give depends on the kind of argument.

Some notions of 'support':

- If the premises are true the conclusion is *probably* true
 e.g. The pavement is wet. Therefore it's been raining.

Here the premise makes it likely that the conclusion is true, but the premise could conceivably be true even if the conclusion is false (e.g. if someone has poured water on the pavement).

- If the premises are true the conclusion *must* be true
 e.g. All cats are mammals and Mu is a cat. Therefore Mu is a mammal.

Here there is *no* possibility of the conclusion being false if the premises are true.

2. Validity

An argument is **valid** if and only if it is not possible for the premises all to be true and the conclusion false.

An argument is **invalid** if and only if it is not valid, i.e. if and only if it is possible for the premises all to be true and the conclusion false.

Here is one familiar kind of invalid argument:

1. If it is raining in St Andrews, it is raining in Leuchars
2. It is raining in Leuchars
3. Therefore, it is raining in St Andrews.

Remember that an argument can be valid even if its premises are false, and invalid even if its premises are all true.

A valid argument with all-true premises is called a **sound** argument.

3. Logic

With simple arguments, a bit of common sense enables us to distinguish the valid arguments above from the invalid ones. But sometimes it is a lot more difficult.

Consider, for instance, the following argument:

If we stay up till three at the party it'll be fun, even though there won't be more than twenty people there because the flat's not big enough. If you come to the party then if you don't get tired and no-one complains about the noise we can stay up till three. But if you are going to get tired you won't come to the party in the first place. Someone will complain about the noise if there are more than twenty people present. So the party will be fun.

Is this argument or invalid? At least at first glance, it is pretty difficult to tell.

Logic is the study of systematic and reliable methods for distinguishing between valid and invalid arguments.

How do we use logic here? First we determine an argument's **logical form**, or shape, and then we check whether this form is a valid form or not.

Logical form is what the following arguments have in common:

1. The sky is pink or the sky is green
 2. It is not the case that the sky is pink
 3. Therefore, the sky is green
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1. Either I'm tired or this lecture's too long
 2. But I'm not tired
 3. Therefore, this lecture's too long.
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1. Quarks have flavour or quarks have spin.
 2. It is not the case that quarks have flavour
 3. Therefore, quarks have spin.

Here's a simple way of representing the logical form these arguments share:

- 1*. A or B
- 2*. It is not the case that A
- 3*. Therefore, B

And here's the same form represented in the language of Sentential Logic (also known as Propositional Logic):

$$\frac{A \vee B \quad \neg A}{B}$$

How can we check that arguments of a certain form are valid?

Well, consider the above form. If the first premise is true, then either A or B must be true. If the second premise is true, then A is not true. So if both premises are true, one of A or B is true and it is not A, so it's B. This means that in any possible situation in which the two premises are both true, B must be true, which is just what the conclusion says. So the argument is valid: whenever the premises are true, the conclusion must be true too.

We can represent all this information using a truth table (a device that should be somewhat familiar to you, but we will be revising truth tables over the next few lectures):

A	B	$A \vee B$	$\neg A$	B
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

This table (particularly line 3) shows that there's no way for all the premises to be true while the conclusion is false.

Here are the Sentential *connectives* that we'll be using in this course:

It is not the case that A	$\neg A$	negation
A and B	$A \wedge B$	conjunction
A or B	$A \vee B$	disjunction
If A then B	$A \rightarrow B$	conditional
A if and only if B	$A \leftrightarrow B$	biconditional

Further reading for this week:

Colin Howson, *Logic with Trees*, pp. 3-5
 Graeme Forbes, *Modern Logic*, pp.3-11