

Spin current and rectification in one-dimensional electronic systems

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Spin and charge currents can be generated by an ac voltage through a one-channel quantum wire with strong electron interactions in a static uniform magnetic field. In a certain range of low voltages, the spin current can grow as a negative power of the voltage bias as the voltage decreases. The spin current expressed in units of $\hbar/2$ per second can become much larger than the charge current in units of the electron charge per second. The system requires neither spin-polarized particle injection nor time-dependent magnetic fields.

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The pioneering paper by Christen and Büttiker [1] has stimulated much interest to rectification in quantum wires and other mesoscopic systems. Most attention was focused on the simplest case of Fermi liquids [2, 3]. Recently this research was extended to strongly interacting systems where Luttinger liquids are formed [4–7]. One of the topics of current interest is the rectification effect in mesoscopic conductors in a magnetic field [3, 6, 7]. In the presence of a magnetic field, both spin and charge currents can be generated. However, only charge currents have been studied so far. In this paper we consider the generation of a dc spin current by an ac voltage bias in a one-channel quantum wire.

In recent years many approaches to the generation of spin currents in quantum wires were put forward. Typically both, spin and charge currents are generated and the spin current expressed in units of $\hbar/2$ per second is smaller than the electric current in units of e per second (e is the electron charge). Such a situation naturally emerges in partially polarized systems since each electron carries the charge e and its spin projection on the z -axis is $\pm\hbar/2$. A proposal how to obtain a spin current exceeding the charge current in a quantum wire was published by Sharma and Chamon [8, 9] who considered a Luttinger liquid in the presence of a time-dependent magnetic field in a region of the size of an electron wavelength. We show that the generation of a dc spin current exceeding the charge current is also possible without time-dependent magnetic fields on the nanoscale. The spin current can be generated in a spatially asymmetric system in the presence of an ac bias. Interestingly, in a certain interval of low voltages the dc spin current grows as a negative power of the ac voltage when the voltage decreases.

The rectifying quantum wire is sketched in Fig. 1. It consists of a one-dimensional conductor with a scatterer in the center of the system at $x = 0$. The scatterer creates an asymmetric potential $U(x) \neq U(-x)$. The size of the scatterer $a_U \sim 1/k_F$ is of the order of the electron wavelength. The wire is placed in a uniform magnetic field \mathbf{H} . The field defines the S_z direction of the electron spins.

At its two ends, the wire is connected to nonmagnetic electrodes, labeled by $i = 1, 2$. The left electrode, $i = 1$, is controlled by an ac voltage source, while the right electrode, $i = 2$, is kept on ground. The magnetic field \mathbf{H} breaks the symmetry between the two orientations of the electron spin. In a uniform wire this would not result in a net spin current since the conductances of the spin-up and -down channels would be the same, e^2/h [11], and the spin currents of the spin-up and -down electrons would be opposite. In the presence of a potential barrier such cancellation does not occur [12]. In a system with strong electron interaction, the spatial asymmetry of the wire leads to an asymmetric $I - V$ curve, $I(V) \neq I(-V)$ [4]. Thus, an ac voltage bias generates spin and charge dc currents, I_s^r and I_c^r . In this paper we focus on the low-frequency ac bias. We define the rectification current as the dc response to a low-frequency square voltage wave of amplitude V : $I_s^r(V) = [I_s(V) + I_s(-V)]/2$ and $I_c^r(V) = [I_c(V) + I_c(-V)]/2$. The above dc currents express via the currents of spin-up and -down electrons: $I_c^r = I_{\uparrow}^r + I_{\downarrow}^r$, $I_s^r = (\hbar/2e)[I_{\uparrow}^r - I_{\downarrow}^r]$. The spin current exceeds the charge current if the signs of I_{\uparrow}^r and I_{\downarrow}^r are opposite.

The calculation of the rectification currents reduces to the calculation of the contributions even in the voltage V to the dc $I - V$ curves $I_s(V)$ and $I_c(V)$. We assume that the Coulomb interaction between distant charges is screened by the gates. This will allow us to use the standard Tomonaga-Luttinger model with short range interactions [13]. Electric fields of external charges are also assumed to be screened. Thus, the applied voltage reveals itself only as the difference of the electrochemical potentials E_1 and E_2 of the particles injected from the left and right reservoirs. We assume that one lead is connected to the ground so that its electrochemical potential $E_2 = E_F$ is fixed. The electrochemical potential of the second lead $E_1 = E_F + eV$ is controlled by the voltage source. Since the Tomonaga-Luttinger model captures only low-energy physics, we assume that $eV \ll E_F$, where E_F is of the order of the bandwidth.

Rectification occurs due to backscattering off the asymmetric potential $U(x)$. We will assume that the asymmetric potential is weak, $U(x) \ll E_F$. This will enable us to use perturbation theory. As shown in Ref. [4], the leading contribution to the rectification effect emerges in the third order in $U(x)$. This fact allows for a simple explanation of our main prediction that the spin rectification current can significantly exceed the charge current. To illustrate the principle, let us consider the following toy problem: There is no uniform magnetic field \mathbf{H} and no asymmetric potential $U(x)$. Instead, both right \leftrightarrow left and spin-up \leftrightarrow spin-down symmetries are broken by a weak coordinate-dependent magnetic field $B_z(x) \neq B_z(-x)$, which is localized in a small region of size $\sim 1/k_F$ (we do not include the components $B_{x,y}$ in the toy model). Let us also assume that the spin-up and -down electrons do not interact with the electrons of the opposite spin. Then the system can be described as the combination of two spin-polarized one-channel wires with opposite spin-dependent potentials $\pm\mu B_z(x)$, where μ is the electron magnetic moment. According to Ref. [4] an ac bias generates a rectification current in each of those two systems and the currents are proportional to the cubes of the potentials $(\pm\mu B_z)^3$. Thus, $I_\uparrow^r = -I_\downarrow^r$. Hence, no net charge current $I^r = I_\uparrow^r + I_\downarrow^r$ is generated in the leading order. At the same time, there is a nonzero spin current in the third order in B_z .

Let us now calculate the currents in the presence of the asymmetric potential $U(x)$ and the field \mathbf{H} . We assume that the magnetic field couples only to the electron spin and neglect the correction $-e\mathbf{A}/c$ to the momentum in the electron kinetic energy. Indeed, for a uniform field one can choose $\mathbf{A} \sim y$, where the y -axis is orthogonal to the wire, and y is small inside a narrow wire. As shown in Ref. [12], such a system allows a formulation within the bosonization language and, in the absence of the asymmetric potential, can be described by a quadratic bosonic Hamiltonian

$$H_0 = \sum_{\nu,\nu'=L,R} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int dx (\partial_x \phi_{\nu\sigma}) \mathcal{H}_{\nu\sigma,\nu'\sigma'} (\partial_x \phi_{\nu'\sigma'}), \quad (1)$$

where σ is the spin projection and $\nu = R, L$ labels the left and right moving electrons, which are related to the boson fields $\phi_{\nu\sigma}$ as $\psi_{\nu\sigma}^\dagger(x) \sim \eta_{\nu\sigma}^\dagger e^{\pm i(k_F \nu \sigma x + \phi_{\nu\sigma}(x))}$ with \pm for $\nu = R, L$. The operators $\eta_{\nu\sigma}^\dagger$ are the Klein factors adding a particle of type (ν, σ) to the system, and $k_F \nu \sigma / \pi$ is the density of (ν, σ) particles in the system. The densities of the spin-up and -down electrons are different since the system is polarized by the external magnetic field. The 4×4 matrix \mathcal{H} describes the electron-electron interactions. In the absence of spin-orbit interactions, $L \leftrightarrow R$ parity is conserved and we can introduce the quantities $\phi_\sigma = \phi_{L\sigma} + \phi_{R\sigma}$ and $\Pi_\sigma = \phi_{L\sigma} - \phi_{R\sigma}$ such that the Hamiltonian decouples into two terms depending on ϕ_σ and Π_σ only. In the absence of the external field, this Hamilto-

nian would further be diagonalized by the combinations $\phi_{c,s} \propto \phi_\uparrow \pm \phi_\downarrow$, and similarly for $\Pi_{c,s}$, expressing the spin and charge separation. This is here no longer the case. If we focus on the ϕ fields only (as Π will not appear in the operators describing backscattering off $U(x)$), the fields diagonalizing the Hamiltonian, $\tilde{\phi}_{c,s}$, have a more complicated linear relation to $\phi_{\uparrow,\downarrow}$, which we can write as

$$\begin{pmatrix} \phi_\uparrow \\ \phi_\downarrow \end{pmatrix} = \begin{pmatrix} \sqrt{g_c}[1+\alpha] & \sqrt{g_s}[1+\beta] \\ \sqrt{g_c}[1-\alpha] & -\sqrt{g_s}[1-\beta] \end{pmatrix} \begin{pmatrix} \tilde{\phi}_c \\ \tilde{\phi}_s \end{pmatrix}, \quad (2)$$

and which corresponds to the matrix A of [12]. The normalization has been chosen such that the propagator of the $\tilde{\phi}$ fields with respect to the Hamiltonian (1) evaluates to $\langle \tilde{\phi}_{c,s}(t_1) \tilde{\phi}_{c,s}(t_2) \rangle = -2 \ln(i(t_1 - t_2) + \delta)$, where $\delta > 0$ is an infinitesimal quantity. For non-interacting electrons without a magnetic field, $g_c = g_s = 1/2$. $g_c < 1/2$ ($> 1/2$) for repulsive (attractive) interactions. In the absence of the magnetic field, terms in (1) in the form of $\exp(\pm 2i\sqrt{g_s}\phi_s)$ may become relevant and open a spin gap for $g_s < 1/2$. In our model they can be neglected since they are suppressed by the rapidly oscillating factors $\exp(\pm 2i[k_{F\uparrow} - k_{F\downarrow}]x)$. It is convenient to model the leads as the regions near the right and left ends of the wire without electron interaction [11].

Backscattering off the impurity potential $U(x)$ is described by the following contribution to the Hamiltonian $H = H_0 + H'$ [13]:

$$H' = \sum_{n_\uparrow, n_\downarrow} U(n_\uparrow, n_\downarrow) e^{in_\uparrow \phi_\uparrow(0) + in_\downarrow \phi_\downarrow(0)}, \quad (3)$$

where the fields are evaluated at position $x = 0$ and $U(n_\uparrow, n_\downarrow) = U^*(-n_\uparrow, -n_\downarrow)$ since the Hamiltonian is Hermitian. The fields Π do not enter the above equation due to the conservation of the electric charge and the z -projection of the spin. The Klein factors are not written because they drop out in the perturbative expansion. $U(n_\uparrow, n_\downarrow)$ are the amplitudes of backscattering of n_\uparrow spin-up and n_\downarrow spin-down particles with $n_\sigma > 0$ for $L \rightarrow R$ and $n_\sigma < 0$ for $R \rightarrow L$ scattering. $U(n_\uparrow, n_\downarrow)$ can be estimated as $U(n_\uparrow, n_\downarrow) \sim \int dx U(x) e^{in_\uparrow 2k_{F\uparrow} x + in_\downarrow 2k_{F\downarrow} x} \sim U/k_F$, where U is the maximum of $U(x)$ (cf. [13]). In the case of a symmetric potential $U(x) = U(-x)$ the coefficients $U(n_\uparrow, n_\downarrow)$ are real.

The spin and charge current can be expressed as

$$I_{s,c} = L_{s,c}^1 + R_{s,c}^1 = L_{s,c}^2 + R_{s,c}^2, \quad (4)$$

where $L_{s,c}^i$ and $R_{s,c}^i$ denote the current of the left- and right-movers near electrode i , respectively. For a clean system ($U(x) = 0$), the currents $R_{s,c}^1 = R_{s,c}^2$, $L_{s,c}^1 = L_{s,c}^2$ and $I_c = 2e^2 V/h$, $I_s = 0$ [11]. With backscattering off $U(x)$, particles are transferred between L and R in the wire, and hence $R_s^2 = R_s^1 + dS_R/dt$, $R_c^2 = R_c^1 + dQ_R/dt$, where Q_R and S_R denote the total charge and

the z -projection of the spin of the right-moving electrons [14]. The currents $L_{s,c}^2$ and $R_{s,c}^1$ are determined by the leads (i.e. the regions without electron interaction in our model [11]) and remain the same as in the absence of the asymmetric potential. Thus, the spin and charge current can be represented as $I_c = 2e^2V/h + I_c^{bs}$ and $I_s = I_s^{bs}$, where the backscattering current operators are [4, 5, 13]

$$\begin{aligned} \hat{I}_c^{bs} &= d\hat{Q}_R/dt = ie[H, \hat{Q}_R]/\hbar \\ &= \frac{-ie}{\hbar} \sum_{n_\uparrow, n_\downarrow} (n_\uparrow + n_\downarrow) U(n_\uparrow, n_\downarrow) e^{in_\uparrow\phi_\uparrow(0) + in_\downarrow\phi_\downarrow(0)}, \end{aligned} \quad (5)$$

$$\hat{I}_s^{bs} = d\hat{S}_R/dt = -\frac{i}{2} \sum_{n_\uparrow, n_\downarrow} (n_\uparrow - n_\downarrow) U(n_\uparrow, n_\downarrow) e^{in_\uparrow\phi_\uparrow + in_\downarrow\phi_\downarrow}. \quad (6)$$

The calculation of the rectification currents reduces to the calculation of the currents (5), (6) at two opposite values of the dc voltage.

To find the backscattered current we use the Keldysh technique [15]. We assume that at $t = -\infty$ there is no backscattering in the Hamiltonian ($U(x) = 0$), and then the backscattering is gradually turned on. Thus, at $t = -\infty$, the numbers N_L and N_R of the left- and right-moving electrons conserve separately: The system can be described by a partition function with two chemical potentials $E_1 = E_F + eV$ and $E_2 = E_F$ conjugated with the particle numbers N_R and N_L . This initial state determines the bare Keldysh Green functions.

We will consider only the zero temperature limit. It is convenient to switch [14] to the interaction representation $H_0 \rightarrow H_0 - E_1 N_R - E_2 N_L$. This transformation induces a time dependence in the electron creation and annihilation operators. As the result each exponent in Eq. (3) is multiplied by $\exp(ieVt[n_\uparrow + n_\downarrow]/\hbar)$.

In the Keldysh formulation [15] the backscattering currents (5), (6) are evaluated as

$$I_{c,s}^{bs} = \langle 0 | \mathcal{S}(-\infty, 0) \hat{I}_{c,s}^{bs} \mathcal{S}(0, -\infty) | 0 \rangle, \quad (7)$$

where $|0\rangle$ is the ground state for the Hamiltonian H_0 , Eq. (1), and $\mathcal{S}(t, t')$ the evolution operator for H' from t' to t in the interaction representation with respect to H_0 . The result of this calculation depends on the elements of the matrix (2), which describe the low-energy degrees of freedom and depend on the microscopic details. Several regimes are possible [5] at different values of the parameters $g_s > 0$, $g_c > 0$, α and β . In this paper we focus on one particular regime:

$$1 \gg g_c > g_s \gg g_c - g_s \gg \alpha, \beta. \quad (8)$$

We will see that in such a regime the spin current can be much greater than the charge current. We will also assume that $[g_c\alpha + g_s\beta] > 0$ but our results do not depend significantly on this assumption. The behavior which we

find in the regime (8) persists for g_c, g_s, α and β smaller than some constants of the order of unity. In this paper we focus on the limit in which the current can be calculated analytically. A numerical investigation of the boundary of the region where the spin current exceeds the charge current is beyond the scope of the present paper.

The currents (7) can be estimated using a renormalization group procedure [13]. As we change the energy scale E , the backscattering amplitudes $U(n_\uparrow, n_\downarrow)$ scale as $U(n, m; E) \sim E^{z(n,m)}$, where the scaling dimensions are

$$z(n, m) = n^2[g_c(1 + \alpha)^2 + g_s(1 + \beta)^2] + m^2[g_c(1 - \alpha)^2 + g_s(1 - \beta)^2] + 2nm[g_c(1 - \alpha^2) - g_s(1 - \beta^2)] - 1. \quad (9)$$

The renormalization group stops on the scale of the order $E \sim eV$. At this scale the backscattering current can be represented as $I_{c,s}^{bs} = Vr_{c,s}(V)$, where the effective reflection coefficient $r_{c,s}(V)$ [4, 13] is given by the sum of contributions of the form $(\text{const})U(n_1, m_1)U(n_2, m_2) \dots U(n_p, m_p)V^{\sum_{k=1}^p z(n_k, m_k)}$. Such a perturbative expansion can be used as long as U is small. In the regime (8) it is sufficient to require that $U(n, m) < (eV)^{1+\delta}/E_F^\delta$, where $\delta > g_c$. The contributions to the backscattering current of the n th order in U scale as $I_{c,s}^{(n)} \sim U^n V^{1-n+O(g_c)}$. Hence, they exceed the contributions of the $(n+1)$ th order for $U(n, m) < (eV)^{1+\delta}/E_F^\delta$. The leading non-zero contribution to the backscattered current, therefore, emerges in the second order. The *rectification current*, however, is dominated by a third order contribution for $V^{1+\delta} > U \gg V^2/E_F$.

Indeed, the second order contributions to the charge current were computed in Ref. [13]. The spin current can be found in exactly the same way. The result is

$$I_{c,s}^{(2)bs}(V) \sim \sum (\text{const}) |U(n, m)|^2 |V|^{2z(n,m)+1} \text{sign} V. \quad (10)$$

If the $U(n, m)$ were independent of the voltage, the above current would be an odd function of the bias and hence would not contribute to the rectification current. The backscattering amplitudes depend [13] on the charge densities $k_{F\nu\sigma}$ though, which in turn depend on the voltage in our model [4]. The voltage-dependent corrections to the amplitudes are linear in the voltage at low bias. Hence, the second order contributions to the rectification currents scale as $U^2|V|^{2z(n,m)+2}$. The additional factor of V makes the second order contribution smaller than the leading third order contribution (13) at $U \gg V^2/E_F$. Note that the second order contribution to the rectification current is nonzero even for a symmetric potential $U(x)$. The leading third order contribution emerges solely due to the asymmetry of the scatterer.

It is easy to find the most relevant backscattering operators in the renormalization group sense using Eqs. (8) and (9). The most relevant operator is $U(1, 0)$, the second most relevant $U(0, -1)$, and the third most relevant

$U(-1, 1)$. The leading non-zero third order contributions to the spin and charge currents come from the product of the above three operators in the Keldysh perturbation theory. This leads to

$$I_{c,s}^{bs} \sim U^3 V^{g_c(2+6\alpha^2)+g_s(6+2\beta^2)-2}. \quad (11)$$

This contribution dominates at $(eV)^2/E_F \ll U \ll (eV)$. Interestingly, the current (11) grows as the voltage decreases.

Does the current (11) contribute to the rectification current? In general, (11) is the sum of odd and even functions of the voltage and only the even part is important for us. One might naively expect that such a contribution has the same order of magnitude for the spin and charge currents. A direct calculation shows, however, that this is not the case and the spin rectification current is much greater than the charge rectification current.

In order to calculate the prefactors in the right hand side of Eq. (11) one has to employ the Keldysh formalism. The third order Keldysh contribution reduces to the integral of $P(t_1, t_2, t_3) = \langle T_c \exp(i\phi_\uparrow(t_1) + ieVt_1/\hbar) \exp(-i\phi_\downarrow(t_2) - ieVt_2/\hbar) \exp(i[-\phi_\uparrow(t_3) + \phi_\downarrow(t_3)]) \rangle$ over $(t_1 - t_3)$ and $(t_2 - t_3)$, where T_c denotes time ordering along the Keldysh contour $-\infty \rightarrow 0 \rightarrow -\infty$ and the angular brackets denote the average with respect to the ground state of the non-interacting Hamiltonian (1). For the purpose of comparing the prefactors in Eq. (11) for I_c^{bs} and I_s^{bs} in the region of parameters (8), it is sufficient to perform the calculation in the limit $1 \gg g_c = g_s$, $\alpha = \beta = 0$, which corresponds to the boundary of the region (8). A small change of the parameters g_c, g_s, α and β will not significantly affect the prefactors (as well as the exponent in Eq. (11)). In the above limit the calculation considerably simplifies and can be performed analytically. The correlation function $P(t_1, t_2, t_3)$ factorizes into the product of a function of $(t_1 - t_3)$ and a function of $(t_2 - t_3)$. One finds

$$I_c^{bs} = -\frac{8e\tau_c^2}{\hbar^3} \text{sign}(eV) \left| \frac{eV\tau_c}{\hbar} \right|^{8g-2} \Gamma^2(1-4g) \sin(4\pi g) \\ \times [1 - \cos(4\pi g)] \text{Re}[U(1,0)U(-1,1)U(0,-1)], \quad (12)$$

$$I_s^{bs} = -\frac{4\tau_c^2}{\hbar^2} \left| \frac{eV\tau_c}{\hbar} \right|^{8g-2} \Gamma^2(1-4g) \\ \times \sin^2(4\pi g) \text{Im}[U(1,0)U(-1,1)U(0,-1)], \quad (13)$$

where $g = g_c = g_s$ and $\tau_c \sim \hbar/E_F$ is the ultraviolet cutoff time. The above results apply at $g < 1/4$, i.e., when the Keldysh integral converges. The charge current (12) is an odd function of the voltage and hence does not contribute to the rectification effect. The spin current (13) is an even function and hence determines the spin rectification current. It is non-zero if $\text{Im}[U(1,0)U(-1,1)U(0,-1)] \neq 0$,

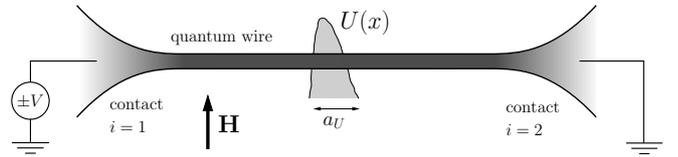


FIG. 1: Sketch of the one-dimensional conductor connected to two electrodes on both ends. Currents are driven through a voltage bias V that is applied on the left electrode while the right electrode is kept on ground. The system is magnetized by the field \mathbf{H} . Electrons are backscattered off the asymmetric potential $U(x)$. $U(x) \neq 0$ in the region of size $a_U \sim 1/k_F$.

which is satisfied for asymmetric potentials. Thus, we expect that in the region (8), the spin rectification current exceeds the charge rectification current in an appropriate interval of low voltages $\sqrt{U E_F} \gg eV \gg U$.

The voltage dependence of the spin rectification current is illustrated in Fig. 2. The expression (11) describes the current in the voltage interval $V^{**} > V > V^*$. In this interval the current *increases* as the voltage decreases. At lower voltages the perturbation theory breaks down. The current must decrease as the voltage decreases below V^* and eventually reach 0 at $V = 0$. At higher voltages, $E_F \gg eV \gg eV^{**}$, the second order rectification current (10) dominates. The leading second order contribution $I_s^r \sim |U(1,0)|^2 V^{2z(1,0)+2}$ grows as the voltage increases. The charge rectification current has the same order of magnitude as the spin current. The Tomonaga-Luttinger model cannot be used for the highest voltage region $eV \sim E_F$.

In conclusion, we have shown that rectification in magnetized quantum wires can lead to a spin current that largely exceeds the charge current. The effect is solely due to the properties of the wire and does not require spin polarized injection as from magnetic electrodes. The currents are driven by the voltage source only. In an interval of low voltages the spin current grows as the voltage decreases.

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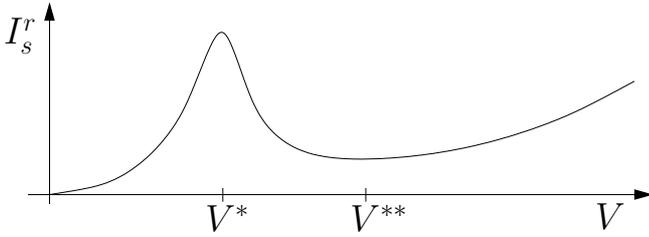


FIG. 2: Qualitative representation of the spin rectification current. The spin current exceeds the charge current and follows a power-law dependence on the voltage with a negative exponent in the interval of voltages $V^* < V < V^{**}$.

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